UNIT 1 THE WORLD OF SYMMETRIES

Structure

1.1 INTRODUCTION

We observe symmetry almost everywhere around us – in nature, in arts and architecture, in paintings and sculptures, in engineering and science. You are familiar with the symmetry of a butterfly, a snowflake and a flower. You are also familiar with symmetry in paintings, patterns and borders, carvings on stones in monuments and places of worship; such wonderful patterns are the best manifestations of beauty of nature and human imagination, intellect and creativity. Even in the microscopic world, we find remarkable symmetry of electron orbitals in an atom, atomic structure of the molecules and solid crystals. In fact, well-developed symmetry is common to crystals such as rock salt, quartz, diamond, calcite, etc.

The symmetry of forms, positions and structures constitutes the subject of *geometrical symmetry*. It can be observed by unaided eye directly or through certain tools/devices indirectly. Symmetry plays an important role in scientific comprehension of the world around us. 'Through symmetry man always tried to perceive and create order, beauty and perfection.' These words of German Mathematician Hermann Weyl (1885-1955) emphasise why symmetry (and asymmetry) form the foundations of the theory of relativity, quantum mechanics, solid state physics and atomic, nuclear and elementary particle physics. The unchangeability or invariance of physical laws lies in the principles of symmetry. In solid state, the crystals are characterized by the ordered symmetric arrangement of atoms. This ordering is governed by the geometrical symmetry. While learning about the crystal structure, which constitutes an important part of Solid State Physics, it is apt to start with learning about symmetry. In this unit you will learn about different types of geometrical symmetries.

Symmetry has many causes. It is related to the orderliness and equilibrium, proportionality and harmony, purpose and usefulness. In Sec. 1.2 you will learn the basic vocabulary used in the world of geometrical symmetries. You will realise that in doing so, not only our discussion is simplified but also it is possible to visualise similarities/regularities! Sec. 1.3 is devoted to the discussion of geometrical symmetries in nature, in molecules and in the world of solids.

Objectives

After studying this unit, you should be able to:

- identify different types of geometrical symmetries;
- state characteristic features of different symmetries; and
- state the importance of symmetry in science.

1.2 SYMMETRY: BASIC VOCABULARY

The term *symmetry* is the Greek word for *proportionality, similarity in arrangement (of parts) and elegance*. According to Weyl, 'An object is called symmetrical if it can be changed somehow to obtain the same object.' An operation (movement or transformation) is said to be symmetric if a body/system looks exactly the same after the operation as it did before. In other words, an operation, which transforms the body into itself, i.e. leaves it invariant is called a **symmetry operation**. The symmetry operations are classified as **point symmetry** and **translation symmetry**. The point symmetry is further categorised as reflection in a plane, inversion about a point and rotation about an axis. The operations such as *reflection*, *inversion*, *rotation* and *translation* constitute **elements of symmetry**. You will now learn about these elements of symmetry and the associated basic vocabulary.

a. Reflection symmetry

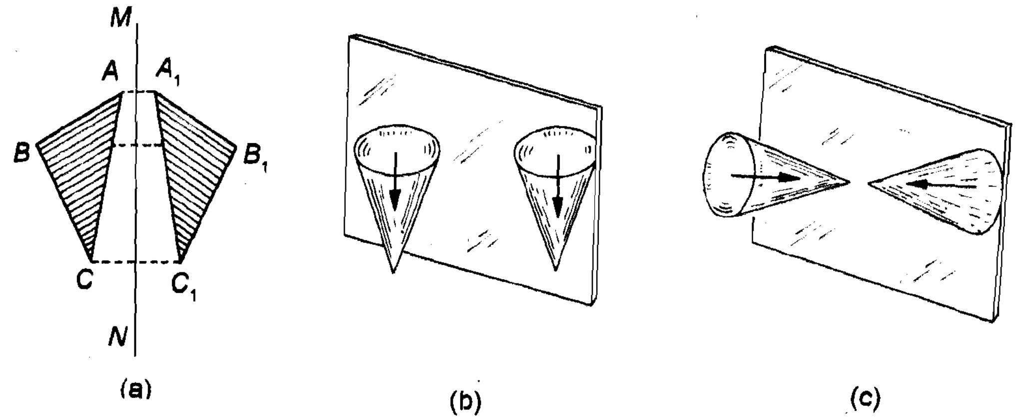


Fig.1.1: Reflection symmetry for a) a 2-D object; b) a 3-D object placed vertically; and c) placed horizontally

Refer to Fig.1.1a. It shows a simple triangular object *ABC* and its mirror image $A_1B_1C_1$. (*MN* is the intersection of the mirror plane with the plane of the drawing.) Note that for each point on the triangle, there is a corresponding point on its mirror image. These points lie on the normal to *MN* drawn from either side and are at the same distance from it. Note that here we have considered a two-dimensional object. In general, physical objects are three-dimensional. In Fig. 1.1b we have shown a cone and its mirror image. From the above discussion you may be tempted to conclude that if we place an object in front of a plane mirror and look into it, the mirror image is an exact replica of the object. However, the mirror transposes the front and the back (with respect to the mirror plane) parts of the object (see Fig. 1.1c). That is to say, the mirror turns image of an object inside out along a direction perpendicular to the plane of the mirror.

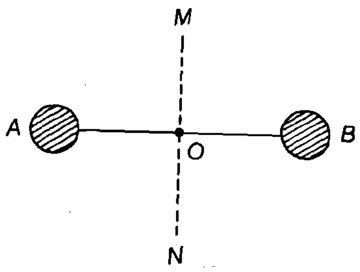


Fig.1.2: Mirror symmetry in linear objects

If one-half of an object is mirror reflection of the other half about a plane, the object is said to be **mirror-symmetrical** and the plane is said to be the **plane of reflection symmetry**.

Let us understand this with the help of few examples. For one-dimensional, i.e. linear object, such as *a dumb-bell* shown in Fig.1.2, the mirror symmetry is with respect to *O*, the point of intersection of the line joining two balls *A* and *B* and the plane of symmetry *MN*. Point *O* is known as **centre of symmetry**. Note that there exists only one centre of symmetry for linear objects.

For a two-dimensional, i.e. flat object, we define an **axis of symmetry** – the line of intersection of the plane of symmetry with the plane of the object. For example, for letters such as **U** and **V** shown in Fig.1.3a & b, one-half of the letters are reflected in the mirror plane (shown by dash-dot line) and the reflection coincides exactly with the other half. That is, both the letters possess one axis of reflection symmetry.

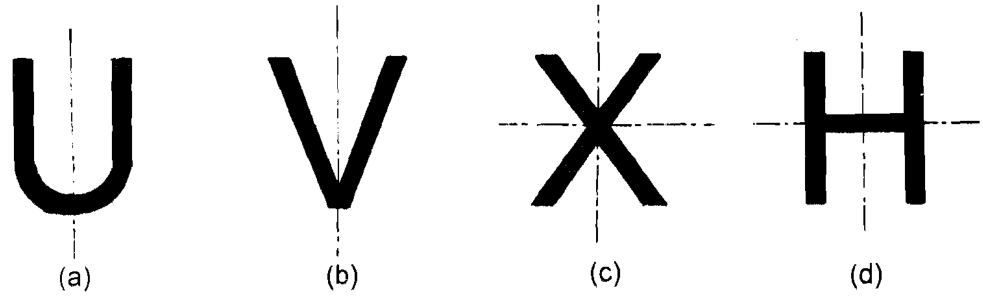


Fig.1.3: Axis of symmetry in 2-D

You may now like to answer an SAQ.

SAQ 1

Spend 2 min.

Write the words **COOK** and **BOOK** on a sheet of paper in capital letters. Place a mirror vertically so that the line of intersection of the plane of the mirror and the plane of the sheet divides the words in two halves along a horizontal line. How will the words look after placing the mirror?

While answering SAQ 1, you must have noted that we need to define only one axis of symmetry for letters **COOK** and **BOOK**. You may now logically ask: Can an object have more than one axis of symmetry? The answer to this question is affirmative; objects can have many axes of symmetry. For example consider the letters **X** and **H**. These letters are symmetric to higher degree as these can be reflected in two plane mirrors. Upon reflection, left and right as well as the upper and lower halves coincide exactly with corresponding mirror reflections as seen in Fig.1.3c & d. Further, as shown in Fig.1.4, a square has four axes of symmetry, two (marked *P* and *Q*) along diagonals and two (marked *R* and *S*) along right bisectors of its sides. A regular hexagon has six axes of symmetry and so on.

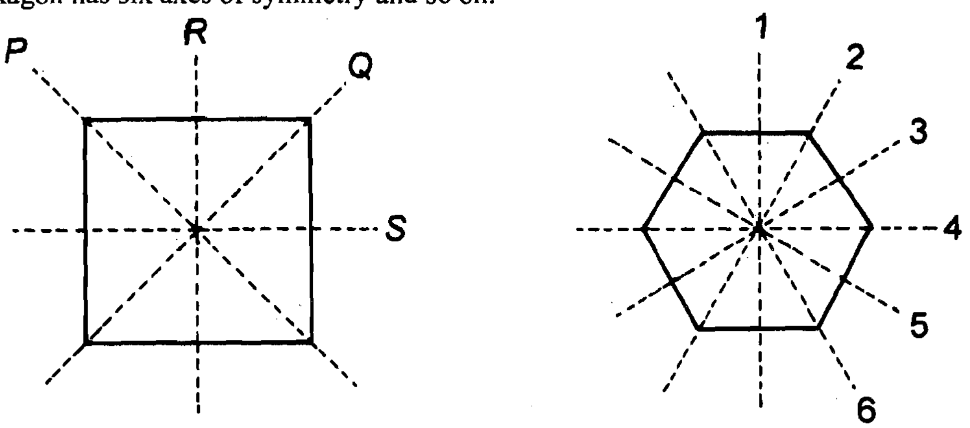


Fig.1.4: Axes of symmetry for a square and a hexagon

Let us now consider a three-dimensional object, say, a cube.

In case of 3-D object we have **plane of reflection symmetry** rather than **point** or **axis of symmetry**, as in case of 1- and 2-D, respectively. A three dimensional object can have multiple planes of symmetry. For a cube, typical planes are shown in Fig. 1.5a & b. There are three planes of type (a) and six of type (b). You should draw other planes of symmetry yourself and in case of difficulty discuss with your peers or your counsellor.

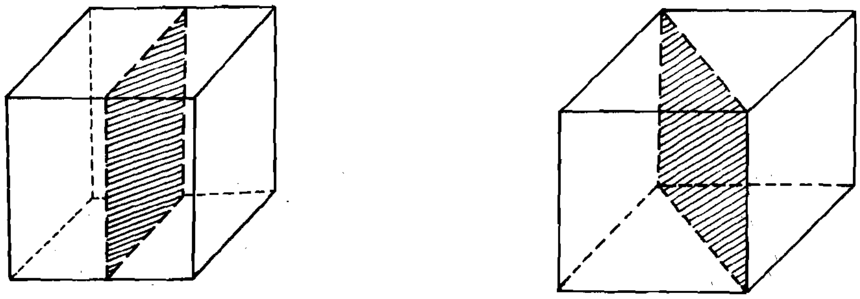

(a) (b)

Fig.1.5: Planes of symmetry for a cube

A mirror reflection of an object has another important consequence. To discover this, you should do the following activity.

Activity 1

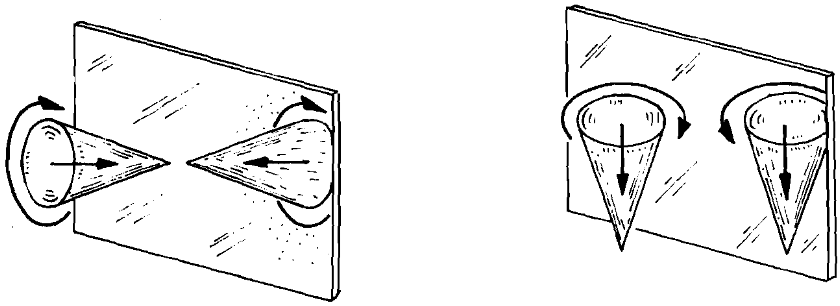

(a) (b)

Fig.1.6: Reflection of a) a horizontal cone; and b) a vertical cone

Take a cone-like object, even your pen or pencil, hold it in front of a mirror and rotate it clockwise holding the axis of rotation normal to the plane of the mirror. What do you observe? Does the direction of rotation of the object, as seen in the mirror, change (Fig.1.6a)? Next, take the axis of rotation parallel to the plane of the mirror and again rotate it clockwise (Fig.1.6b). Is the direction of rotation reversed by reflection?

In this activity you have observed that when the axis of rotation of an object is

- normal to the plane of the mirror, the directions of rotation of the object and its image seen in the mirror are the same; and
- parallel to the plane of the mirror, the directions of rotation of object and its image are opposite.

Hence the directions of rotation of object and its mirror image entirely depend on its orientation with respect to the mirror.

We come across many objects like our own hands that exhibit right and left-handed forms, i.e. they are like mirror images of each other. What appears on left side of one object appears on the right side of other one. This is known as **enantiomorphism** and the two forms are said to be **enantiomorphs**.

You will notice that whatever be the amount of similarity, it is possible that an object and its reflection could be different and incompatible. Note that *enantiomorphs are pairs of mirror-asymmetrical objects* (figures) *that are mirror reflections of each other*. To distinguish between enantiomorphs of a given pair, these are referred to as left and right. It is immaterial which is called as left (right). It is simply a matter of convention. Fig.1.7 shows two-dimensional enantiomorphs.

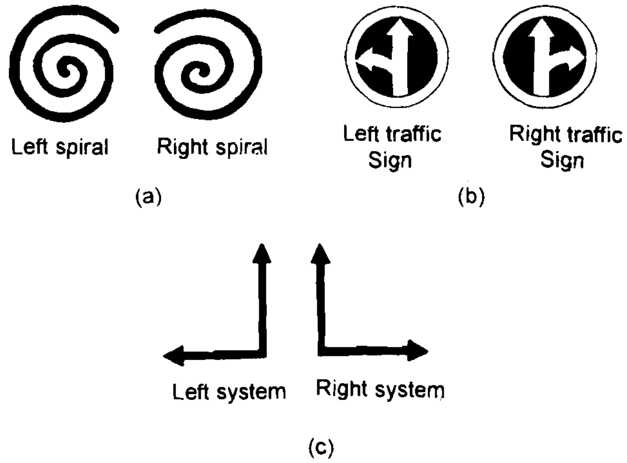


Fig.1.7: 2-D enantiomorphs a) left and right spirals; b) left and right traffic sign; and c) left and right reference frames

Examples of three-dimensional enantiomorphs are given in Fig.1.8.

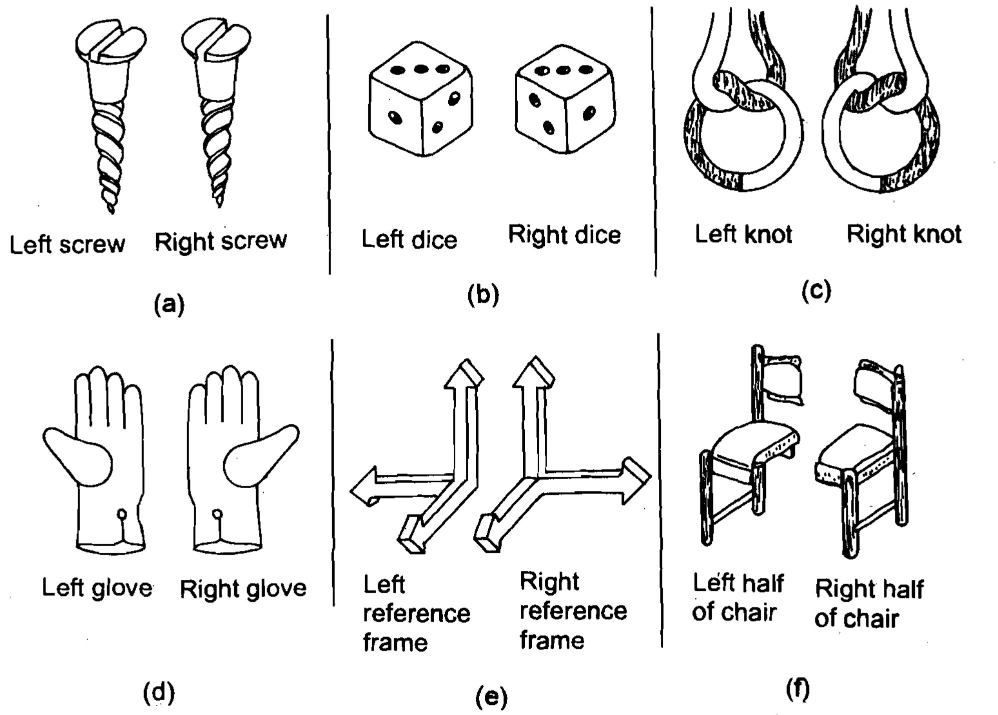


Fig.1.8: 3-D enantiomorphs a) left and right screws; b) left and right dice; c) left and right knots; d) left and right gloves; e) left and right reference frames; and f) left and right halves of a chair cut along the symmetry plane

Two-dimensional enantiomorphs cannot be superimposed by placing them on one another or by any translation or turn in their own plane. It is possible to make them to coincide only if a turn in the three-dimensional space is made, as shown in Fig.1.9.

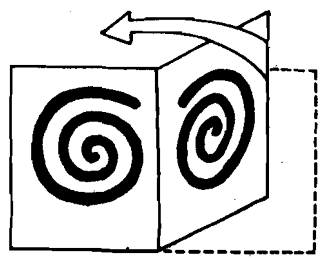


Fig.1.9: Coinciding enantiomorphs in 2-D

For three-dimensional enantiomorphs, no translation or turn can convert a left enantiomorph into a right enantiomorph and vice-versa. For example, you may turn your left shoe as you like, it will never fit your right foot. Similarly, a left dice can never turn into a right dice whatever way we cast it.

The naturally occurring crystals like quartz exhibit enantiomorphism. This gives quartz the interesting optical properties, which are used in polarisers.

Similar to reflection symmetry, another type of symmetry is *inversion symmetry*. We describe it now.

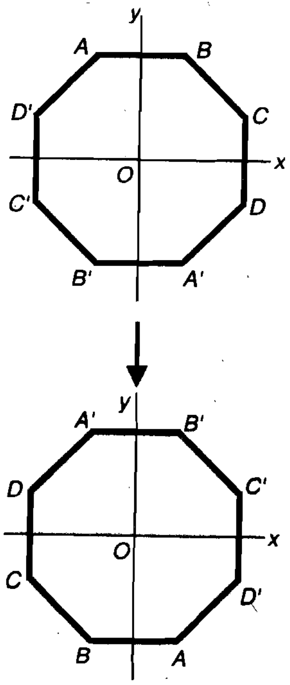


Fig.1.10: Inversion symmetry in octagon

b. Inversion symmetry

When an object is inverted in space with respect to a point (within it) and still remains invariant, it is said to have **inversion symmetry**. Let us understand it with some examples. Fig.1.10 shows an octagon in *xy*-plane. If we consider the centre of the octagon as the origin *O* and take co-ordinate axes as shown, then for every point (x, y) on this figure, there exists a point $(-x, -y)$ that also lies on the figure. Points *A, B, C, D* have their equivalent counterparts *A', B', C'* and *D'*, lying on the octagon. Hence, we can invert this figure entirely around *O* and still obtain a similar octagon. In this process, we have performed **inversion** operation around *O*. During octagon inversion, all the points inside and on the boundary of the octagon, except point *O*, have changed their places.

Fig.1.11a shows a cube with its corners *A, B, C, D, E, F, G* and *H*. All these points are equivalent and *O* is in the centre of this cube. If we invert (in 3-D space) this cube around *O*, the resultant will be as shown in Fig.1.11b, which is also a cube of same dimensions.

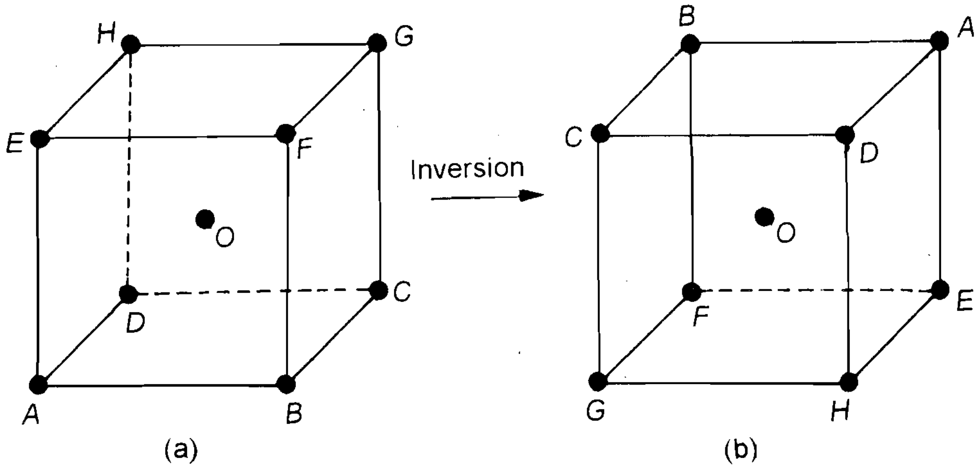


Fig.1.11: Inversion symmetry in cube

As you notice, in this case also, similar to 2-D example, all the points in Fig.1.11a have interchanged their positions in 3-D except the point *O* to result into the cube shown in Fig.1.11b. This point is called **centre of inversion** and it is unique in any inversion symmetric object. In case of 1-D objects, the point of reflection symmetry is the centre of inversion. You must have noticed that in case of 2-D objects, the inversion is simply a rotation by 180° around the body centre.

You will learn about the rotational symmetry in the next subsection. But before that, you may like to answer the following SAQ.

Spend 3 min.

SAQ 2

Look at Fig.1.12 and identify the objects with inversion symmetry. Also locate their centres of inversion.

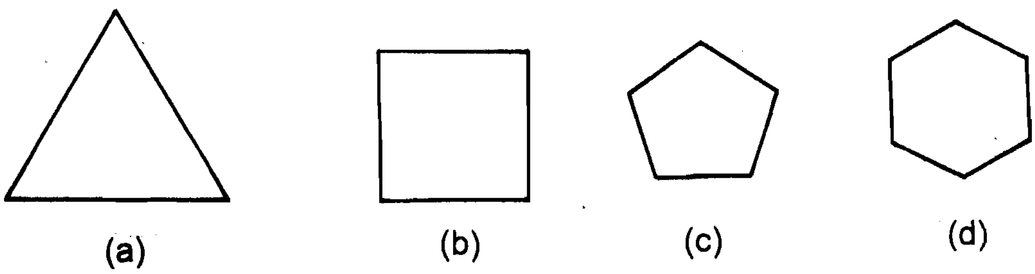


Fig.1.12: Determining centre of symmetry of 2-D objects

c. Rotational symmetry

Consider the letter **N** as shown in Fig.1.13. It does not possess any mirror symmetry, but if it is rotated by 180° about an axis normal to the paper (i.e. plane of letter **N**) and passing through its centre *O*, the letter would coincide with itself. The letter **N** is, therefore, symmetric in relation to a rotation by 180°. That is, it possesses rotational symmetry. You can easily convince yourself that rotational symmetry is inherent in letter **S** also, when rotated in the plane of paper.

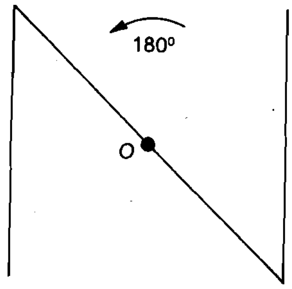


Fig.1.13: Rotational symmetry

Now look at the cube in Fig.1.14. Suppose we rotate the cube about a vertical line passing through the centre of the horizontal faces at *I* and *J*. You will notice that on rotation through 90° from its initial position, the cube coincides with its original position. That is, in one complete rotation, the cube comes into self-coincidence four times. If an object coincides with itself exactly when rotated about an axis passing through it by an angle $\theta = 2\pi/n$ (or its multiples), where n = 1,2,3,4, ..., it is said to possess ***n*-fold rotational symmetry**. The corresponding axis of rotation is then said to be an ***n*-fold axis** of symmetry. If $n = 1$, the object must be rotated through 2π or 360° to achieve self-coincidence. Such an axis is also termed as *identity axis* and every object possesses an infinite number of such axes. If $n = 2$, the object must be rotated through $(2\pi/2)$ or 180° to achieve congruence and the axis is termed a 2-fold axis. For n = 3, 4, 5 and 6, the corresponding angles of rotation are 120°, 90°, 72° and 60° and the axes are called 3-, 4-, 5- and 6-fold axes, respectively. Fig.1.15 depicts simple objects with rotation axes passing through centre and normal to object plane for two-fold to six-fold symmetry.

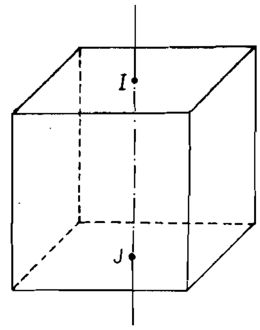


Fig.1.14: Rotational symmetry in cube

In the next unit you will learn that in real crystals only 1-, 2-, 3-, 4- and 6-fold rotational symmetry axes are possible.

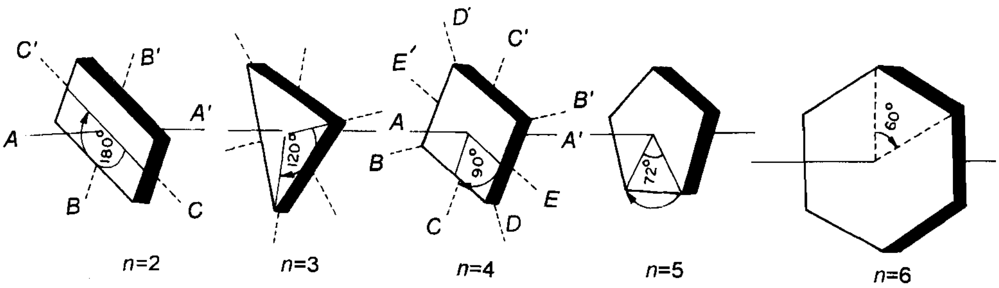


Fig.1.15: Objects with different folds of rotation axes symmetry

A three-dimensional object may have several rotation axes. For example, the first object in Fig.1.15 has three two-fold axes, indicated by *AA'*, *BB'* and *CC '*. Third object in this figure, in addition to a four-fold axis (*AA'*), has four two-fold axes (*BB'*, *CC '*, *DD '* and *EE'*). Consider the cube shown in Fig.1.16a, it has three four-fold rotation axes (1-1′, 2-2′ and 3-3′). Further examination reveals six two-fold axes passing through the centres of diagonally opposite parallel edges (1-1′, 2-2′ 3-3′, 4-4′, 5-5′ and 6-6′) (Fig. 1.16b) and four three-fold axes which coincide with the inner diagonals of the cube (1-1′, 2-2′ 3-3′ and 4-4′) (Fig. 1.16c). We may mention that a cube is a four-fold symmetric object.

An object is characterized by the maximum fold axis of its rotational symmetry.

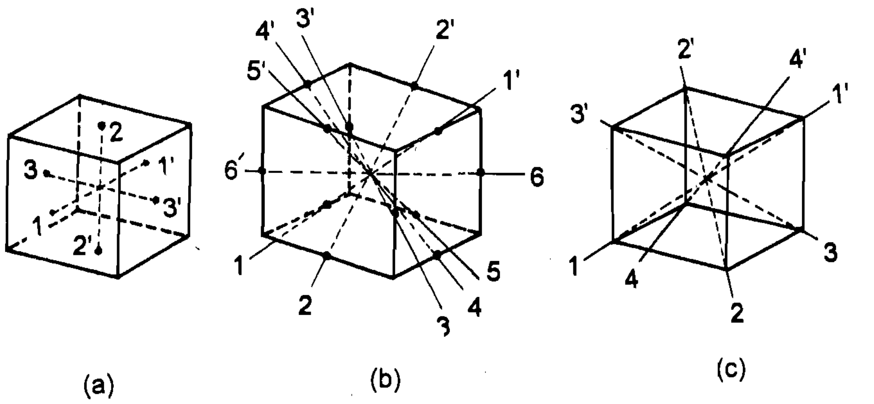


Fig.1.16: Rotation axes of a cube

Fig.1.17a shows the rotational symmetry of a cylinder. It has an infinite number of two-fold axes and one infinite-fold axis. Further, if we consider a (geometrical) structure composed of two identical regular pyramids (Fig.1.17b), we find that it has one four-fold rotation axis (*AB*), and four two-fold axes (*CE*, *DF*, *MP* and *NQ*).

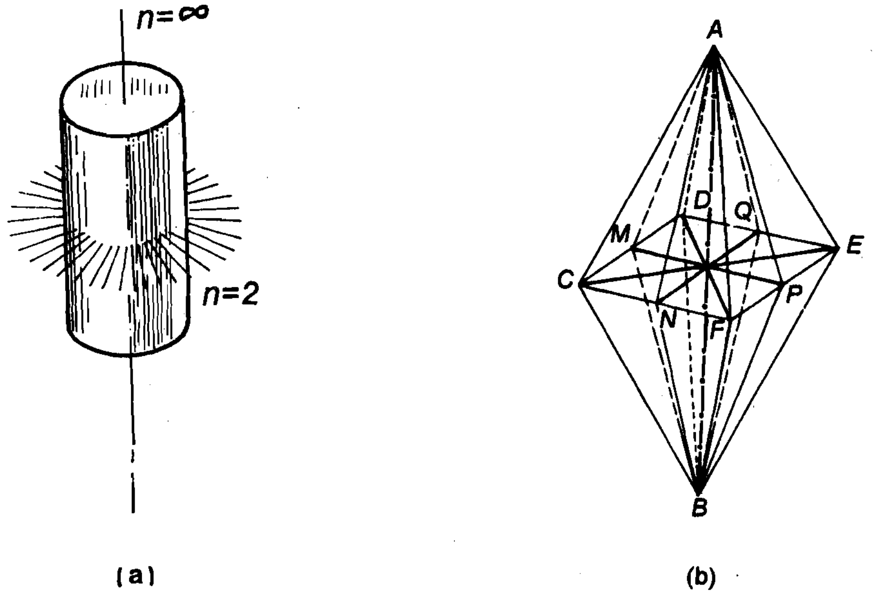


Fig.1.17: Rotation axes of a) a cylinder; and b) a body made of two regular pyramids

d. Translational symmetry and periodicity

Refer to Fig.1.18. You can easily convince yourself that a translation along the line *AB* by a distance *a* (or its multiples) keeps the figure invariant. We express this observation by saying that the figure possesses translational symmetry.

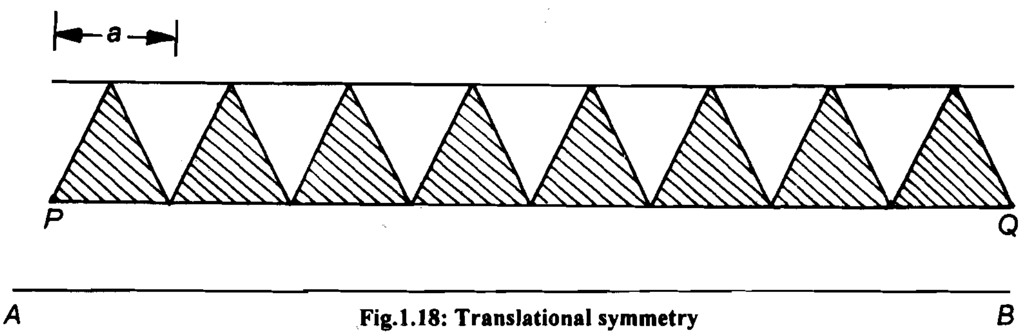


Fig.1.18: Translational symmetry

The line *AB* is called the **direction of translation** and distance *a* defines **fundamental translation** or **period**. Strictly speaking, a body/pattern must be infinitely long in the direction of translation. However, in practice the concept of translational symmetry is applied to bodies of finite size. Pattern *PQ* can be constructed by repeating the period *a* eight times in the direction *AB*. In this way, the pattern constructed by repetition of a period is said to possess **translational symmetry** or **periodicity**. The period is repeated by integral multiple values like ma; $m = 1,2,3, \ldots$ The translational symmetry can be present in 1-D (i.e. along an axis) or in 2-D (i.e. expanding in a plane) or in 3-D (expandable in all three direction to cover a volume). In the solid crystals, as you will study in next unit, the translational symmetry in 3-D is inherent.

Till now, we discussed various symmetry elements. It is possible to have objects, which exhibit symmetry arising out of combination of more than one symmetry elements. As an example, we now discuss one such combination of reflection and rotation symmetry.

Reflection-rotational symmetry

Let us cut a square out of a thick paper and inscribe into it obliquely another square as shown in Fig.1.19. Bend the corners along the periphery of inner square as shown in the figure. You will note that the resultant object has a two-fold symmetry axis (*AB*)

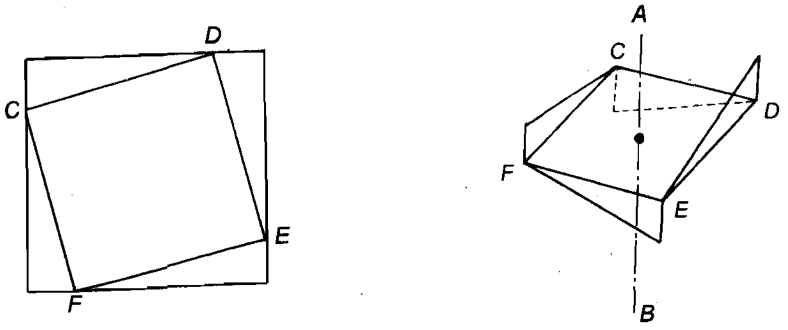


Fig.1.19: Reflection-rotational symmetry

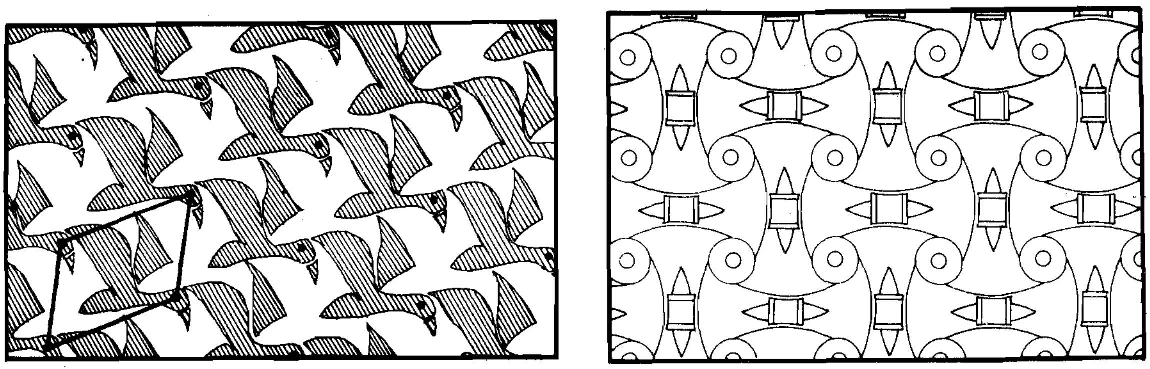
and there are no planes of reflection symmetry. Besides its two-fold rotational symmetry, the object coincides with itself when turned through 90° about *AB* and then reflected from plane *CDEF*. i.e. rotated by 90° and then seen from the other side of the sheet. This new symmetry of the object is said to be *reflection-rotational* (or *mirror-rotational*) *symmetry*. Axis *AB* is called the four-fold mirror-rotational axis. We thus have a symmetry relative to two consecutive operations: a turn by 90° and a reflection in a plane normal to the rotation axis.

We hope, now you can identify different symmetry operations. To convince yourself, you should answer the following SAQ.

SAQ 3

Spend 6 min.

Look at the patterns shown in Fig.1.20a & b and name the type(s) of symmetries.



(a) (b)

Fig.1.20: Symmetries in patterns

1.3 SYMMETRIES IN SCIENCE

There are various symmetric objects found in nature. After studying the basic elements of symmetry, you may like to re-look at some of the naturally occurring symmetric objects.

1.3.1 Symmetries in Nature

Fauna and flora exhibit remarkable symmetry. Now let us discuss the elements of symmetry in some of them. A number of rotational axes of symmetry occur in the rich world of flowers. The most widespread symmetry is a five fold rotational symmetry with respect to stem. Five-fold symmetry is seen in flowers of hibiscus, *Nerium odorum* (Kaner), *Vinca rosia* (Sadabahar) etc, flowers of fruit trees like apple, oranges, cherry etc., flowers of berry plants (strawberry, raspberry, blackberry etc.) also show five-fold symmetry. In several flowers, rotational symmetry is often accompanied by reflection symmetry. Reflection symmetry is remarkably inherent in the leaves also. A perfect reflection symmetry in a leaf is shown in Fig.1.21.

Fig.1.21: Reflection symmetry in a leaf

Reflection symmetry is the most prominent characteristic of animal kingdom. The left half is the perfect mirror image of its right half. This is clearly visible in a butterfly (Fig.1.22). Even human face possesses mirror symmetry. You will note that the symmetry of left and right halves display near mathematical accuracy.

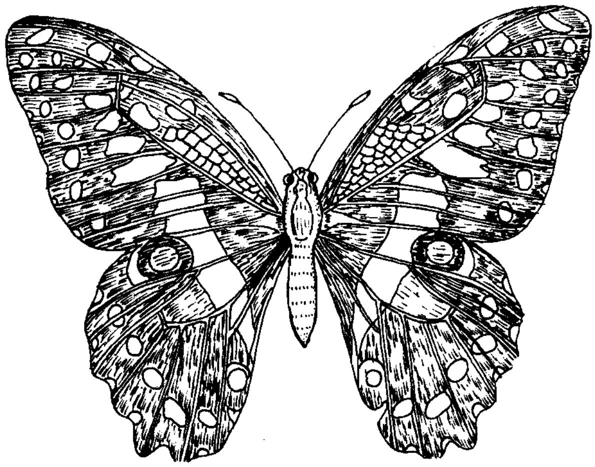


Fig.1.22: Perfect symmetry in a butterfly

When a system displays symmetry relative to a combination of a rotation and a translation along the rotational axis, it is said to possess *helical or screw symmetry.* In plants, we find numerous examples of helical symmetry in the arrangement of leaves and branches. Creeping plants are remarkable helices. Other familiar examples of natural helices include shell of a snail, umbilical cord of a newborn baby (which is a triple left handed spiral formed of two veins and one artery) and horns of a Pamir sheep.

Snowflakes, which are crystals of ice, are of hexagonal shape (Fig.1.23). These exhibit a 6-fold rotational symmetry.

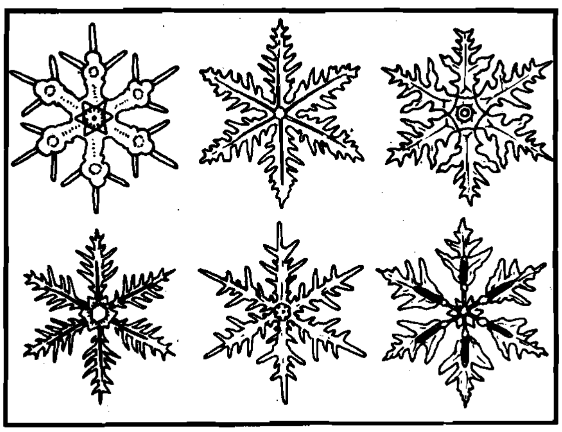


Fig.1.23: 6-fold hexagonal symmetry in snowflakes

You may now like to answer an SAQ

SAQ 4

Spend 2 min.

Refer to Fig.1.24 which shows an acacia leaf. Name the symmetries it possesses.

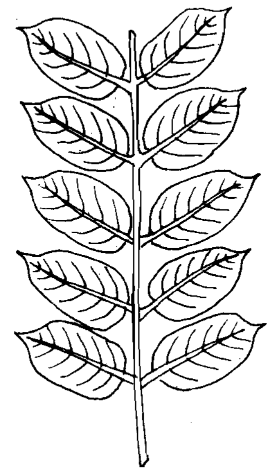


Fig.1.24: Acacia leaf

After revisiting the symmetries observed macroscopically in nature, let us look into the symmetries involved in the microscopic systems like molecules.

1.3.2 Symmetries in Molecules

You know that macroscopic systems can be understood through laws of classical physics. But to explain the behaviour of sub-atomic particles (atoms and molecules), we resort to quantum mechanics. From your earlier courses you may recall that microscopic world exhibits wonderful symmetry. For instance, symmetry manifests itself in ordered structure of electron orbitals in an atom. Symmetry of these orbitals, in turn, manifests in geometrically ordered atomic structure of molecules. By referring to Fig.1.25, you will note that both carbon dioxide and water molecules have a plane of reflection symmetry (the vertical line in figure). Even if you interchange the paired atoms, no change will be observed; this will amount to mirror reflection. Both these molecules exhibit 2-fold rotational symmetry also.

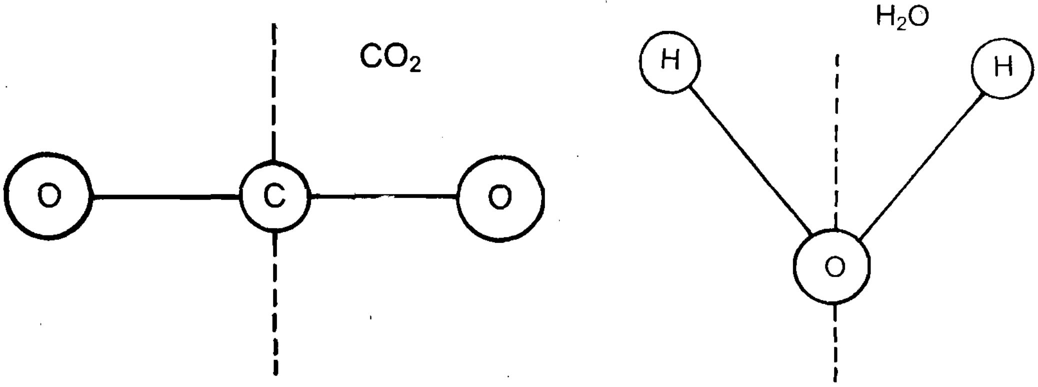


Fig.1.25: Symmetry in molecules of CO_2 and water

In methane (CH_4) molecule, carbon atom is covalently bonded to four identical hydrogen atoms. The spatial structure of CH_4 molecule is a tetrahedron. Four identical C–H bonds determine the spatial tetrahedral structure of methane; hydrogen atoms are at the corners and carbon atom is at the centre (Fig.1.26). Can you list the symmetry elements of a methane molecule? It has six planes of symmetry (each passing through the carbon atom and two H-atoms), four 3-fold rotation axes (each passing through the carbon atom and one of the H-atoms) and three 2-fold rotation axes (the mutually perpendicular Cartesian axes).

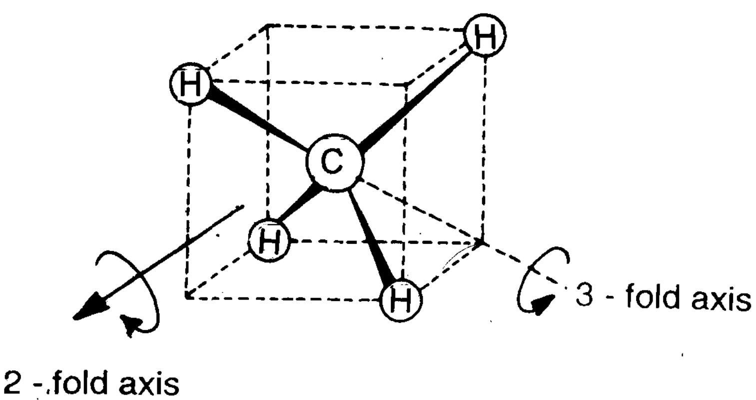


Fig.1.26: Symmetry in CH_4 molecule

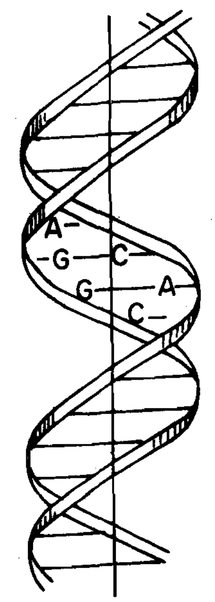


Fig.1.27: Helical structure of a DNA molecule

Living molecules that play an important role in biological processes are natural spirals. A protein molecule consisting of a large number of atoms of hydrogen, carbon, oxygen and nitrogen is a right-handed spiral called as alpha-spiral. Molecules of deoxyribonucleic acid (DNA), one of the most basic blocks of life known so far, have the structure of a double right-handed helix (Fig.1.27).

You may now like to answer an SAQ.

Spend 3 *min.*

SAQ 5

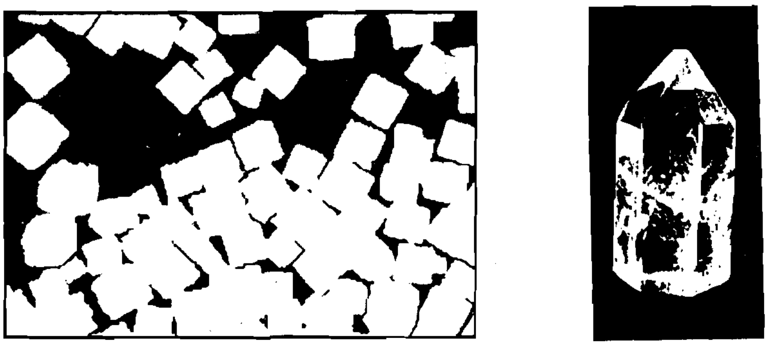
We replace one hydrogen atom with an OH-radical in methane. How will the symmetry be affected?

1.3.3 Symmetries in Solids

Matter around us exists in gaseous, liquid and solid states. In the gaseous state, molecules move randomly and each molecule continuously changes its position with respect to other molecules. The inter-atomic forces between the molecules are very weak. If temperature is lowered, the motion becomes less vigorous. At very low temperatures, the atoms may begin to condense and become more closely packed. Cohesive forces then come into play. In the liquid state, the molecules are free to roam around while in contact with the neighbours. As the temperature is further lowered, molecular motion is slowed down. At the freezing point, unrestricted thermal motion ceases and the molecules are held in position in an orderly 3-dimensional pattern.

You will appreciate that the distances we are dealing with here are in Å and the long-range order can refer to anything between a few 100Å to a few mm.

To an unaided eye, a solid appears as a continuous rigid body, notwithstanding the fact that all solids are composed of discrete units – atoms. The atoms within a solid have fixed locations, separated from each other by less than 10Å. Solids exist in two states: *crystalline and amorphous*. In a crystalline solid, perfect periodicity is obtained in all the three directions over long distances and a long-range order is said to exist in it. (In fact, this order is an important characteristic of the crystalline state.) However, when constituent units fail to obtain the configuration of minimum potential energy, a short-range order or no order sets in and the solid is said to be in amorphous state. You will learn the differences in details in the next unit, but you must note here that most solids are crystalline (either as a single crystal or as an aggregate of several small crystals). Common salt is the most familiar example of a crystalline solid. Fig.1.28a shows a magnified view of several NaCl pieces. Each piece exhibits a well-defined cubic shape with sharp edges. Note that it is an external expression of an orderly internal arrangement of atoms.



(a) (b)

Fig.1.28: (a) NaCl crystals; and (b) quartz crystal through a microscope

In Fig.1.28b we have shown optical micrograph of naturally grown crystal of quartz i.e. SiO_2, which is the principal ingredient of most sands. You will note that in both these examples, crystals exhibit a characteristic shape with several faces and sharp edges. Gem stones such as yellow topaz, blue sapphire, red ruby, green emerald and colourless diamond also possess perfect order. If you hold a polished crystal of ruby or diamond, you will observe lustre, smoothness and pleasing symmetry of arrangement with sharp edges. Due to regularity of faces (and angles), solid crystals can be identified as possessing some symmetry properties. In fact, most crystals display three basic elements of symmetry – symmetry planes, symmetry axes and symmetry centres – or their combination.

You can easily convince yourself that a study table, matchbox or a chair have reflection symmetry. Similarly, a cube has 4-fold rotational symmetry axis passing normally through the centres of any pair of its opposite faces. Any long diagonal of the cube is a 3-fold axis of symmetry. A brick has three 2-fold axes of symmetry passing through the centres of opposite faces. However, these symmetries have to be self-consistent due to finite sizes of these objects.

Let us now sum up what you have learnt in this unit.

1.4 SUMMARY

- A **symmetry operation** leaves the object invariant.
- **Reflection symmetry** in 2-D objects is defined with respect to an axis while in 3-D objects it is with respect to a plane.
- **Enantiomorphs** are objects having right and left-handed symmetries.
- **Inversion symmetry** leaves an object unaltered after inversion about a point within it.
- **Rotational symmetry** is defined around a point in 2-D and an axis in 3-D. An object is said to have n-fold symmetry if the rotation through an angle $(2\pi/n)$ leaves it invariant.
- **Translational symmetry** deals with displacement along a line, which leaves the object invariant.
- Many **natural objects** such as flowers, shapes of leaves, arrangement of leaves on the stem of a tree, butterflies, snowflakes exhibit symmetries.

1.5 TERMINAL QUESTIONS *Spend* 15 *min.*

1. List the symmetries observed in an ellipse.
2. List the symmetries observed in case of a writing pencil.
3. List at least two objects each that exhibit (a) only 1-fold rotational symmetry and (b) ∞-fold rotational symmetry.
4. Determine the symmetry observed in case of (a) ammonia and (b) benzene molecules.

1.6 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. With one horizontal mirror axis, we can write

   ~~COOK~~ ~~BOOK~~

2. The square and hexagon are inversion symmetric with centre of pattern as centre of inversion.

3. a) Translational symmetry of the parallelogram marked in the Fig.1.20a.

   b) Two-fold rotation axis, passing through *O* normal to plane, reflection symmetry with respect to axes *AB* and *CD* and translational symmetry of the unit cell marked in the Fig.1.29.

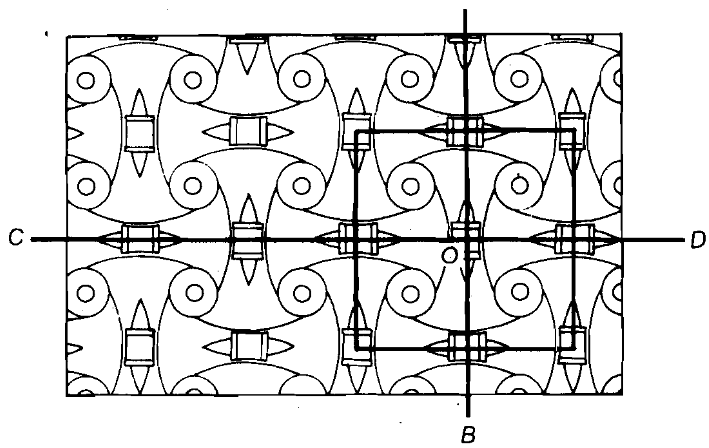


Fig.1.29: Symmetry elements in a pattern

4. Acacia leaf has mirror axis across its stem, 2-fold rotation symmetry around the stem and translational symmetry for every pair of leaflet.

5. When one H is replaced by OH-radical in the methane molecule, there is only one three-fold axis (passing through C and OH), there are 3 mirror planes (passing through C, OH and any one H).

Terminal Questions

1. As shown in Fig.1.30, an ellipse has 2 mirror reflection axes, *AB* and *CD*; centre of inversion *O*, and one 2-fold rotational axis passing through *O* and perpendicular to the plane of the paper.

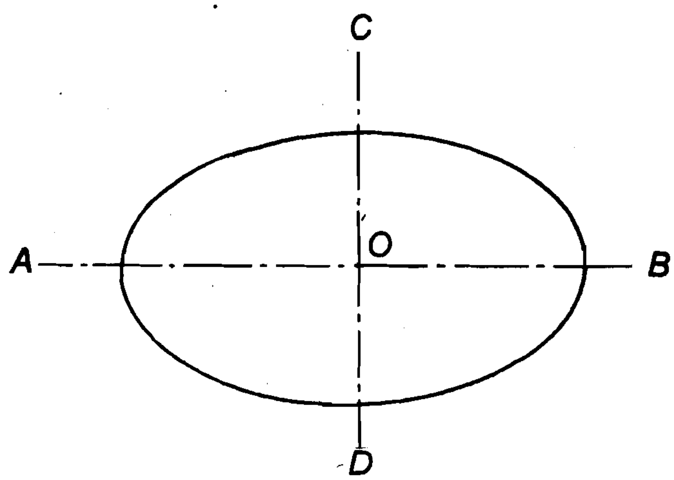


Fig.1.30: Symmetry elements of ellipse

2. One 6-fold rotational axis and six mirror planes; or one infinite fold rotational axis.

3. a) Any irregular shaped object like stone, clay cluster.

   b) Sphere, circle, cone, cylinder.

4. a) Ammonia has a 3-dimensional structure as shown in the Fig.1.31.

This molecule has (i) 3 fold rotational axis passing through N and centre of the triangle made by 3-H. (ii) 3 mirror planes each passing through N, one H and bisector of the line joining remaining 2 H.

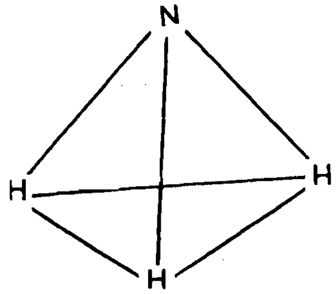


Fig.1.31: Structure of ammonia

(b) Benzene is a planar molecule with 6 carbons at the corners of the hexagon and 3 double bonds as shown in Fig.1.32

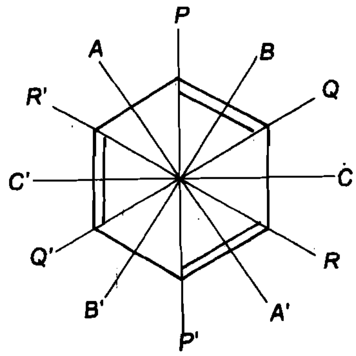


Fig.1.32: Structure of benzene

This possesses (i) 3-fold rotation axis passing through the centre of the molecule and normal to the molecular plane; (ii) three mirror planes (*AA'*, *BB'*, *CC'*) each perpendicular bisecting the opposite single and double bonds; (iii) three combinations of reflection and rotation symmetry planes (*PP'*, *QQ'*, *RR'*) passing through diagonally opposite corners and rotation around principal axis by 60°.