
UNIT 13 MAGNETIC PROPERTIES

Structure

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13.1 INTRODUCTION

So far in this course you have learnt various characteristics of solids starting from their crystal structure and properties up to superconductivity. In this unit, you will learn about another important property of solids viz. **magnetism**. In the past few decades, we have witnessed several processes and properties of magnetic materials which find useful applications. Production of ultra-low temperatures by adiabatic demagnetisation of a paramagnetic salt, magnetic levitation trains using superconductors, high density memory devices for computer applications are some of the recent developments. (You may be aware of the conventional applications of magnetic materials in chokes, transformer cores, electromagnets etc.) These advances in technology became possible when we could make necessary modifications in the structure of materials at the microscopic level for desired applications.

You may recall that a study of magnetic properties helps us in understanding the constitution of matter and interactions involved amongst its atoms/molecules. In this unit you will learn how modern quantum mechanical ideas enable us to explain the properties of magnetic materials. From Block 3 of PHE-07 course on Electric and Magnetic Phenomena, you may recall that the macroscopic properties of magnetic materials can be explained in terms of the motion of electrons inside an atom, and the interactions amongst electrons. Working within the framework of classical physics, you learnt that materials can be classified as diamagnetic, paramagnetic or ferromagnetic depending on the sign and magnitude of the magnetic susceptibility. Though classical ideas have provided a lot of useful information about magnetic properties of materials, we now realize that this information is not complete; magnetism is a quantum mechanical phenomenon and should be treated as such.

In Sec.13.2, you will revisit the classical theories of magnetism. In Sec.13.3 you will learn the basics of diamagnetism, which is a universal phenomenon. In Sec. 13.4 we have discussed quantum mechanical theory of paramagnetism, which stipulates that the magnetic moments of an atom/ion, as determined by a combination of spin and orbital magnetic moments, can have only discrete orientations relative to the applied magnetic field. Sec. 13.5 is devoted to ferromagnetism. In Sec. 13.6 you will learn about some special magnetic materials which have found important technological applications. To be able to appreciate this fascinating area of physics, you are advised to revise units 11 and 12 of PHE-07 course.

Objectives

After studying this unit, you should be able to:

- recall classical explanations of the properties of magnetic materials;
- state the relation between the magnetic dipole moment and angular momentum of a paramagnetic material and apply it to understand the behaviour of such materials at different fields and temperatures;
- distinguish between the paramagnetic behaviour of rare earth metals and transition metals;
- explain the domain structure in ferromagnets;
- describe the processes involved at the microscopic level when a ferromagnetic substance is magnetised; and
- explain the technological use of magnetic materials.

13.2 CLASSICAL THEORIES REVISITED

In this section, we review how classical theories enable us to distinguish between different magnetic materials.

When a magnetic substance is placed in a magnetic field, it acquires magnetism by induction. Its ends, in general, exhibit polarities opposite to the polarities of the applied field. The lines of force starting from the *N*-pole of the magnetising field get concentrated inside the material through the *S*-pole. The degree to which the medium can permeate the lines of force is known as its **permeability**. We denote it by the symbol μ . It is measured in **tesla meters per ampere** (TmA^{-1}). The permeability of a medium is measured as the ratio of the number of magnetic lines of force per unit area in the medium to the number of lines of force per unit area that would be present if the medium is replaced by vacuum.

If μ is (slightly) smaller than μ_0 , the permeability of free space, then we say that the substance is **diamagnetic**. On the other hand, if μ is slightly greater than or equal to μ_0 , the material is said to be **paramagnetic**. If $\mu \gg \mu_0$, the material is said to be **ferromagnetic**. The ratio μ/μ_0 (denoted by μ_r) is called the **relative permeability** of a material.

The permeability can also be defined as the ratio of the magnetic flux density ϕ to the applied magnetising field B_0 . Mathematically, we can write

$$\mu = \frac{\phi}{B_0} \quad (13.1)$$

When a material is placed in a magnetic field, its atoms or molecules acquire magnetic moment. The total magnetic moment acquired per unit volume is termed as **magnetisation**, M . Can you suggest a relation between magnetisation and magnetising field? Well, the two are linearly related:

$$M = \chi B_0, \quad (13.2)$$

where the constant of proportionality χ is called **magnetic susceptibility**. Obviously it is a measure of the ease with which a material takes up magnetisation. Since M and B_0 have the same units, χ is a dimensionless quantity. The resultant magnetic field B in this case becomes

$$B = B_0 + M = B_0 + \chi B_0. \quad (13.3)$$

The values of susceptibility of a few materials at 293 K are given in Table 13.1.

Table 13.1: Magnetic susceptibility of some materials

Materials	χ
Diamagnetic	
Mercury	-3.2×10^{-5}
Silver	-2.6×10^{-5}
Carbon (diamond)	-2.1×10^{-5}
Copper	-1.0×10^{-5}
Paramagnetic	
Magnesium	1.2×10^{-5}
Platinum	26×10^{-5}
Aluminium	2.2×10^{-5}
Oxygen gas	0.19×10^{-5}

Since electrons are charged particles, the orbital motion of electrons in an atom is analogous to a current carrying coil. When magnetic field is applied, the motion of the orbiting electrons gets modified and a weak magnetic moment is induced which opposes the applied field. This gives rise to **diamagnetism** in the material. It is observed that due to diamagnetism, the magnetic lines of force are repelled as shown in Fig. 13.1a. For a material in which induced magnetic moment dominates all other changes, the value of magnetic susceptibility is negative. Physically it means that induced moment opposes the applied field and $\mathbf{B} < \mathbf{B}_0$. From Table 13.1 you can note that the value of susceptibility for diamagnetic materials is very small. However superconductors are an important exception to this. From Unit 12 you will recall that superconducting materials are perfectly diamagnetic and $\chi = -1$.

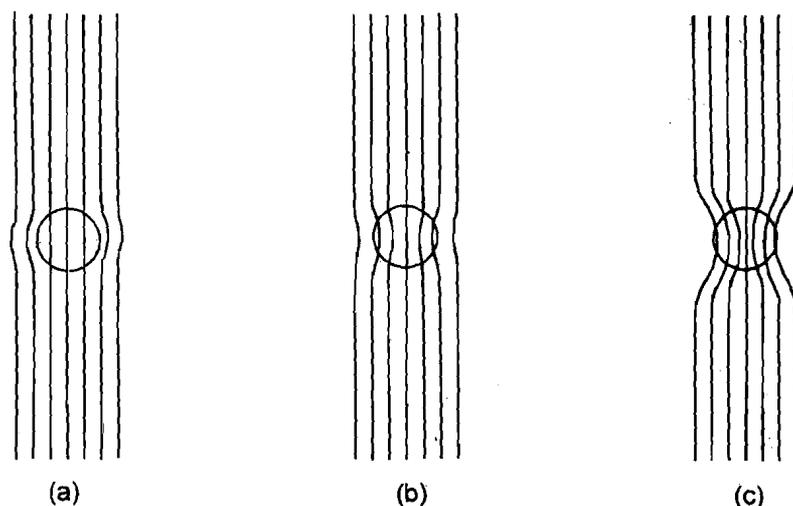


Fig.13.1: Behaviour of various magnetic substances in a magnetic field a) diamagnetic materials; b) paramagnetic materials; and c) ferromagnetic materials

In some materials, atoms and molecules possess intrinsic permanent magnetic moment due to interaction between spin and angular momentum of electrons occupying partially filled electronic shells. Such materials are called **paramagnetic** substances. In such substances, under normal conditions, the magnetic moment vectors are randomly oriented with respect to one another and there is no net magnetic moment. When a paramagnetic material is placed in an external magnetic field, the atoms tend to align their magnetic moments along the applied field. As a result, the intensity of the lines of force along the field direction increases (Fig. 13.1b). In general, the paramagnetic effect is weak and the magnetic susceptibility is small, of the order of 10^{-5} , and positive. If the temperature is increased, thermal effects tend to randomize the alignment of the magnetic moment vectors resulting in a decrease in susceptibility. This is called **Curie's law**.

The intrinsic magnetic moment in some magnetic materials is very large and the magnetic susceptibility is in the range of 10^2 to 10^5 . These materials strongly concentrate the magnetic lines of force when placed in an external magnetic field (Fig. 13.1c). Such materials are called **ferromagnetic** substances. You may note here that in dia- and paramagnetic substances, the effect of induced magnetic moment and alignment of intrinsic dipoles is effective so long as the material is in an external magnetic field. However, ferromagnets do not require any magnetic field to sustain their magnetism.

We hope that with this discussion you understand the basic difference in different types of magnetic materials. Now you will learn these in details.

13.3 DIAMAGNETISM

Diamagnetism is universal since all atoms and molecules in a solid produce a diamagnetic contribution to total susceptibility. However, due to small value, its effect is visible only in the absence of any other type of magnetic activity.

In Unit 11 of PHE-07 course, you have learnt about the Larmor's precession, which signifies the influence of magnetic field on the motion of an electron within an atom quantitatively. The Larmor frequency ω_L (in SI units) is expressed as

In CGS units Larmor frequency is given as

$$\omega_L = \frac{eB_0}{2mc}$$

where c is speed of light.

$$\omega_L = \frac{eB_0}{2m}, \quad (13.4)$$

where e is electronic charge, B_0 is magnetic field strength and m is mass of electron.

There could be many electrons revolving around the nucleus of an atom. Under normal conditions, average current due to orbiting electrons around the nucleus is zero. When external magnetic field is applied, a small current is produced around the nucleus due to induced magnetic moment.

The Larmor precession of Z electrons around an atom will give rise to a current which can be expressed as a product of total charge and revolutions made by it per second. Mathematically, we write

$$I = \frac{\omega_L}{2\pi} (-Ze) = \frac{e}{2\pi} \frac{B_0}{2m} (-Ze) = -\frac{Ze^2 B_0}{4\pi m} \quad (13.5)$$

In literature, μ is used quite frequently to denote magnetic moment. However, we are using symbol \mathfrak{R} for magnetic moment.

The magnetic moment \mathfrak{R} of such a system will be product of current and the area of the current carrying loop, which in this case is the electronic orbital. If we assume it to be circular, we can write

$$\mathfrak{R} = -\frac{Ze^2 B_0}{4\pi m} \pi \overline{a^2} = -\frac{Ze^2 B_0}{4m} \overline{a^2}, \quad (13.6)$$

where $\overline{a^2}$ is mean square distance of an electron from the nucleus (taken as origin) as projected onto a plane perpendicular to the field direction. This is shown in Fig. 13.2. Suppose that the number of electrons in an atom is Z . The radial distance of i th electron from the nucleus (considered as origin) is given by

$$r_i = \sqrt{x_i^2 + y_i^2 + z_i^2}$$

Therefore, the mean square radial distance of all electrons

$$\begin{aligned} \overline{r^2} &= \frac{\sum r_i^2}{Z} = \frac{\sum (x_i^2 + y_i^2 + z_i^2)}{Z} \\ &= \frac{\sum x_i^2}{Z} + \frac{\sum y_i^2}{Z} + \frac{\sum z_i^2}{Z} \\ &= \overline{x^2} + \overline{y^2} + \overline{z^2} \end{aligned} \quad (13.7)$$

Now consider that the field is applied along the z -axis. Then,

$$\overline{a^2} = \overline{x^2} + \overline{y^2}. \quad (13.7a)$$

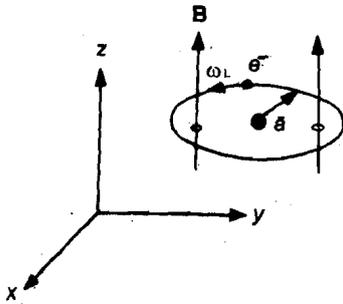


Fig.13.2: Electron orbit perpendicular to the applied field B_0

For a spherically symmetric charge distribution, $\overline{x^2} = \overline{y^2} = \overline{z^2}$ and we can write $\overline{a^2}$ in terms of $\overline{r^2}$ as:

$$\overline{a^2} = \frac{2}{3} \overline{r^2}. \quad (13.8)$$

For a single atom, the magnetisation will simply be the magnetic moment. Hence the diamagnetic susceptibility is given by

$$\chi_{\text{dia}} = \frac{\mathfrak{R}}{B_0} = -\frac{Ze^2}{4m} \frac{2}{3} \overline{r^2} = -\frac{Ze^2}{6m} \overline{r^2}. \quad (13.9)$$

The magnetisation M is a product of individual magnetic moment and total number of atoms i.e.
 $M = \mathfrak{R} N$.

If the number of atoms is N , the expression for susceptibility becomes

$$\chi = \frac{N\mathfrak{R}}{B_0} = -\frac{NZe^2}{6m} \overline{r^2}. \quad (13.10)$$

Let us consider one gram-mole of a solid with atomic number Z . It will contain N_A atoms, where N_A is Avagadro's number. The molar diamagnetic susceptibility is then given by

$$\chi_{\text{dia}}^{\text{mol}} = N_A \chi_{\text{dia}} = -\frac{N_A Ze^2}{6m} \overline{r^2}. \quad (13.11)$$

The value of $\overline{r^2}$ will depend on the charge distribution within an atom. If we take its value to be $\sim 10^{-16} \text{ cm}^2$ and $N_A = 6.02 \times 10^{23}$ per mole, we get $\chi_{\text{dia}}^{\text{mol}} \sim -10^{-5}$. As χ_{dia} is small, the diamagnetic effect in most atoms is masked by other effects. The typical materials exhibiting diamagnetism are inert gases like helium, neon, krypton, argon etc. Also, the molecules of diatomic gases like H_2 , N_2 and compounds like NaCl show diamagnetism. This is because each atom has completely filled outermost electronic shell due to charge sharing/ transfer.

You must note here that the nature of bonding in a molecule also contributes to its magnetic properties. Let us consider the case of oxygen molecule. In oxygen molecule, each atom has 8 electrons, which are configured as $1s^2 2s^2 2p^4$. In molecular state, the outer 4 electrons in $2p$ orbitals of both the atoms are shared mutually. For this purpose, the $2p$ orbitals of oxygen atoms combine to form molecular orbitals as shown in Fig. 13.3.

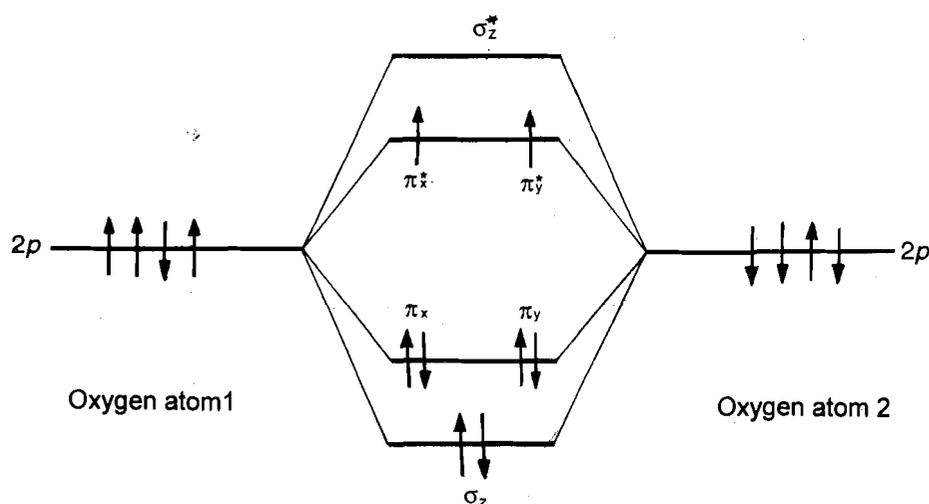


Fig.13.3: Charge sharing by $2p$ molecular orbitals in an O_2 molecule

According to Pauli's exclusion principle, no two electrons having all the quantum numbers same can occupy the same level.

Here σ_z , π_x , π_y are bonding orbitals, which correspond to lower energy and are preferred by electrons. These 3 orbitals can accommodate 6 electrons. But the total number of electrons is 8 (4 from each atom). This means that two of these are still to be placed. These two electrons occupy the antibonding orbitals denoted by π^* . If these electrons have opposite spins, they can occupy the same empty orbital π_y^* (or π_x^*).

But if they have the same spin, they will occupy π_y^* and π_x^* orbitals, in conformity with Pauli's exclusion principle. This is shown in Fig. 13.3. When these electrons are in separate orbitals, the $e^- - e^-$ Coulomb interaction is minimum and energetically more favourable. This suggests that effectively O_2 molecule has two unpaired electrons in its outermost shell and possesses an intrinsic magnetic moment. What magnetic behaviour will it exhibit? In the following section you will discover that such molecules exhibit *paramagnetism*.

But before proceeding further, you may like to answer an SAQ.

Spend
2 min.

SAQ 1

What type of magnetism will be exhibited by an NO molecule?

13.4 PARAMAGNETISM

You know that paramagnetism is shown by the materials having atoms/ions/molecules with incomplete electronic shells. Typical examples of paramagnetic substances are transition metals and rare earth materials. In metals, the spins of conduction electrons contribute to their paramagnetic behaviour.

From Sec. 11.6 of PHE-07 course, you would recall that intensity of magnetisation M and Langevin's function $L(a)$ are related by the expression

$$M = \mathfrak{R} N L(a), \quad (13.12)$$

where \mathfrak{R} is magnetic moment of an atom and N is total number of atoms. The Langevin's function is given by

$$L(a) = \coth a - \frac{1}{a} \quad (13.13a)$$

with

$$a = \mathfrak{R} B_0 / k_B T. \quad (13.13b)$$

At low temperatures and/or strong fields, $a \rightarrow \infty$ and $L(a) \rightarrow 1$. It means that the magnetisation is determined by the number of atoms present. But at high temperature and/or weak fields, a is small and $\coth a \approx \frac{1}{a} + \frac{a}{3}$ so that $L(a) = \frac{a}{3}$. Using this result in Eq. (13.12), we obtain

$$M = \frac{\mathfrak{R}^2 N B_0}{3 k_B T} \quad (13.14a)$$

so that

$$\chi_{\text{para}} = \frac{M}{B_0} = \frac{\mathfrak{R}^2 N}{3 k_B T}. \quad (13.14b)$$

From these results we note that magnetisation and susceptibility are inversely proportional to the temperature.

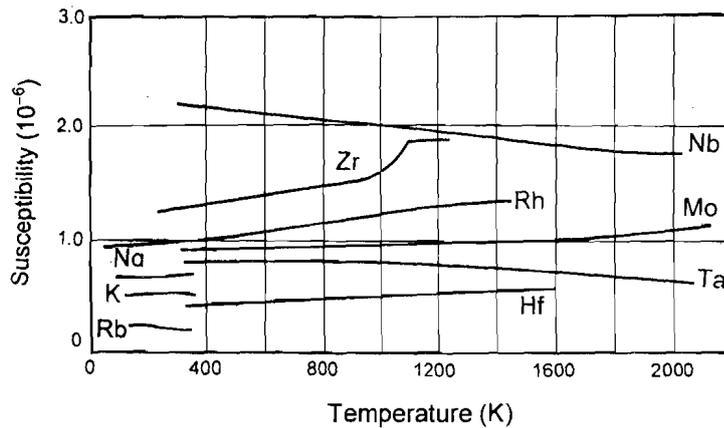


Fig.13.4: Variation of magnetic susceptibility with temperature for some typical metals

Let us now consider conduction of free electrons in a metal at room temperature. If $N \sim 10^{22} \text{ cm}^{-3}$, Eq. (13.14b) gives χ_{para} to be $\sim 10^{-4}$. The experimentally observed value of χ_{para} in metals is $\sim 10^{-6}$ and practically it shows no temperature dependence at all (see Fig.13.4). Obviously the weak temperature dependence of χ_{para} in metals can not be explained by the classical free electron theory. Pauli explained this in his theory of spin paramagnetism, which we shall discuss in the following subsection. Before that, however, you may answer an SAQ.

SAQ 2

Can we use diamagnetic and paramagnetic materials to manufacture permanent magnets? Justify your answer.

Spend
3 min.

13.4.1 Pauli's Theory

We know that electrons are spin $\frac{1}{2}$ particles and obey Fermi-Dirac statistics. For simplicity, let us consider that the substance is at low temperature ($T \approx 0 \text{ K}$) and there is no magnetic field. If Fermi-Dirac distribution is used, the number of electrons in the energy interval E and $E + dE$ is given by Eq. (9.35) of this course:

$$N(E)dE = \frac{\pi}{2} \left(\frac{8mL^2}{h^2} \right)^{3/2} E^{1/2} dE.$$

A plot of number of particles per unit energy as a function of energy based on this result is shown in Fig. 13.5a. Note that all the energy levels below the Fermi energy E_F are occupied while all those above E_F are empty. We know that two electrons having opposite spin vectors only can occupy the same level. It means that below E_F , there are equal number of spin-up and spin-down electrons. This is indicated by the arrows in the figure.

When an external magnetic field \mathbf{B}_0 is applied, the potential energy of a dipole (having magnetic moment \mathfrak{R}) in the direction of field is given by $-\mathfrak{R}\mathbf{B}_0$. And the energy of electrons with \mathfrak{R} antiparallel to the field direction is $+\mathfrak{R}\mathbf{B}_0$, as shown in Fig. 13.5b. The energy of electrons with parallel spins would, on application of field, change to $E - \mathfrak{R}\mathbf{B}_0$, while for electrons with antiparallel spins will increase to $E + \mathfrak{R}\mathbf{B}_0$. That is, the energy of electrons with antiparallel spins is greater than those with parallel spins by $2\mathfrak{R}\mathbf{B}_0$. However, Fig. 13.5b depicts an unstable situation as the highest filled

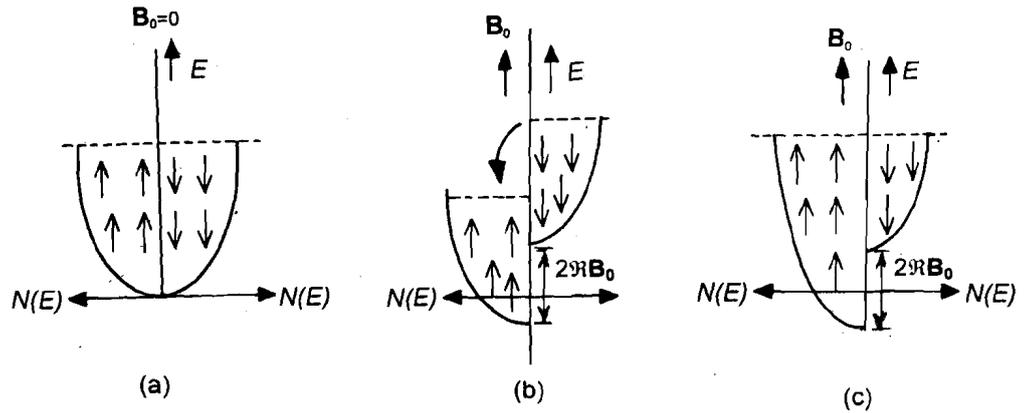


Fig. 13.5: Fermi-Dirac distribution of electrons with parallel and antiparallel spins: a) when there is no external field; b) unstable distribution caused by applied field; and c) transfer of electrons from antiparallel to parallel states restore equilibrium

energy levels of the two halves are not equal. As a result, some electrons in energy states with spins antiparallel to \mathbf{B}_0 will cross over to the energy states with spins parallel to \mathbf{B}_0 by flipping their spins. This results in an equilibrium condition and both the halves are filled to the same energy level, as shown in Fig. 13.5c. But this transfer of electrons from one type of spin to another causes unequal distribution of spins and an atom gains net magnetic moment.

You will appreciate here that most of the orbitals near the Fermi level with parallel spin are already occupied. Hence only a few electrons within small range ($k_B T$) of the Fermi energy can flip their spins to shift to parallel spin orientation. The fraction of electrons that can flip the spin is determined by the ratio (T/T_F) , where T_F is the Fermi temperature. You will recall from Unit 9 of this course that Fermi energy E_F can be expressed in terms of Fermi temperature by the relation $E_F = k_B T_F$.

Since the magnetic moment in this case is induced by electrons changing their spin, the resultant magnetisation is given by

$$\mathbf{M} = \frac{N\mu_B^2 \mathbf{B}_0}{k_B T} \frac{T}{T_F} = \frac{N\mu_B^2 \mathbf{B}_0}{k_B T_F} \tag{13.15}$$

and the paramagnetic susceptibility is given by the expression

$$\chi_{\text{para}} = \frac{\mathbf{M}}{\mathbf{B}_0} = \frac{N\mu_B^2}{k_B T_F} \tag{13.16}$$

Note that the value of susceptibility is independent of temperature, which is in agreement with experimental results. In this discussion, we have considered that magnetism arises due to free conduction electrons. For a free electron in an external

magnetic field, the component of magnetic moment along the field direction is $\frac{e\hbar}{2m}$

and is termed as *Bohr magneton*. It is denoted by the symbol μ_B and its magnitude is $9.274 \times 10^{-24} \text{ JT}^{-1}$.

If you calculate χ_{para} with $N = 10^{22} \text{ cm}^{-3}$, you will see that paramagnetic susceptibility of free electrons is of the order of 10^{-6} , which is in fairly good agreement with the experimental results. So we can say that Pauli explained the anomaly observed in the temperature dependence of susceptibility of paramagnetic materials by treating electrons as fermions. However, it is also observed that the magnetic moments cannot align arbitrarily; these follow the direction resulting from a combination of spin and

orbital magnetic moments. This could also be explained by the quantum theory of paramagnetism. Let us see how.

13.4.2 Quantum Theory

Suppose that a material having N atoms or molecules per unit volume is placed in a constant magnetic field \mathbf{B}_0 . Let us assume that the interaction between the atoms is so weak that an individual atom can be considered to be affected only by the applied field. The atoms, each of which has a magnetic moment, will tend to get oriented parallel to the the magnetic field. However, according to quantum theory, the magnetic moment of an atom or ion cannot freely rotate and align parallel to the applied field; it can have only certain finite orientations relative to the field.

Let $J (= L+S)$ be the total angular momentum of each atom. Then the magnetic moment can take the values

$$\mathfrak{R} = m_J g \mu_B, \quad (13.17)$$

where g is the Lande g -factor and m_J is magnetic quantum number. It takes half integral values between $-J$ and $+J$. Since m_J can take discrete values, \mathfrak{R} takes only discrete values.

If we assume that dipole moments behave as a classical system, i.e. obey Maxwell-Boltzmann statistics, the expression for average dipole moment \mathfrak{R} in the direction of the field can be written as

$$\mathfrak{R} = \frac{\sum_{m_J=-J}^{-J} (m_J g \mu_B) e^{m_J g \mu_B B_0 / k_B T}}{\sum_{m_J=-J}^{-J} e^{m_J g \mu_B B_0 / k_B T}}. \quad (13.18)$$

On simplification (see Appendix A), we obtain

$$\mathfrak{R} = g \mu_B J B(y), \quad (13.19a)$$

where $B(y)$ is the *Brillouin function*

$$B(y) = \frac{(2J+1)}{2J} \coth \frac{(2J+1)}{2J} y - \frac{1}{2J} \coth \frac{y}{2J} \quad (13.19b)$$

and

$$y = \frac{g \mu_B J B_0}{k_B T}.$$

Note that according to quantum theory, magnetic moment in atoms/ions with partially filled electronic shells is governed by a combination of spin and orbital quantum numbers.

From Eq. (3.19a) it readily follows that magnetisation in paramagnetic substances is given by

$$M = N \mathfrak{R} = N g \mu_B J B(y). \quad (13.20)$$

You have learnt about various quantum numbers in the course on Modern Physics (PHE-11: Units 8,9, and 10).

The Lande g -factor is characteristic of the state of an atom. Its values is 1 if the atomic state is determined purely by the orbital angular momentum and 2 for pure spin case.

The average value of a variable is given by

$$\bar{x} = \frac{\sum_{i=1}^n x_i f(x_i)}{\sum_{i=1}^n f(x_i)},$$

where $f(x)$ is the probability function. For discrete nature of a variable, we consider summation instead of integration.

You should not confuse Brillouin function with magnetic field.

Let us now consider two special cases of Eq. (13.20).

Case I: For strong magnetic field and low temperature, the value of y will be large. Then the hyperbolic function in Eq. (13.19b) tends to unity and

$$B(y) \rightarrow \frac{2J+1}{2J} - \frac{1}{2J} = 1$$

and

$$M = Ng\mu_B J. \quad (13.21)$$

Using series expansion of the function we can show that for small values of y ,

$$\coth y = \frac{1}{y} + \frac{y}{3} \quad (i)$$

$$\therefore B(y) = \frac{2J+1}{2J} \coth \frac{2J+1}{2J} y - \frac{1}{2J} \coth \frac{y}{2J}$$

$$= \frac{2J+1}{2J} \frac{2J}{2J+1} \frac{1}{y}$$

$$+ \frac{2J+1}{2J} \frac{1}{3} \frac{2J+1}{2J} y$$

$$- \frac{1}{2J} \frac{2J}{y} - \frac{1}{2J} \frac{1}{3} \frac{y}{2J}$$

$$= \frac{1}{3} \frac{1}{(2J)^2} [(2J+1)^2 - 1] y$$

$$= \frac{1}{3} \frac{1}{(2J)^2} 4J(J+1)y$$

$$= \frac{1}{3} \frac{J+1}{J} y$$

$$= \frac{1}{3} \frac{J+1}{J} \frac{g\mu_B J B_0}{k_B T} \quad (ii)$$

Hence, magnetisation

$$M = Ng\mu_B J B(y)$$

$$= Ng\mu_B J \frac{1}{3} \frac{J+1}{J} \frac{gJ\mu_B B_0}{k_B T}$$

$$= \frac{NJ(J+1)(g\mu_B)^2}{3k_B T} B_0 \quad (iii)$$

This means that saturation in magnetisation is reached at low temperatures, i.e. all the dipoles ultimately get directed in the direction of applied magnetic field. You may say that this expression is analogous to the Langevin's expression $M = \mathcal{R}N$. The important difference to note is that whereas Langevin's expression is true for freely rotating dipoles with all possible orientations, with quantum theory only a finite set of orientations of dipoles is possible.

Case II: For low magnetic fields and high temperatures, $\frac{\mu_B B_0}{k_B T} \ll 1$, i.e. $y \ll 1$. The magnetization in this case is given by (see M.R.)

$$M = \frac{1}{3} \frac{NJ(J+1)(g\mu_B)^2}{k_B T} B_0. \quad (13.22)$$

It means that magnetization in ions and metals will be governed by temperature. The paramagnetic susceptibility is given by the expression

$$\chi_{\text{para}} = \frac{M}{B_0} = \frac{NJ(J+1)(g\mu_B)^2}{3k_B T}. \quad (13.23)$$

It can also be written as

$$\chi_{\text{para}} = \frac{N p_{\text{eff}}^2 \mu_B^2}{3k_B T} = \frac{C}{T}, \quad (13.23a)$$

where $C = \frac{N p_{\text{eff}}^2 \mu_B^2}{3k_B}$ and $p_{\text{eff}} = g[J(J+1)]^{1/2}$ is called *effective Bohr magneton number*. This is **Curie's law**. With $N \sim 10^{22} \text{ cm}^{-3}$ and a dipole moment of one Bohr magneton, we obtain $\chi_{\text{para}} = \frac{1}{300 T}$. At room temperature, $\chi_{\text{para}} \approx 10^{-5}$ and at 1 K,

χ_{para} lies in the range 10^{-3} to 10^{-2} . These values match fairly well with experimental results suggesting that paramagnetism is a quantum phenomenon.

Now let us study some examples of metals which behave as paramagnetic materials.

13.4.3 Paramagnetism in metal and ions

You now know that paramagnetism is observed in atoms or ions having partially filled electronic shells. From Eq. (13.19a) we recall that magnetic moment in these cases is determined by a combination of orbital and spin quantum numbers. Paramagnetism is most commonly observed in transition metals containing partially filled d shell and rare earth metals with partially filled f shell.

The rare earth metals classified as Lanthanides possess partially filled $4f$ shell. The valence electronic configuration of these metals is generally of the form $4f^n 5p^6 5s^2 5d^1 6s^2$ where n varies from 1 for cerium to 14 for lutecium. Three electrons in $5d^1 6s^2$ shells are shed by these atoms and electronic configuration of their outer-most shell is of the form $4f^n 5p^6 5s^2$. Since the $4f$ shell is situated inside $5p$ and $5s$ shells (see Fig. 13.6), it is completely screened from external interactions and the experimentally observed value of p_{eff} compares well with its calculated value.

In transition metals, in contrast to screened f shell of lanthanides, the d shells are the outer-most shells and exposed directly to the crystal environment. As a result, these atoms (or ions) in the crystal are influenced by neighbouring atoms (ions), and this effect is prominent only in the directions in which neighbouring atoms are situated. Now, due to the inhomogeneous crystal field, the spin-orbit (L-S) coupling in the atom is broken and the magnetic moment of the atom is determined only by the spin quantum number i.e. the value of p_{eff} is effectively decided by $g\sqrt{S(S+1)}$. It is as if the orbital momentum does not contribute at all. This is referred to as **quenching** of orbital angular momentum.

Before we proceed further, we would like you to answer an SAQ.

SAQ 3

Identify which of the following ions will exhibit angular momentum quenching:

(a) Ti^{3+} , (b) Cr^{3+} , (c) Gd^{3+} , (d) Er^{3+} , (e) Ni^{2+} , (f) V^{3+} , (g) Nd^{3+} , (h) Dy^{3+}

As you know, ferromagnets constitute another important class of magnetic materials with wide commercial applications. These materials possess large value of magnetic susceptibility and are easily magnetised. You may like to know as to what characteristics differentiate them from diamagnetic and paramagnetic substances at the microscopic level. Let us learn about these now.

13.5 FERROMAGNETISM

We know that when a piece of iron is kept in an external magnetic field, it shows strong magnetisation. Yet at room temperature, an ordinary piece of iron is not magnetic in the absence of a magnetic field! To explain this discrepancy, Weiss conceived the idea of domains (Fig. 13.7). According to him, in an unmagnetised iron crystal, there are a large number of randomly oriented domains (see Unit 12 of PHE-07). Obviously spins in all domains are not aligned in the same direction.

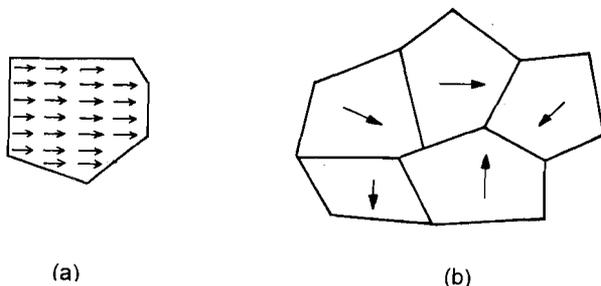


Fig.13.7: Magnetic domains in a ferromagnetic materials: a) orientation of spins in a single domain; and b) random arrangement of domains

Due to random orientation of domains, their magnetic moments cancel out and the crystal as a whole possesses no magnetism. However, when a magnetic field is applied, the domains tend to align themselves along the field (due to rotation of domains or displacement of domain walls) resulting in a large net magnetisation of the specimen. Even after removal of the external field, the effect of magnetisation persists for a long time. Is it not different from that observed in the cases of para- and diamagnets? The answer is in the affirmative. Note that the process of magnetisation

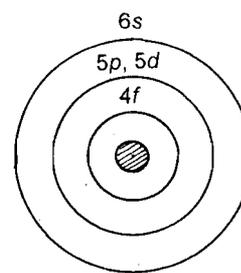


Fig 13.6: Outer electronic shells of lanthanide ions

You have learnt about various groups of elements in the Periodic Table in Unit 2 of CHE-01 course.

The field experienced by atoms (ions) because of neighbouring atoms (ions) in the crystal is called **crystal field**.

*Spend
3 min.*

A single crystal ferromagnetic specimen can contain large number of domains. Existence of domains does not disturb the geometrical arrangement of atoms in a crystal.

of a ferromagnetic material is the alignment of various domains along the applied field. You can experimentally verify the existence of domains by spreading finely powdered magnetic material on a ferromagnetic specimen and observing tiny collections around the domain boundaries.

Origin of domain walls

The magnetic energy E_B is determined by the field B and volume of the spread of B . It is given as

$$E_B = \frac{1}{8\pi} \int B^2 dV$$

where dV is the volume element.

An obvious question that may come to your mind is: Why do all the magnetic dipoles not align in the same direction in the specimen? Why should there be a large number of domains? To know the answer to the first question, refer to Fig. 13.8a. It shows that the material is magnetised in the direction shown by the arrow. The magnetic field lines are spread over a large volume around the material. In Fig. 13.8b and c, the material is seen to consist of 2 and 4 domains, respectively. From these figures, you

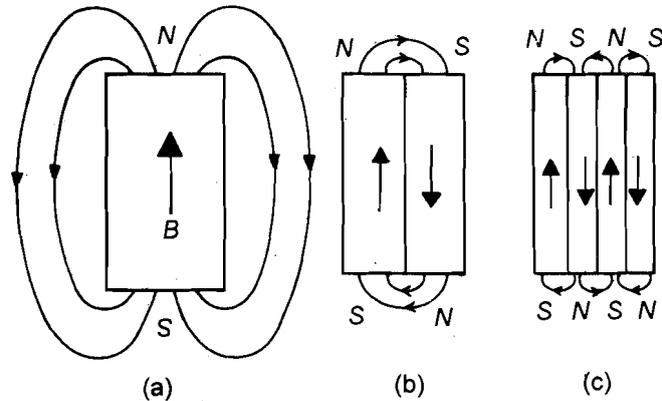


Fig.13.8: Development of magnetic domains a) single domain; b) two domains; and c) four domains

can see that the volume over which B spreads decreases as the number of domains increases. Due to this reduction in the spread of B , the energy stored in the field decreases as the number of domains increases.

You must appreciate the fact that in going from single domain to two domains, one domain wall has been introduced, separating the atoms having oppositely directed spins. Since, the state of parallel spins corresponds to minimum energy, some energy goes into the system to create this wall with a state of antiparallel spins. That is, some energy is required to create domains. This energy is called *wall energy*. The total energy of the system is therefore the sum of the stored magnetic energy and the wall energy. In going from (a) to (c), the total energy decreases further. Since, a system would prefer to reside in a state of minimum energy, it is clear that the situation shown in (c) is more likely to occur than the situations in (a) and (b). This argument therefore suggests that the sample should consist of a large number of domains. However it cannot go on and on continuously because ultimately a state is reached when the decrease in magnetic energy equals the energy required to put on additional domain wall. The dimensions of a domain depend on the shape and size of the bulk sample as well as on the nature of the material. Generally, the volume of the domain may lie between 10^{-18} m^3 to 10^{-12} m^3 and it may contain about 10^{10} to 10^{16} atoms. The field developed within the domain is known as **molecular field**.

The salient features of ferromagnetism are:

- All magnetic moments are aligned parallel to the field like paramagnetism. But the value of ferromagnetic susceptibility (χ_{ferro}) is much greater than paramagnetic susceptibility (χ_{para}). This results in spontaneous magnetisation.
- For ferromagnetic substances, linear relations like $M = \chi B_0$, and $B = \mu B_0$ do not hold except for a certain small value of B_0 . These rather depend on the applied field. Hence we can express them as functions of B_0 as $\chi = \chi(B_0)$, and $\mu = \mu(B_0)$.
- Net magnetic moment of a ferromagnet depends upon the magnetic history of the material. This implies that B depends on the value of B_0 applied previously. The

specimen exhibits net magnetic moment even after removal of the field; so if we want to get zero magnetic moment we have to apply some negative field.

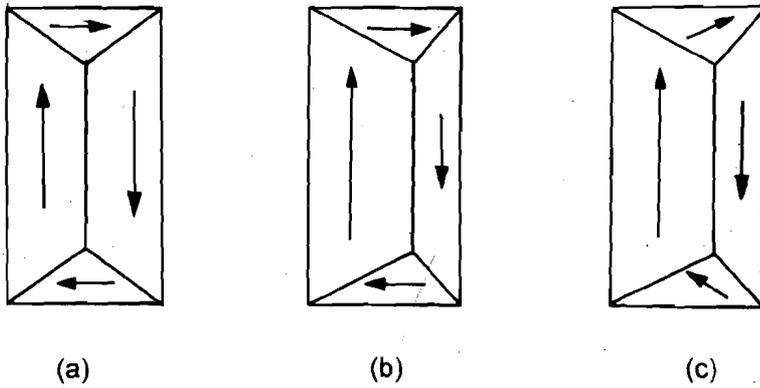


Fig.13.9: Role of domains in magnetisation of ferromagnetic substance: a) unmagnetised; b) domain wall movement; and c) domain rotation

The magnetisation of the bulk specimen consisting of domains may occur by various mechanisms. Fig. 13.9 shows these mechanisms. Fig. 13.9a shows an unmagnetised specimen. Under small field, growth of one domain occurs at the expense of another domain or by the motion of domain walls as seen in Fig. 13.9b. Under a strong field, the domains rotate to align themselves, as shown in Fig. 13.9c.

You have already learnt about the relationship between applied magnetic field (B_0) and internal magnetic field (B) in PHE-07 course. Now we revisit this topic in light of these magnetisation processes described just now. In Fig. 13.10 the part of $B-B_0$ curve is shown indicating dominant processes in various regions.

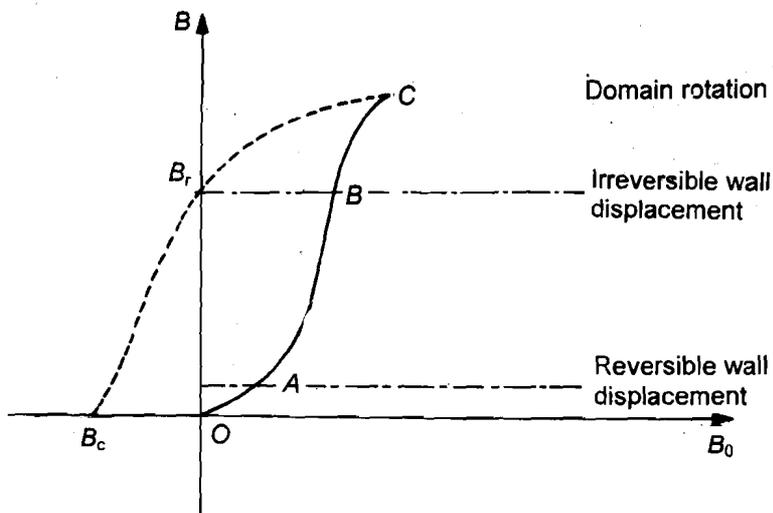


Fig. 13.10: Magnetisation due to various processes

Under small fields (region $O-A$ in the figure), the domain wall displacement occurs but it can be restored by removing the field. Further increase in the field displaces the wall irreversibly (region $A-B$). At still higher field values, (region $B-C$) the domains rotate irreversibly. When the field is reversed at C , the dashed curve is obtained. When applied field is made zero, there is still some magnetisation B , observed in the sample. This is called **remanence**. To nullify this magnetisation, negative field B_c needs to be applied. It is called **coercivity**.

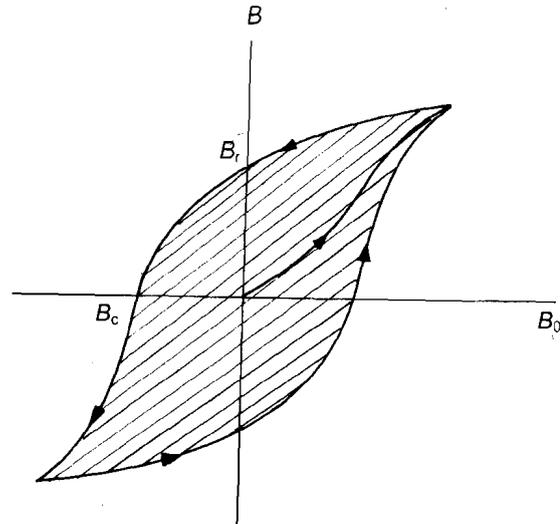


Fig. 13.11: Magnetisation cycle

A complete cycle of magnetisation is shown in Fig. 13.11. As you can see, the major part of the curve is irreversible. Due to this irreversibility, if one complete cycle is undertaken, i.e. from a particular value of B , we attain a value $-B$ (with the same magnetic field in the reverse direction) and come back to B again, we loose some energy.

The area of B - B_0 loop is the energy loss per unit volume per cycle. So if we plot B with corresponding B_0 value, after completing the whole cycle, we have an area in B - B_0 curve which is the energy loss per unit volume per cycle, more commonly known as **Hysteresis Loss**.

The shape of the hysteresis loop as well as the values of B_r and B_c are determined by the properties of a material.

Materials like Alnico, Ferroxdur ($BaFe_{12}O_{19}$) and rare earth-cobalt are used to make permanent magnets. In these materials the hysteresis loop is very broad, and coercivity values are large. But large values of instantaneous energy product $(BB_0)_{max}$ are the true indicator of hard magnets. Rare earth-cobalt magnets are the best choice.

The materials like pure iron, silicon ferrite (transformer sheet), permalloy (Ni-Fe), superpermalloy (Ni-Fe-Mo) etc. are used for electromagnetic application (like transformer cores, solenoid cores). In this case the hysteresis loop is very narrow with large remanence but small coercivity. This means that with small amount of applied field, large magnetisation is induced. Even demagnetisation can be achieved by a very small negative reverse field. Effectively, the B - B_0 curve is vertical (Fig 13.12) and its slope is very large. This slope is a measure of **magnetic permeability**. Typical values of permeability for silicon-ferrite transformer sheet is 15,000 while that of superpermalloy is 100,000. Hence these materials can reach saturation induction in both positive and negative directions very fast. That is why they find applications where field direction continuously changes, say in a line ac transformer in which the field changes at 50 Hz frequency, or in high frequency communication circuits.

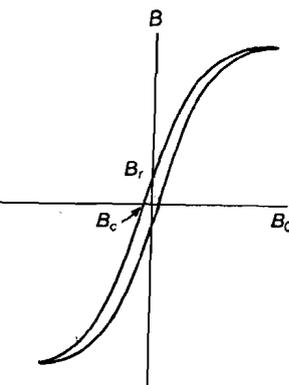


Fig. 13.12: B - B_0 loop for soft magnets

Magnetic permanence can be indexed by the coercive field B_c , that is required to return the induction to zero. The maximum instantaneous energy product $(BB_0)_{max}$ is a better measure because it represents the critical energy barrier that must be exceeded for demagnetisation.

You may now like to answer an SAQ.

SAQ 4

A transformer core of volume 0.01 m^3 is wound with a coil carrying an ac at a frequency of 50Hz. Assuming the magnetisation to be uniform throughout the core volume, calculate the hysteresis loss. The hysteresis loop has an area of 60,000 units, when the axes are drawn in unit of $10^{-4} \text{ Wb m}^{-2}$ and 10^2 A m^{-1} .

Spend 5 min.

Every ferromagnetic substance has a characteristic temperature called *Curie Temperature* (T_C) above which it is converted into a paramagnetic substance and magnetic domain structure breaks down. For ferromagnetic metals like iron, cobalt and nickel, the values of T_C are 1043 K, 1388 K and 627 K, respectively. Above T_C the susceptibility of a ferromagnet is given by *Curie-Weiss* law:

$$\chi_{\text{ferro}} = \frac{C}{T - T_C}, \quad (13.39)$$

where C is called Curie constant and is given by $N(g\mu_B)^2 S(S+1)/3k_B$. For comparison, note that for paramagnetic materials, C is determined by $J(J+1)$.

Below Curie temperature, ferromagnets show spontaneous magnetisation and χ_{ferro} increases as T decreases (Fig. 13.13).

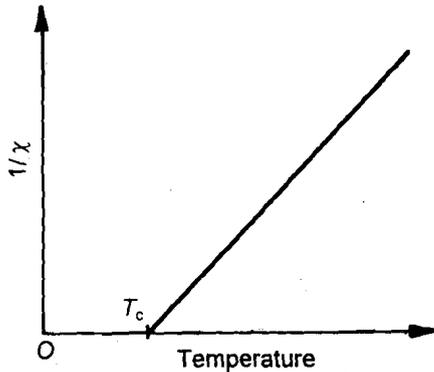


Fig.13.13: Temperature variation of magnetic susceptibility in ferromagnetic materials

We hope you now understand the physical processes at work in different types of magnetic materials. Let us now consider some technologically important magnetic substances.

13.6 SPECIAL MAGNETIC MATERIALS

13.6.1 Ferrites

You know that the magnetic properties of materials are determined by the alignment and magnitudes of spin vectors of its constituent atoms. Refer to Fig 13.14a-c.

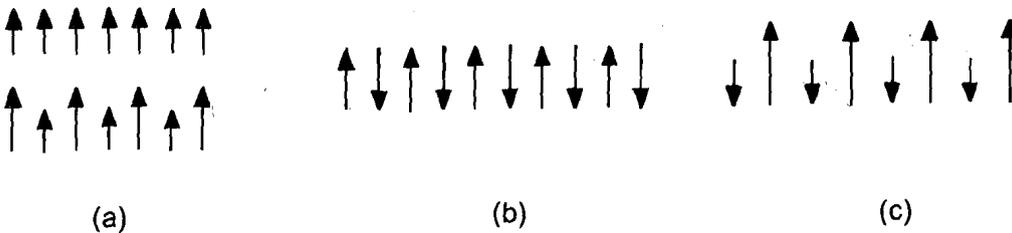


Fig.13.14: Linear array of spins depicting a) ferromagnets; b) antiferromagnets; and c) ferrites

From Fig. 13.14a you will note that all spins are oriented in the same direction. Such materials exhibit strong magnetic moment. Fig. 13.14b show equal number of spin up and spin down orientations of equal magnitude and the resultant moment is zero. Such materials are called *antiferromagnetic*. In Fig. 13.14c, the spin orientations are similar to those for antiferromagnets but the magnetic moments of spin-up atoms and spin-down atoms are not equal. So the net magnetic moment is non-zero. Such materials are called *ferrites*. The resistivity of ferrites is very large; 10^{-2} to $10^5 \Omega \text{ cm}$, which is 10^4 to 10^{11} times that of iron. This property controls the eddy currents so that not much heat is generated when such materials are placed in a frequently changing magnetic field. That is why ferrites find applications in telecommunication systems

Eddy currents are generated in magnetic materials due to rapidly fluctuating magnetic field and result in loss of energy in form of heat.

or in high frequency transformers. Due to their high resistivity and oxide nature, the ferrites are also termed as **ceramic magnets**. The ferrites are iron oxides with the unit cell structure $(M^{2+} Fe_2^{3+} O_4)_8$ where M indicates a divalent metal ion like Fe^{2+} , Ni^{2+} , Co^{2+} , Mo^{2+} , Cu^{2+} , Mg^{2+} , Zn^{2+} . These atoms are arranged in a typical crystal structure called **spinel**. This structure is similar to that of $MgAl_2O_4$ commonly known as spinel compound. A unit cell of typical spinel structure consists of 8 units, each comprising 4 oxygen, one divalent metal and two trivalent iron ions.

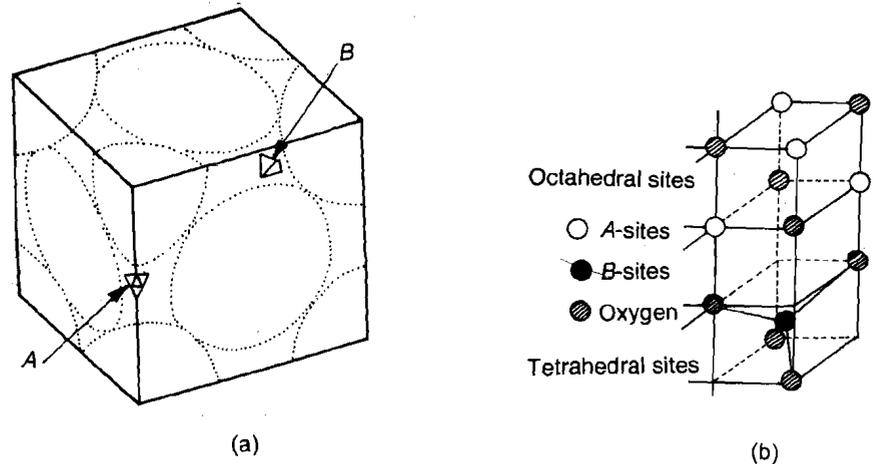


Fig.13.15: a) Schematic diagram of spinel structure; and b) a part of spinel structure indicating the octahedral (A) and tetrahedral (B) sites in oxygen lattice.

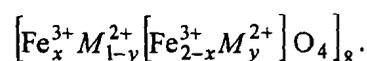
In this structure, oxygen ions (O^{2-}) are arranged in the fcc structure. The Fe^{3+} ions take up the octahedral sites (shown as A) of edge centre with 6 O^{2-} ions as nearest neighbours. M^{2+} ions take up the tetrahedral sites (shown as B) with 4 O^{2-} neighbours. The number of occupied A sites is double the number of occupied B sites. The ions occupying the octahedral sites are normally enclosed in brackets, i.e. they are represented in the formula as $(M^{2+} [Fe_2^{3+}] O_4)_8$.

A naturally occurring ferrite material is magnetite. It has iron both in bi- and tri-valent states. Its chemical formula is $[Fe^{2+} Fe_2^{3+} O_4]_8$. This material possesses a structure called **inverse spinel**. Here one Fe^{3+} takes up the tetrahedral site while one Fe^{3+} and one Fe^{2+} take the octahedral sites. Hence it is represented as $[Fe^{3+} [Fe^{3+} Fe^{2+}] O_4]_8$.

The unit cell of ferrites exhibits magnetic moment because the magnetic moment vectors of the ions in octahedral sites are all aligned in the same direction and those in tetrahedral sites are all aligned in the opposite direction.

We summarize the ferrite structures to enable you to remember them easily.

- Ferrite structures are represented as 8 units of basic structure $(M^{2+} Fe_2^{3+} O_4)_8$.
- In the *normal* spinel structure of a ferrite, 8 divalent metal ions occupy tetrahedral positions while 16 trivalent iron ions occupy octahedral positions. It is represented as $[M^{2+} [Fe_2^{3+}] O_4]_8$.
- In the *inverse* spinel structure of a ferrite, divalent M^{2+} ions occupy octahedral sites; Fe^{3+} ions are distributed in equal numbers over the tetrahedral and octahedral sites. This structure is represented by $[Fe^{3+} [Fe^{3+} M^{2+}] O_4]_8$.
- In the intermediate case, the arrangement may be of the type



When we account for the magnetic moments of magnetite $\left[\text{Fe}^{3+} \left[\text{Fe}^{3+} \text{Fe}^{2+} \right] \text{O}_4 \right]_8$,

we have to take note of eight Fe^{2+} and eight Fe^{3+} ions at octahedral sites with magnetic moments in parallel (\uparrow) direction while there are eight Fe^{3+} ions in tetrahedral sites with moments in antiparallel (\downarrow) direction. We know that Fe^{2+} has valence electronic configuration $4s^0 3d^6$ while Fe^{3+} has the configuration $4s^0 3d^5$. This means that Fe^{2+} has 4 unpaired electrons in the $3d$ valence shell while Fe^{3+} has 5. Every unpaired electron possesses magnetic moment equivalent to Bohr magneton ($\mu_B = 9.274 \times 10^{-24} \text{ JT}^{-1}$). When we tabulate various ions according to their spins, the result is as shown in Table 13.2.

Table 13.2: Calculation of Magnetic Moment of Magnetite

Interstitial site	Spin alignment	Fe^{2+}	Fe^{3+}	Total number of unpaired electrons	Magnetic moment
Tetrahedral	\downarrow	-	8	$8 \times 5 = 40$	$-40 \mu_B$
Octahedral	\uparrow	8	-	$8 \times 4 = 32$	$32 \mu_B$
Octahedral	\uparrow	-	8	$8 \times 5 = 40$	$40 \mu_B$
Total					$32 \mu_B$

The term parallel (\uparrow) and antiparallel (\downarrow) moments are relative. You can take any one site as corresponding to parallel moment site, and the other as antiparallel sites.

In Unit 11 of PHE-07 course, you have learnt to calculate the value of Bohr magneton

In this way, you can determine the magnetic moment of the unit cell of a ferrite. This value determines the amount of ferrite material required to design a magnet of necessary strength.

You may now like to answer the following SAQ.

SAQ 5

Determine the magnetic moment of $\text{Ni}^{2+}\text{Fe}_2^{3+}\text{O}_4$. Note that it has inverse spinel structure.

Hint: Ni^{2+} has $4s^0 3d^8$ valence configuration.

*Spend
5 min.*

13.6.2 Superconductors

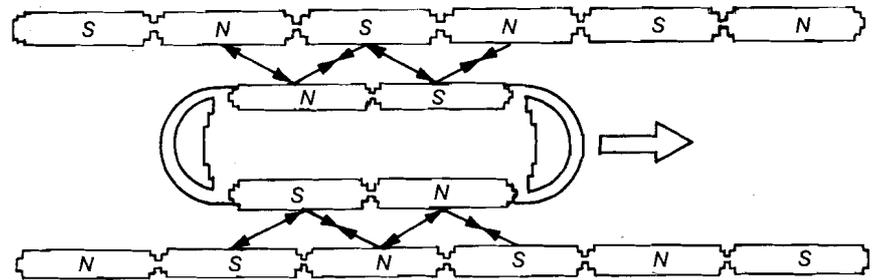
Apart from ferrites, superconductors constitute another category of technologically important materials. In Unit 12 you have studied the properties of superconductors. These are perfect diamagnetic materials with very high magnetic susceptibility (~ 1) and exhibit Meissner effect. Their resistance is negligibly small below the transition temperature.

In case of electromagnets, the strength of the magnetic field produced is determined by the current flowing through the coil. In case of normal conductors like copper wire, due to Joule heating, the value of maximum current that could be passed through the coil is limited. This puts limitation on the maximum field produced by these electromagnets. But, if the coil is made of superconducting material and is operated below its transition temperature, very large current can be passed through it. This increases the strength of the magnetic field produced by an electromagnet.

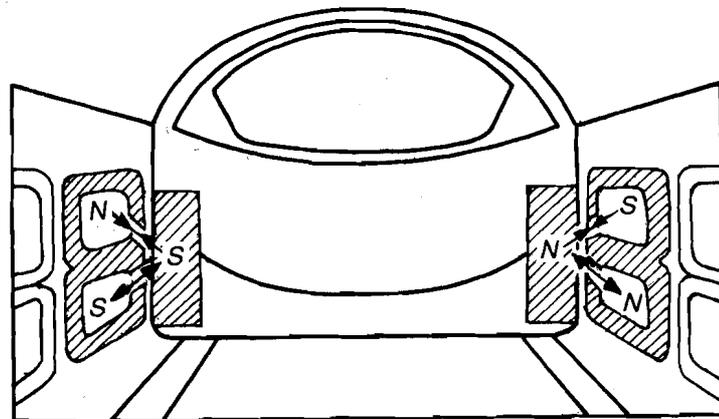
Another interesting application of superconductors is based on levitation. This is finding use in modern super fast trains. Due to levitation, the trains are lifted few centimeters above the track and hence there is an air gap between the train and the track. Since there is no friction between the train and track, the trains can reach speeds

upto 500 km per hour. Such Maglev trains (magnetic levitation trains) are presently being developed in Germany, Japan and US.

Maglev is a system in which the vehicle runs levitated from the guideway (corresponding to the rail tracks of conventional railways) by using electromagnetic forces between superconducting magnets on board the vehicle and coils on the ground. Fig. 13.16 depicts the schematic principle of magnetic levitation train.



(a)



(b)

Fig.13.16: a) Propulsion of maglev; and b) Levitation of maglev

For forward motion of the train, the principle of linear motor is used. This is illustrated in Fig. 13.16a. It shows one train carriage along with the guideway. Superconducting magnets are placed at the bottom of the carriage with polarities as indicated. On the guideway wall, electromagnetic coils, called *propulsion coils*, are fitted. These coils are powered by three phase alternating current which creates shifting magnetic field on the guideway. Any on-board superconducting magnet experiences attraction as well as repulsion by two consecutive propulsion coils simultaneously indicated by the arrows. This action helps to push the train ahead and set it in motion.

When on board magnets move due to propulsion action, a current in 8 shaped magnetic coils fitted on the guideway wall is induced (Fig. 13.16b). This induced current makes these coils to act as temporary magnets and on-board superconducting magnets begin to experience attractive repulsive forces as indicated by the arrows. These forces lift the superconducting magnets and thereby the whole carriage from the ground. This is called the *levitation action*. Note that a levitated train does not experience any friction and can attain large speeds.

After studying about various magnetic properties of solids and their technological applications, let us now summarize the points you learnt in this unit.

13.7 SUMMARY

- **Magnetic materials** can be categorised into three categories depending on their magnetic behaviour: **diamagnetic**, **paramagnetic** and **ferromagnetic**.
- A **diamagnetic substance** generates a magnetic moment that opposes the applied external magnetic field. All substances possess diamagnetic susceptibility, but its value is very small.
- In **paramagnetic substances**, magnetic moment vector is oriented parallel to the applied field. However, quantum theory tells us that the orientation angle can take values determined by a combination of spin and orbital angular momentum quantum numbers.
- In case of transition metals, the paramagnetic moment orientation is determined only by spin quantum number due to **quenching effect**.
- **Ferromagnets** have very high positive value of magnetic susceptibility. In spite of this, these materials do not exhibit magnetism in the absence of a magnetic field due to **domain structure**. By application of external field, the domains reorient themselves to give magnetic property to the ferromagnetic materials.
- Due to irreversible reorientation of magnetic domains, ferromagnets exhibit **hysteresis loss**.
- **Ferrites** have $(M^{2+}Fe^{3+}O_4)_8$ structure and possess magnetism due to non-equivalent sites of metallic ions. These materials have very high resistance and hence can be used in high frequency applications.
- Magnetic applications of **superconductors** can range from high strength electromagnets to magnetic levitation trains.

13.8 TERMINAL QUESTIONS

Spend 15 min.

1. The saturation magnetisation of bcc iron is 1750 kA m^{-1} . Calculate the net magnetic moment per iron atom in the crystal in terms of Bohr magneton when the lattice constant of iron is 2.87 \AA .
2. A bar magnet made of steel has a magnetic moment of 5 Am^2 and a mass of $13.2 \times 10^{-3} \text{ kg}$. If the density of steel is $7.9 \times 10^3 \text{ kg m}^{-3}$, find the intensity of magnetisation of the magnet.
3. Below the superconductivity transition temperature a piece of permanent magnet floats above the superconductor pallet. This is called Meissner effect. Can you explain the phenomenon?
4. Though Ni^{2+} and Fe^{2+} , both are transition metals with same valences, the magnetic moments of their ferrites are different. Why?

13.8 ANSWERS AND SOLUTIONS

Self-Assessment Questions

1. In case of NO molecule, there is 1 unpaired electron in the antibonding molecular orbital. Hence it exhibits paramagnetism.
2. Both diamagnets and paramagnets get magnetised only under the effect of external magnetic field. The susceptibility values in both cases are negligibly small. Hence they cannot be used for preparing permanent magnets.

- Ti^{3+} , Cr^{3+} , Ni^{2+} and V^{3+} will show quenching of angular momentum.
- Energy loss during each cycle = Area of the loop.
 $= 60,000 \times (10^{-4} \text{ Wb m}^{-2}) \times (10^2 \text{ A m}^{-1})$
 $= 600 \text{ J m}^{-3}$.
 Volume of the transformer core = 0.01 m^3 .

Hence the energy lost in core of each cycle = $(600 \text{ J m}^{-3}) \times (0.01 \text{ m}^3) = 6 \text{ J}$
 and
 the power loss due to hysteresis = $6 \text{ J} \times 50 \text{ Hz} = 300 \text{ W}$.

- Ni^{2+} has $4s^0 3d^8$ electronic configuration, means it has two unpaired electrons in the d shell contributing to magnetisation.

Interstitial	Spin alignment	Ni^{2+}	Fe^{3+}	No. of unpaired electrons	Magnetic moment
Tetrahedral	↓	-	8	$8 \times 5 = 40$	$40 \mu_B$
Octahedral	↑	8	-	$8 \times 2 = 16$	$16 \mu_B$
Octahedral	↑	-	8	$8 \times 5 = 40$	$40 \mu_B$
Total					$16 \mu_B$ $= 16 \times 9.274 \times 10^{-24} \text{ JT}^{-1}$

Terminal Questions

- Lattice constant for iron is 2.87 \AA .

This means that

$$\text{volume of the unit cell} = (2.87)^3 \times 10^{-30} \text{ m}^3$$

Number of atoms in the unit cell in case of bcc structure is 2.

Hence, net magnetic moment per atom

$$= 1750 \times 1000 \times (2.87 \times 10^{-10})^3 \times \frac{1}{2}$$

$$= 2.068 \times 10^{-23} \text{ A m}^2$$

In units of Bohr magneton with $\mu_B = 9.274 \times 10^{-24} \text{ J T}^{-1}$,

$$\text{the net magnetic moment per atom} = \frac{2.068 \times 10^{-23}}{9.274 \times 10^{-24}} = 2.23 \mu_B.$$

- Volume of the bar magnet is:

$$V = \text{mass/density} = \frac{13.2 \times 10^{-3} \text{ kg}}{7.9 \times 10^3 \text{ kg m}^{-3}} = 1.67 \times 10^{-6} \text{ m}^3$$

The intensity of magnetisation is given by the ratio of magnetic moment and volume. We have magnetic moment = 5 A m^2 and volume = $1.67 \times 10^{-6} \text{ m}^3$, so that, intensity of magnetisation is $= 5 \text{ A m}^2 / 1.67 \times 10^{-6} \text{ m}^3 = 2.99 \times 10^6 \text{ A m}^{-1}$.

- In superconducting state, the material is diamagnetic with very large magnetic susceptibility (~ -1). Hence it strongly repels the external magnetic field. This repulsion causes the small magnetic piece to float away from the superconducting material.
- Though both Ni^{2+} and Fe^{2+} have same valency, the number of unpaired electrons, in $3d$ shell for both cases are different. (Fe^{2+} has 4 unpaired electrons while Ni^{2+} has 2 unpaired electrons). As these unpaired electrons contribute to the magnetic moment, the values are different for Fe and Ni.

$$\mathfrak{R} = g\mu_B J \frac{d}{dy} \ln \left[\frac{e^{-\frac{y}{2J}} \left(\frac{Jy}{e^J e^{2J}} - \frac{Jy}{e^J e^{2J}} e^{-\frac{(2J+1)y}{J}} \right)}{e^{-\frac{y}{2J}} \left(e^{\frac{y}{2J}} - e^{-\frac{y}{2J}} \right)} \right]$$

On simplification, we obtain

$$\mathfrak{R} = g\mu_B J \frac{d}{dy} \ln \left[\frac{\left(e^{\frac{(2J+1)y}{2J}} - e^{-\frac{(2J+1)y}{2J}} \right)}{e^{\frac{y}{2J}} - e^{-\frac{y}{2J}}} \right]$$

$$= g\mu_B J \frac{d}{dy} \ln \left[\frac{\sinh \left((2J+1) \frac{y}{2J} \right)}{\sinh \frac{y}{2J}} \right]$$

$$= g\mu_B J \frac{d}{dy} \left[\ln \sinh \left((2J+1) \frac{y}{2J} \right) - \ln \sinh \frac{y}{2J} \right]$$

On differentiation with respect to y , we get

$$\mathfrak{R} = g\mu_B J \left[\frac{1}{\sinh \left((2J+1) \frac{y}{2J} \right)} \left(\cosh \left((2J+1) \frac{y}{2J} \right) \frac{(2J+1)}{2J} - \frac{\cosh \frac{y}{2J}}{\sinh \frac{y}{2J}} \left(\frac{1}{2J} \right) \right) \right]$$

$$= g\mu_B J \left[\frac{(2J+1)}{2J} \coth \left((2J+1) \frac{y}{2J} \right) - \frac{1}{2J} \coth \frac{y}{2J} \right] \quad . \text{(A.2)}$$