UNIT 12 RADIOACTIVITY

Structure

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12.1 INTRODUCTION

Towards the end of nineteenth century, physicists thought that the era of exciting discoveries in physics was over. However, the chance discovery of radioactivity by Becquerel in 1896 overcame this pessimism and opened the flood gates for new discoveries. This became possible because spontaneously emitted radiations — alpha, beta and gamma-rays — could be used as convenient tools to probe matter, For instance, the alpha- particles from radioactive nuclei were used by Rutherford to propose the nuclear model of atom. (You will learn about it in the next unit.) It further led to discoveries of artificial transmutation and production of transuranic elements as well as radioisotopes, which find wide use in medical diagnosis and therapy, research, agriculture, carbon dating of archaeological specimen etc. The studies of beta decay led lo the discovery of neutrino. In short, discovery of radioactivity acted as precursor of fundamental developments in nuclear physics in the early part of this century.

In Sec. 12.3 we have discussed the theory of radioactive decay. We have applied this theory to explain the growth and decay of radioactivity in a given radioactive sample in Sec. 12.4. Successive radioactive disintegrations and the condition of radioactive equilibrium amongst the different members of a radioactive series is discussed in Sec. 12.5.

Objectives

After going through this unit, you should be able lo

- identify the three types of radioactive radiations
- formulate the laws of radioactive transformation
- compute the half-life and disintegration constant of a radioactive substance
- explain the growth and decay of radioactivity in a given sample
- explain the radioactive equilibrium amongst the different members of a radioactive series and
- list different elements belonging to naturally occurring radioactive series.

12.2 DISCOVERY OF RADIOACTIVITY AND PRELIMINARY STUDIES

The story of discovery of radioactivity is very interesting. In 1896, Henri Becqucrel was working on the phenomenon of fluorescence in which certain substances emit visible light when they are exposed to ultra-violetradiations, say from the sun. In one of the drawers of his desk, Henry Becquerel had kept a collection of various minerals, which also included uranium salts, along with several cardboard boxes of photographic plates wrapped with thick black paper. A few days later he used one of the boxes of photographic plates. When he developed the plate, he was amazed to observe that it was heavily fogged. He tried other plates and found them also to be exposed. This puzzled him because all boxes were unused. Can you guess what had affected these photographic plates?

Becquerel conjectured very rightly that uranium salts, placed beside the photographic plates must have emitted some new type of radiation(s). After extensive experimentation, Becquerel also proved that these radiations, if passed through m e gas, make it conducting by virtue of their ionising power. The emission of ionising and penetrating radiation(s) from uranium was named radioactivity.

Following an exhaustive study, Madame Marie Curie found evidence of radioactivity in elements like thorium, polonium and radium as well. She also obtained convincing evidences that radioactivity is a nuclear property of the element concerned and remains unaffected by physical or chemical changes.

By studying the ionising and penetrating power of these radiations, Rutherford established the existence of two distinct components, a-rays and β-rays. α -rays are more easily absorbed in matter compared to β-rays but have comrelatively greater ionising power. However, the penetrating power of β-rays is about 100 times more than that of a-rays. The existence of the third component, called gamma rays, which are much more penetrating than the other two, was established by P. Villars. By subjecting these radiations to a magnetic field, it was also established that the gamma rays are electrically neutral, whereas the alpha-rays are positively charged and the beta-rays are negatively charged particles. We now know that a- rays can be identified with helium nuclei and β-rays with electrons. Moreover, α-particles affect photographic films and excite flourescence in many substances. Also, they produce phosphorescence and their speed is of the order of 11100th of the speed of light On the other hand, the speed of β-particles is about 1/10th of the speed of light. Gamma rays (γ) are very short wavelength electromagnetic radiations emitted from nuclei of radioactive substances and travel with speed of light. Further studies revealed that radioactive emanations have energies in the MeV range. To enable you to comprehend these concepts, we give below some examples of radioactive decays which involve emission of α and β-rays:

$$^{238}U \rightarrow {}^{234}Th + a \qquad (4.20\ MeV)$$

$$^{226}Ra \rightarrow {}^{222}Rn + a \qquad (4.78\ MeV)$$

$$^{60}Co \rightarrow {}^{60}Ni + \beta^- \qquad (0.318\ MeV)$$

$$^{90}Sr \rightarrow {}^{90}Y + \beta^- \qquad (0.546\ MeV)$$

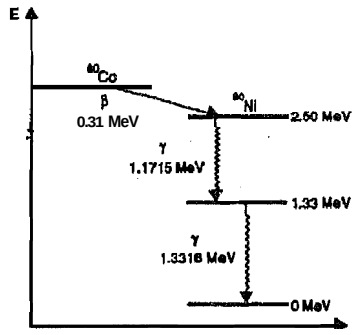

Fig.12.1 : Energy-level diagram for the ^{60}Ni nucleus formed in the decay $^{60}Co \rightarrow {}^{60}Ni + \beta^-$

(The β-decay is accompanied by the emission of neutrinos. But these are very difficult Lo detect and we will discuss about'them later.) You will agree that all these reactions involve transmutation of one element'to another: uranium decays to thorium, radium to ruthenium, cobalt to nickel and strontium to ytrium. In many cases of a and β decays, the daughter nucleus emerges in an excited state and subsequently undergoes a transition to a lower/ground state by emitting a γ-ray. In Fig. 12.1 we have shown the energy-level diagram for the ^{60}Ni nuclei formed in the β-decay of ^{60}Co. A nucleus in an excited state can also make a transition to a lower state by transferring its excitation energy to an atomic electmn. Such an energy transfer is called internal conversion. The electron gaining this energy gets ejected from the atom as a β-ray.

12.3 RADIOACTIVE DECAY

You now know that when a radioactive element disintegrates spontaneously, its nucleus emits either an a or a β-particle. The nucleus of new element formed could also be unstable. The first decay is then succeeded by another and another, ... , resulting in a sequential series. Refer to the decay of ^{238}U shown below:

$$^{238}U \xrightarrow{\alpha} {}^{234}Th \xrightarrow{\beta} {}^{234}Pa \xrightarrow{\beta} {}^{234}U \xrightarrow{\alpha} {}^{230}Th \xrightarrow{\alpha} {}^{226}Ra \ldots$$

The decays continue until a stable isotope is reached. In this series, the final stable decay product is lead. ^{238}U is called the parent nucleus and the resultant nucleus ^{234}Th is called the daughter nucleus. You may now like to know other such series. We have four such series in all: thorium series, neptunium series, uranium series and actinium series. The mass numbers of the members of these series are given by $4n, 4n+1, 4n+2, 4n+3$ respectively, where n is an integer.

In all the radioactive series, a parent radioactive element of large atomic number and very long half-life gives rise to a series of radioactive elements as a result of successive emissions of a or β-particles. We can express the transformations by expressions of the form:

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$$

$$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}Q + \beta^{-}$$

The decay products may themselves be radioactive and decay by emitting a or β-particles. The successive radioactive transformations continue, until we reach a stable isotope of lead.

The members of the uranium series are listed in Table 12.1, together with the half-life and mode of disintegration. Table 12.2 gives the corresponding information about the actinium series, while Table 12.3 refers to the thorium series. In the uranium series, RaA, RaC and RaF all have atomic number 84 and are isotopes of polonium. Similarly, RaB, RaD and RaG all have atomic number 82 and are isotopes of lead.

Table 12.1: The Uranium Series ($A = 4n + 2$)

Radioactive species	Chemical symbol	Z	A	Half-life	Particles emitted
Uranium I	UI	92	238	4.5×10^{9} yr	a
Uranium X_1	UX_1	90	234	24.1d	β
Uranium X_2	UX_2	91	234	1.18 m	β
Uranium Z	UZ	91	234	6.7 h	β
Uranium II	UII	92	234	2.5×10^{5} yr	a
Ionium	Io	90	230	8.0×10^{4} yr	a
Radium	Ra	88	226	1620 yr	α
Radon	Rn	86	222	3.82 d	a
Radium A	RaA	84	218	3.05 min	α, β
Radium B	RaB	82	214	26.8 min	β
Asiatine-218	At-218	85	218	2 s	a
Radium C	RaC	83	214	19.7 min	β, α
Radium C'	RaC'	84	214	1.64×10^{-4} s	a
Radium C''	RaC''	81	210	1.32 min	β
Radium D	RaD	82	210	19.4 yr	β
Radium E	RaE	83	210	5.0 d	β
Radium F	RaF	84	210	138.3 d	a
Radium G	RaG	82	206	—	stable

—UX_1 exhibits a branching effect; 99.65 percent of UX_1 atoms emit β-particles to form UX_2, and 0.35 percent of the UX_1 atoms emit β-particles to form UZ. Both UX_2 and UZ have the same mass number 234 and the same atomic number 91, but their nuclear energy levels are different Such pairs of radioactive species are known as nuclear isomers.

We note that a few isotopes in each of the radioactive series have two alternative modes of decay: these nuclides decay both by a-emission and by β- emission. This type of disintegration is called branching decay and is characterised by a definite branching ratio for each mode of decay. For example, we find branching decays for RaA and RaC in the uranium series, for Ac, AcA and AcC in the actinium series, and for ThA and ThC in the thorium series. In most cases, one mode of decay is more probable than the other. Thus RaA and AcA decay almost entirely by α-emission (branching ratio > 99%); only a small fraction (< 1%) of the atoms disintegrate by β-emission. On the other hand, RaC, Ac and AcC decay almost entirely by β-emission, with < 1% of the atoms decaying by a-emission, The only exception is ThC, which has a branching ratio 66.3% for a -disintegration and 33.7% for β-disintegration. The neptunium series starts with plutonium (Pu) and the final stable end product is an isotope of bismuth (Bi).

Table 12.2 : The Actinium Series ($A = 4n + 3$)

Radioactive species	Symbol	Z	A	Half-life	Particles emitted
Actinouranium	AcU	92	235	7.1 x 10^8y	α
Uranium	Y W	90	231	25.6 h	β
Protoactinium	Pa	91	231	3.4 x 10^4yr	α
Actinium	Ac	89	227	22yr	β, α
Radioactinium	RdAc	90	227	18.2d	α
Actinium K	AcK	87	223	22min	β, α
Actinium X	AcX	88	223	11.63 d	α
Actatin 219	At 219	85	219	0.9 min	α, β
Actinon	An	86	219	3.92 s	α
Bismuth 215	Bi-215	83	215	8 min	β
Actinium A	AcA	84	215	1.83 x 10^{-3}s	a, β
Actinium B	AcB	82	211	36.1 min	β
Astatine 215	At 215	85	215	10^{-4} s	a
Actinium C	AcC	83	211	2.16 min	β, α
Actinium C′	AcC′	84	211	0.52s	a
Actinium C″	AcC″	81	207	4.8 min	β
Actinium D	AcD	82	207	—	stable

Table 123 : The Thorium Series (A = 4 n)

Radioactive species	Symbol	Z	A	Half-life	Particles emitted
Thorium	Th	90	232	1.39 x 10^{10}yr	α
Mesothorium I	$MsTh_1$	88	228	6.7 yr	β
Mesothorium II	$MsTh_2$	89	228	6.13 h	β
Radiothorium	RdTh	90	228	1.9 yr	a
Thorium X	ThX	88	224	3.64 d	a
Thoron	Th	86	220	51.5s	a
Thorium A	ThA	84	216	0.16 s	α, β
Thorium B	ThB	82	212	10.6 h	β
Astatine-216	At-216	85	216	3×10^{-4} s	a
Thorium C	ThC	83	212	60.5 min	β, α
Thorium C	ThC′	84	212	3.0 x 10^{-7} s	α
Thorium C″	ThC″	81	208	3.1 min	β
Thorium D	ThD	82	208	—	stable

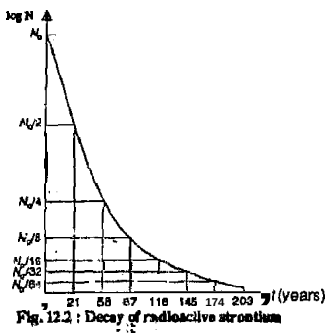

Fig. 12.2 : Decay of radioactive strontium as a function of time

We now know that if we have a given amount of radioisotope, it will gradually decrease with time. Measurements show that the quantitative law describing the decay process is very simple. To understand this, refer to Fig. 12.2. It shows a plot of amount of radioactive strontium (on the logarithmic scale) as a function of time. You will note that

- it takes 29 years for one-half of the initial amount of strontium to decay
- during the next 29 years, one-half of the remainder strontium decays, i.e. we are left with $\frac{1}{2}\left(\frac{N_0}{2}\right) = \frac{N_0}{4}$ nuclei.

Hence, the fraction of parent radioactive substance (left) after 29, 58, 87 years will be

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ of the initial amount. Do you recognise this series? It forms a geometric progression with $r = \frac{1}{2}$. So if $N(t)$ denotes the number of strontium nuclei surviving decay at time t and N_0 is the number at $t = 0$, then we can write

$$N(t) = N_0 \left(\frac{1}{2}\right)^{t/(29\ \text{yr})} \qquad (12.1)$$

The time required for one-half of the parent nuclei (mat.) to decay is called its **half-life**. We will denote it by the symbol $T_{1/2}$. The half-lives of some important radioisotopes are given in Table 12.4. You will note that values of $T_{1/2}$ show a very wide range; from 4.5×10^9 yr for ^{238}U to 3×10^{-9}s for ^{212}Po.

Table 12A : Half-lives of some radioisotopes

Radioisotope	$T_{1/2}$
^{14}C	5730 yr
^{40}K	1.3×10^9 yr
^{60}Co	5.24 yr
^{90}Sr	28.8 yr
^{131}I	8.05 d
^{212}Po	3×10^{-9}s
^{238}U	4.5×10^9 yr

In terms of the half-life, we can rewrite Eq. (12.1) as

$$N(t) = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}} \qquad (12.2)$$

Let us pause for a while and ask: What is the physical implication of this result? This relation tells us how the given quantity of a radioactive sample disintegrates as time passes. You may also like to know: Does this formula *hold* only for times $t = 0, T_{1/2}, 2T_{1/2}, \ldots$ etc.? *The* answer to this question is: Eq. *(12.2)* holds for all times.

It may be emphasized here that the general appearance of the decay curve is essentially the same for all radioactive elements. However, each element takes its own characteristic time to decay.

Using the identity $2 = e^{\ln 2} \cong e^{0.693}$, we can rewrite Eq. *(12.2)* into a more convenient form as

$$N(t) = N_0 \exp(-\ln 2\, t/T_{1/2})$$
$$= N_0 \exp(-t/\tau) \qquad (12.3)$$

where $\tau = \frac{T_{1/2}}{\ln 2}$.

Eq. (12.3) expresses the law of radioactive decay mathematically. It shows that the number of atoms in a radioactive sample decreases exponentially with time with a characteristic time constant τ. Let us now find out the number of nuclei decaying in any time interval dt. From Eq. *(12.3)* it readily follows that

$$dN = -\frac{N_0}{\tau} e^{-t/\tau}\, dt$$

The negative sign signifies that the number of nuclei decreases with time due to their continuous disintegration.

By definition, the average life time, $\bar{t}$, is

$$\bar{t} = \frac{\int_0^\infty t\,|dN|}{\int_0^\infty |dN|} = \frac{1}{N_0}\int_0^\infty t\,|dN|$$

$$= \frac{1}{N_0}\int_0^\infty \frac{t}{\tau} N_0\, e^{-t/\tau}\,dt$$

$$= \frac{1}{\tau}\int_0^\infty t\, e^{-t/\tau}\,dt$$

Using the method of integration by parts, we obtain

$$\bar{t} = \frac{1}{\tau}\left[-\tau e^{-t/\tau}\Big|_0^\infty + \tau\int_0^\infty e^{-t/\tau}\,dt\right]$$

The first term vanishes at both the limits. Therefore, this expression simplifies to

$$\bar{t} = \int_0^\infty e^{-t/\tau}\,dt = \left[-\tau\, e^{-t/\tau}\right]_0^\infty \qquad (12.4)$$

That is, τ is mean life of radioactive nuclei. It may be pointed out here that like $T_{1/2}$, τ will also vary over a very wide range. The measurement techniques over such a wide spread of time are bound to differ vastly. But the law of radioactive decay is common to all radioactive decay processes.

We will now illustrate these concepts through solved examples.

Example 1

The half-life of radon is 3.8 days. After how many days will only 5% of radon be left over?

Solution

We know that $T_{1/2}$ = 3.8 days. Therefore

$$\tau = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693} = \frac{3.8 \text{ days}}{0.693} = 5.48 \text{ days}$$

We are required to calculate the number of days in which only 5% radon is left. Therefore, we can write $\frac{N}{N_0} = 0.05$.

From Eq. (12.3) it readily follows that

$$\frac{N}{N_0} = 0.05 = \exp\left(-\frac{t}{5.48 \text{ days}}\right)$$

We can write it as

$$\exp\left(\frac{t}{5.48 \text{ days}}\right) = 20$$

Taking natural logarithm of both sides, we find that

$$\frac{t}{5.48 \text{ days}} = \ln 20 = 2.303 \log_{10} 20 = 2{,}303 \times 1.3010 = 2.9\%$$

Hence

$$t = 2.9\% \times 5.48 \text{ days} = 16.42 \text{ days}$$

Example 2

Due to accident in a research laboratory a radioactive element got spread inside a room. As a result, the level of radiation became 50 times the permissible level for normal occupancy of the room. After how many days the room would be safe for occupation? The half-life of the radioactive substance is 30 days.

Solution

Here $\frac{N}{N_0} = \frac{1}{50}$ and $T_{1/2} = 30$ days

so that $\tau = \frac{T_{1/2}}{0.693} = \frac{30 \text{ days}}{0.693} = 43.3$ days

Hence $\frac{N}{N_0} = \frac{1}{50} = \exp(-t/43.3 \text{ days})$. This can be rearranged to give

$$\frac{t}{43.3 \text{ days}} = \ln 50 = 3.912$$

and

$$t = 3.912 \times 43.3 \text{ days} = 169.4 \text{ days}$$

Example 3

A sample of pitchblende has a lead-uranium weight ratio of 9/40. Calculate the age of the mineral. The half-life of uranium is 4.5×10^9 yr. The atomic weights of lead and uranium are 206.0 and 238.4, respectively. .

Solution

Since the weight ratio of lead to uranium is 9/40, we can say that if there were 9 kg of lead, the amount of uranium would be 40 kg. The number of atoms in 9 kg of lead $= \frac{9}{206} \times 6 \times 10^{26} = 0.262 \times 10^{26}$ atoms. Similarly, the number of atoms in 40 kg of uranium $= \frac{40}{238.4} \times 6 \times 10^{26} = 1.007 \times 10^{26}$.

$\therefore$ Total number of uranium atoms in the beginning $= 1.269 \times 10^{26}$ atoms

Since $T_{1/2} = 4.5 \times 10^9$ yr, $\tau = \frac{T_{1/2}}{\ln 2} = \frac{4.5 \times 10^9 \text{ yr}}{0.693} = 6.494 \times 10^9$ yr

From Eq. (12.3) we know that

$$N = N_0 e^{-t/\tau}$$

so that

$$\frac{t}{\tau} = \ln\left(\frac{N_0}{N}\right)$$

or

$$t = \tau \ln\left(\frac{N_0}{N}\right)$$

$$= (6.494 \times 10^9 \text{ yr}) \ln$$

$$= (6.494 \times 10^9 \text{ yr}) \times 0.2296$$

$$= 1.49 \times 10^9 \text{ yr}$$

You may now like to solve an SAQ.

SAQ 1

Spend 10 min

The mean life of a radioactive element is 14.43 months. Calculate the time required for 75% of the element to decay.

In practice, we are more interested in the decay rate of the material rather than its amount, since it determines the rate of emission of α, β or γ-rays. (Moreover, this information can also be used to estimate the age of any specimen.) To this end, we note from Eq. (12.3) that

$$\frac{dN}{dt} = -\frac{N_0}{\tau} \exp(-t/\tau) = -\frac{N}{\tau} \qquad (12.5)$$

That is, the decay rate at any time is proportional to the amount of radioactive material present at that instant.

Alternatively, we can write Eq. (12.5) as

$$\frac{dN}{dt} = -\lambda N \tag{12.6}$$

where λ is constant of proportionality and is called ***decay constant.*** It is characteristic of a particular radioactive element or decay process. In terms of λ, the law of radioactive decay can be expressed as

$$N = N_0 \exp(-\lambda t) \tag{12.7}$$

If you compare Eqs. (12.3) and (12.7), you will find that $\lambda = 1/\tau$.

The activity A of a given radioactive substance is defined as the number of atoms disintegrating per unit time. Mathematically

$$A = \left| \frac{dN}{dt} \right| \tag{12.8}$$

From Eqs. (12.6) and (12.8), we have

$$A = \lambda N \tag{12.9}$$

i.e., the activity of a given radioactive substance is directly proportional to the number of radioactive atoms present. If A_0 is the initial activity of the source at $t = 0$, then we have

$$\frac{A}{A_0} = \frac{N}{N_0} = e^{-\lambda t} \tag{12.10}$$

The quantity $\frac{A}{A_0}$ is defined as the relative activity and is a measure of the radioactivity of a given source. From this result you will note that even the relative activity of a given radioactive substance decays exponentially with time.

The most natural way to express decay rate is in disintegrations per second. But the activities encountered in practice are usually so high that a larger unit, the curie, abbreviated as Ci, is more often used. Initially the curie was defined as the activity of 1 g of radium but its value kept changing as improvements in measuring techniques were accomplished. The curie is now defined as

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ disintegrations per second}$$

In the SI system, decay rate is expressed in becquerel:

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ disintegration per second}$$

You may have seen children with swollen thyroid gland. Do you know that to scan the thyroid, radioisotope ^{131}I is used? Let us now calculate its decay rate.

The number of nuclei in 1 g of ^{131}I is

$$1\text{g} \times \frac{1 \text{ mol}}{131 \text{ g}} \times 6.023 \times 10^{23} \text{ nuclei mol}^{-1} = 4.6 \times 10^{21}$$

Hence, the decay rate

$$-\frac{dN}{dt} = \frac{N}{\tau} = \frac{\ln 2}{T_{1/2}} N$$

From Table 12.4 you would note that $T_{1/2}(^{131}I) = 8.05$ days.

$$\therefore \quad -\frac{dN}{dt} = \frac{0.693}{305 \text{ days}} (4.6 \times 10^{21} \text{ atoms})$$

$$= \frac{3.188 \times 10^{21} \text{ atoms}}{6.955 \times 10^{5} \text{ s}}$$

$$= 4.58 \times 10^{15} \text{ disintegrations per second}$$

Expressing this in curies, we find that

$$-\frac{dN}{dt} = (4.58 \times 10^{15} \text{ disintegrations per second}) \times \frac{1 \text{ Ci}}{(3.7 \times 10^{10} \text{ disintegrations per second})}$$

$$= 1.24 \times 10^{5} \text{ Ci}$$

This is an extremely large disintegration rate. The amount of ^{131}I injected into the human body is only 10^{-9} g so that the decay rate is nearly 10^{-4} Ci, which is well below the safely limit of

We will now illustrate how a knowledge of decay rate enables us to estimate the age af specimen using the carbon dating technique.

Example 4

^{14}C isotope of carbon is used for radioactive dating of organic materials. Samples of fresh carbon from trees in equilibrium with the CO_2 of the atmosphere have an abundance of 98.89% ^{12}C, 1.11% ^{13}C and 1.3×10^{-12} % ^{14}C. After a tree dies, the abundance of ^{12}C and ^{13}C in the wood does not change but the abundance of ^{14}C decreases because of radioactive decay. Apiece of wood is taken from an Egyptian tomb. Each gram of carbon exhibits an activity of 3.9×10^{-12} Ci. Estimate the age of the wood.

Solution

The number of nuclei in 1 g of carbon is $1 \text{ g} \times \frac{1 \text{ mol}}{12 \text{ g}} \times 6.02 \times 10^{23} \text{ nuclei mol}^{-1} = 5.02 \times 10^{22}$ nuclei. Thus, 1 g of fresh carbon should contain $5.02 \times 10^{22} \times 1.3 \times 10^{-12} = 6.53 \times 10^{10}$ nuclei of ^{14}C. From Eq. (12.4) the activity at $t = 0$ is given by

$$A_0 = \left| \frac{dN}{dt} \right|_{t=0} = \frac{\ln 2}{T_{1/2}} N_0$$

From Table 12.4 we note that $T_{1/2}$ (^{14}C) is 5730 yr. Hence

$$A_0 = \frac{0.593}{(5730 \times 365 \times 24 \times 3600 \text{ s})} \times (6.53 \times 10^{10} \text{ nuclei})$$

$$= \frac{4.53 \times 10^{10} \text{ nuclei}}{1.807 \times 10^{11} \text{ s}} = 0.251 \text{ disintegrations per second}$$

To express it in curies we note that

$$A_0 = \frac{(0.251 \text{ disintegrations per second}) \times 1 \text{ Ci}}{3.7 \times 10^{10} \text{ disintegrations per second}}$$

$$= 6.78 \times 10^{-12} \text{ Ci}$$

We are told that the measured activity of the sample is 3.9×10^{-12} Ci. This obviously is smaller than the initial activity by a factor of $\frac{3.9 \times 10^{-12} \text{ Ci}}{6.78 \times 10^{-12} \text{ Ci}} = 0.575$

Since the activity is proportional to the amount of radioactive material, from Eq. (12.10) we recall that

$$\frac{A}{A_0} = \frac{N}{N_0} = e^{-(\ln 2)t/T_{1/2}} = 0.575$$

so that by taking logarithm of both sides we find that

$$t = -\frac{T_{1/2}}{\ln 2} \ln (0.575)$$

$$= -\frac{5730 \text{ yr}}{0.693} \times (-0.553)$$

$$= 4572.4 \text{ yr}$$

Spend 10 min.

SAQ 2

The half-life of ^{238}U is known to be 4.51 x 10^9 yr. Compute the disintegration constant (in s^{-1}). Also calculate the number of disintegrations per second from 1 g of uranium. Take Avogadro's number=6.03×10^{23}.

12.4 GROWTH AND DECAY OF RADIOACTIVITY

You now know that naturally occurring radioactive elements disintegrate continuously and their decay is governed by a characteristic decay constant. Physically it means that the number of atoms (of parent nuclei) in a radioactive sample will decrease steadily (according to Eqs. (12.3) and (12.7)). However, if the daughter nuclei happen to be radioactive, their (daughter nuclei) activity will also start to build up with time (and may compensate for the loss of parent nuclei). But this cannot go on indefinitely. Let us now find out more about the observed growth and decay of the activities of the daughter nuclei.

Let us reconsider the decay of ^{238}U. We now know that it decays by emitting an a-particle with a half-life of 4.5×10^9 yr. But ^{234}Th atoms disintegrate by emitting β-particles with a half-life of only 24.1 days. This means that in the original sample of uranium, ^{234}Th atoms will disintegrate at a much faster rate and all the apparent activity of the sample will be practically due to them. However, if ^{234}Th is separated from the uranium, its activity will decay exponentially and reach half its initial value in 24.1 days.

To know how ^{234}Th grows in a freshly separated sample of uranium, we note that if at any instant of time t, the number of atoms of ^{238}U and ^{234}Th are N_U and N_{Th}, then the rate of disintegration of the parent element dN_U

$$\frac{dN_U}{dt} = -\lambda_U N_U$$

Physically it also means that ^{234}Th atoms are produced at the rate $\lambda_U N_U$, where λ_U is the decay constant of uranium. However, ^{234}Th atoms will disintegrate at a rate $\lambda_{Th} N_{Th}$, where N_{Th} is the number of ^{234}Th atoms present at time t and λ_{Th} is their characteristic decay constant Hence, the net rate of increase of ^{234}Th atoms in uranium is given by

$$\frac{dN_{th}}{dt} = \lambda_U N_U - \lambda_{Th} N_{Th}$$

or

$$\frac{dN_{th}}{dt} + \lambda_{Th} N_{Th} = \lambda_U N_U \qquad (12.11)$$

To solve this ODE for N_{Th}, we have to convert its left hand side into an exact differential. We can do so by multiplying throughout by exp ($\lambda_{Th}t$), which acts as integrating factor. This gives

$$\exp(\lambda_{Th}t)\frac{dN_{th}}{dt} + \lambda_{Th}N_{Th} \exp(\lambda_{Th}t) = \lambda_U N_U \exp(\lambda_{Th}t)$$

so that

$$\frac{d}{dt}[N_{Th} \exp(\lambda_{Th}t)] = \lambda_U N_U \exp(\lambda_{Th}t)$$

Integrating we obtain

$$N_{Th} \exp(\lambda_{Th}t) = \frac{\lambda_{th}}{\lambda_{th}} N_U \exp(\lambda_{Th}t) + K$$

or $$N_{Th} = \frac{\lambda_U}{\lambda_{Th}} N_U + K \exp(-\lambda_{Th} t) \quad (12.12)$$

where K is a constant of integration. To evaluate it, we note that at $t = 0$, i.e. in a freshly prepared uranium sample $N_{Th} = 0$. Hence, it readily follows that $K = -\frac{\lambda_U}{\lambda_{th}} N_U$. Therefore Eq. (12.12) takes a compact form:

$$N_{Th} = \frac{\lambda_U}{\lambda_{th}} N_U [1 - \exp(-\lambda_{Th} t) \quad (12.13a)$$

This result shows that in the separated uranium fraction, the number of ^{234}Th atoms ultimately reaches a constant equilibrium value of $N_{Th}^{\infty} (\lambda_U / \lambda_{Th}) N_U$. On combining this result with Eq. (12.13a) we can write

$$N_{Th} = N_{Th}^{\infty} [1 - \exp(-\lambda_{Th} t)] \quad (12.13b)$$

The growth of ^{234}Th are shown in Fig.12.3.

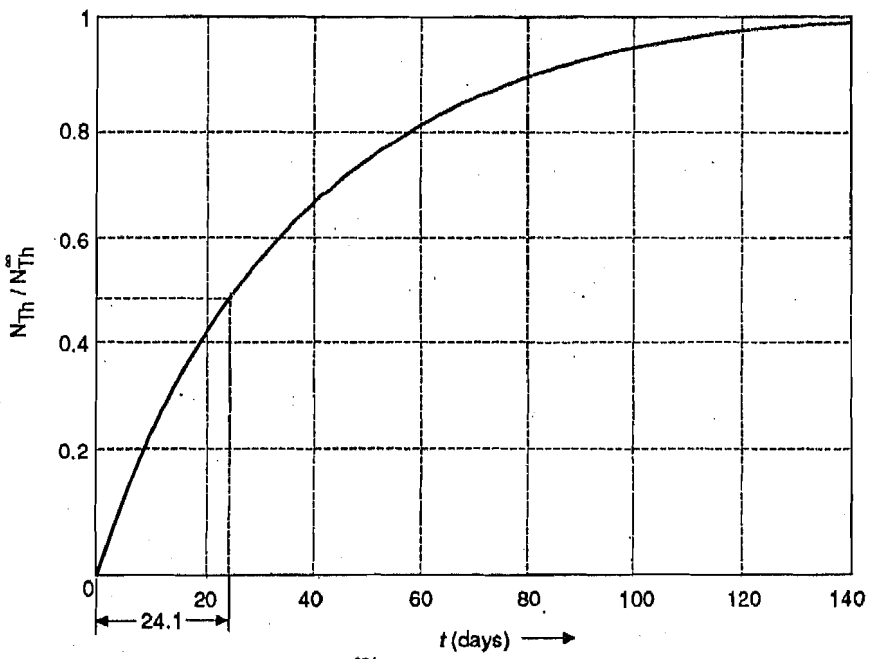

Fig. 12.3 : Growth of ^{234}Th in a freshly prepared uranium sample

12.5 SUCCESSIVE RADIOACTIVE TRANSFORMATIONS

In the preceding section we considered growth and decay of radioactivity when the parent nuclei decay into daughter nuclei, which decay in turn. You now know that naturally occurring radioactive elements undergo successive decays till a stable clement is produced. While studying a particular radioactive series, we often find it necessary to work out the number of atoms of each member of the series at a specified time. We can express the problem as follows:

Suppose that initially we have N_{A0} atoms of a parent element A. It decays to an clement B which in turn decays to C and so on. If λ_A, λ_B, λ_C,... are the respective disintegration constants of A, B, C, ... let us derive expressions for the number of atoms N_A, N_B, N_C at rime t. From Eq. (12.7) we note that the number of atoms of A present at time t is given by

$$N_A = N_{A0} \exp(-\lambda_A t) \quad (12.14)$$

The atoms of element B are produced at the rate $\lambda_A N_A$ and decay at the rate $\lambda_B N_B$. Hence, the net rate at which the number of atoms of B grows is given by

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \tag{12.15}$$

For element C, we can similarly write

$$\frac{dN_C}{dt} = \lambda_B N_B - \lambda_C N_C \tag{12.16}$$

On substituting for N_A from Eq. (12.14) in Eq. (12.15) and rearranging terms in the resultant expression, we get

$$\frac{dN_B}{dt} + \lambda_B N_B = \lambda_A N_{A0} \exp(-\lambda_A t) \tag{12.17}$$

Multiplying both sides by $\exp(\lambda_B t)$ and following the steps outlined in the previous section, you can easily show that (SAQ 3)

$$N_B = \frac{\lambda_A N_{A0}}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) \tag{12.18}$$

Spend 10 min

SAQ 3

Starting from Eq. (12.17), derive Eq. (12.18) using the condition that initially only the parent element was present, i.e. $N_B = 0$ at $t = 0$.

If you now substitute this expression for N_B in Eq. (12.16) and multiply throughout by $\exp(kt)$, you can rearrange the resultant expression as

$$\frac{d}{dt}\exp(kt) = \frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A} N_{A0}\left[e^{(\lambda_C - \lambda_A)t} - e^{(\lambda_C - \lambda_B)t}\right]$$

This can readily be integrated to yield

$$\exp(\lambda_C t) N_C = \frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A} N_{A0}\left[\frac{e^{(\lambda_C - \lambda_A)t}}{(\lambda_C - \lambda_A)} - \frac{e^{(\lambda_C - \lambda_B)t}}{(\lambda_C - \lambda_B)}\right] + K$$

$$N_C = \frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A} N_{A0}\left[\frac{e^{-\lambda_A t}}{(\lambda_C - \lambda_A)} - \frac{e^{-\lambda_B t}}{(\lambda_C - \lambda_B)}\right] + K \exp(-\lambda_C t) \tag{12.19}$$

where K is constant of integration. To evaluate it, we use the fact that at $t = 0$, $N_C = 0$. This gives

$$K = -\frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A} N_{A0}\left[\frac{1}{(\lambda_C - \lambda_A)} - \frac{1}{(\lambda_C - \lambda_B)}\right]$$

$$= \frac{\lambda_A \lambda_B N_{A0}}{(\lambda_C - \lambda_A)(\lambda_C - \lambda_B)}$$

Hence

$$N_C = \lambda_A \lambda_B N_{A0}\left[\frac{\exp(-\lambda_A t)}{(\lambda_B - \lambda_A)(\lambda_C - \lambda_A)} + \frac{\exp(-\lambda_B t)}{(\lambda_C - \lambda_B)(\lambda_A - \lambda_B)} + \frac{\exp(-\lambda_C t)}{(\lambda_A - \lambda_C)(\lambda_B - \lambda_C)}\right] \tag{12.20}$$

In a compact form, we can rewrite it as

$$N_C = N_{A0}\left(a_1 e^{-\lambda_A t} + a_2 e^{-\lambda_B t} + a_3 e^{-\lambda_C t}\right) \tag{12.21}$$

where $a_1 = \dfrac{\lambda_A \lambda_B}{(\lambda_B - \lambda_A)(\lambda_C - \lambda_A)}$

$$a_2 = \frac{\lambda_A \lambda_B}{(\lambda_A - \lambda_B)(\lambda_C - \lambda_B)}$$

and

$$a_3 = \frac{\lambda_A \lambda_B}{(\lambda_A - \lambda_C)(\lambda_B - \lambda_C)} \qquad (12.22)$$

You can now easily extend the procedure outlined above to a chain of radioactive elements. We leave this as an exercise for you.

Let us apply these equations to the specific case of successive radioactive transformations. If a metal wire is exposed for a few seconds to radioactive radon gas, we obtain a deposit of RaA, a decay product of radon. RaA has a half-life of 3.05 min and decays to RaB, whose half-life is 27 min. RaC decays to RaD with a half-life of 20 min. RaD has a half-life of 22 years and for d l practical purposes during the experiment the number of RaD atoms may be taken to be constant.

The number of RaA, RaB, RaC and RaD atoms as functions of time are shown in Fig. 12.4. The number of RaA atoms, initially assumed to be 100 decreases exponentially with time and reaches a value 50 after 3.05 rnin. At time t = 0, there are no atoms of RaB, RaC and RaD present. However, the number of RaB atoms increases with time, passes through a maximum about 11 min later and then decreases with time. The number of RaC atoms passes through a maximum after about 35 min. The number of RaD atoms increases

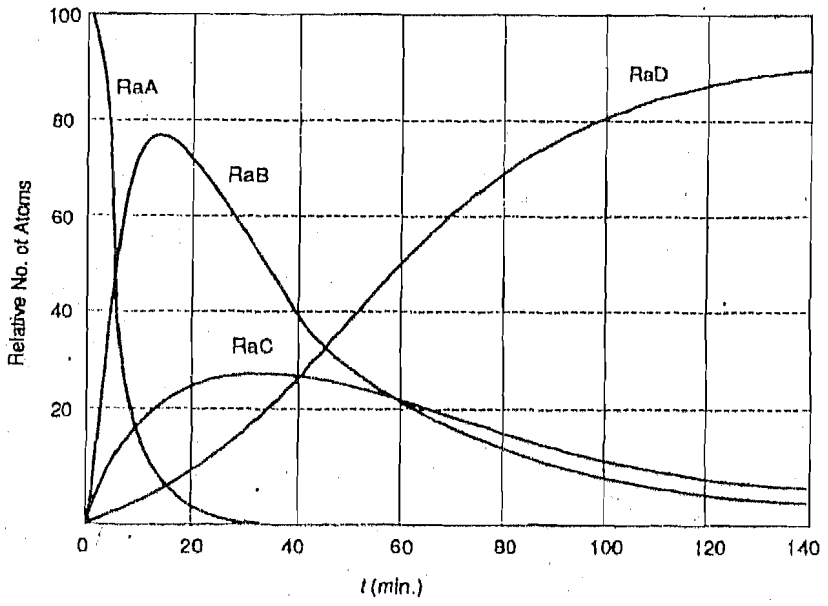

Fig. 12.4 : Variation of the relative numbers of atoms of RaA, RaB, RaC and Ran with time in a radioactive sample.

continually and reaches a maximum when RaB and RaC hove practically disappeared. Eventually, the RaD will decay exponentially with a half-life of 22 years.

SAQA **Spend 10 min**

In the above example, calculate the time at which the number of RaB atoms is a maximum.

12.5.1 Radioactive Equilibrium

You now know that different radonctive elements, either naturally occurring or produced in a decay chain, have their characteristic half-lives. The law governing successive disintegrations deals with the quantity of any given radioactive isotope present at any time. Depending upon the relative magnitudes of the half lives of various nuclei in a decay chain, we may obtain a situation where the number of parent and/or daughter atoms either remain constant or bear a constant ratio. This is blown as radioactive equilibrium. There are two possibilities. We will discuss these in turn.

Again refer to the decay of ^{238}U. You would recall that it is an extremely long-lived nuclei compared to the atoms of its daughter element ^{234}Th ($\lambda_U << \lambda_{Th}$). Let us assume chat the initial sample is pure, i.e, has only ^{238}U. Then Eqs. (12.7) and (32.13) suggest that,

$$N_U \cong N_{U0} \qquad (12.23)$$

and $$N_{Th} = N_{U0} \lambda_{Th} [1 - \exp(-\lambda_{Th} t)] \quad (12.24)$$

since $\exp(-\lambda_U t) \cong 1$.

Let us pause for a while and think as to what have we achieved so far. Eqs. (12.23) and (12.24) suggest that though the number of uranium (parent) nuclei remain essentially constant, the number of daughter atoms increases exponentially with time. After a time t much greater than the half-life of daughter nuclei, $\exp(-\lambda_{Th} t)$ becomes negligibly small and N_{Th} reaches an equilibrium value of

$$N_{Th} = N_U \frac{\lambda_U}{\lambda_{Th}}$$

or $$N_{Th}\, \lambda_{Th} = N_U\, \lambda_U \quad (12.25)$$

This result tells us that in the equilibrium state the rate of decay of the daughter atoms is equal to their rate of production implying that the number of parent and daughter atoms remains constant. Such a long-term equilibrium between the parent and daughter atoms is known as secular equilibrium. Such an equilibrium can be found in the formation of radon ($T_{1/2}$ = 5.5 days) from radium ($T_{1/2}$ = 2300 yr).

Spend 5 min

SAQ 5

Uranium minerals in which secular equilibrium has been obtained contain one atom of radium for every 2.8×10^6 atoms of uranium. If the half-life of radium is 1620 years, calculate the half-life of uranium.

You now know that in case of successive radioactive decays, secular equilibrium is obtained when parent nuclei are longer lived than those of the daughter elcmcnt. You may now like to know: What will happen when the parent is longer-lived than the daughter ($\lambda_A < \lambda_B$), but the half-life of the parent is not very long. That is, the half-life of the parent is greater than the daughter by a small factor. You may like to identify such a situation in the uranium as well as actinium series. In such a situation we cannot use the approximation $\exp(-\lambda_A t) = 1$. If the parent and daughter atoms are initially separated, the number of their atoms are respectively given by Eqs. (12.14) and (12.18). Moreover, if $\lambda_B t \gg 1$, then in Eq. (12.18), the term involving $e^{-\lambda_B t}$ becomes negligible compared with the term involving $e^{-\lambda_A t}$. Then the number of daughter atoms is given by

$$\frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B - \lambda_A} N_{A0} \exp(-\lambda_A t) \quad (12.26)$$

That is, the daughter nuclei eventually decay with the half-lifc of the parent. On combining this result with Eq. (12.14) we find that

$$\frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B - \lambda_A} = \text{constant} \quad (12.27)$$

In words, the ratio of the number of parent atoms and the number of daughter atoms attains a constant value. This constitutes what we term as transient equilibrium.

When the parent has a shorter half-life than the daughter ($\lambda_A > \lambda_B$), no equilibrium is attained. If the parent and daughter are separated initially, then as the parent decays, the number of daughter atoms is given by

$$N_B = N_0 \frac{\lambda_A}{\lambda_B - \lambda_A} \exp(-\lambda_B t) \quad (12.28)$$

That is. the parent substance disappears completely and the daughter atoms eventually decay with their own half-life.

Spend 5 min

SAQ 6

For two radioactive elements A and B in transient equilibrium, show that the daughter activity is greater than the parent activity by a factor $\frac{\lambda_A}{\lambda_B - \lambda_A}$.

12.6 SUMMARY

- A radioactive nucleus disintegrates spontaneously by emitting either an α or a β-particle, usually accompanied by γ radiation.
- The number of atoms disintegrating per unit time is given by $N = N_0 e^{-\lambda t}$, where No is the number of atoms present initially and λ is the disintegration constant of the radioactive element
- The half-life of a radioactive element is the time taken for half of the radioactive atoms to disintegrate; this is related to the disintegrationconstant λ and mean life τ through the relations $T_{1/2} = 0.693/\lambda = 0.693\tau$.
- The standard, unit of radioactivity, the curie, is defined as the quantity of any radioactive material giving 3.7×10^{10} disintegrations per second. The SI unit of radioactivity is rutherford. It is defined as the amount of a radioactive substance giving 10^6 ds^{-1}.
- The naturally occurring radioactive elements conform to three radioactive series, known as the uranium, actinium and thorium series. Each series starts with an element having an extremely long half- life and terminates in a stable isotope of lead. With the discovery of transuranic elements, a fourth radioactive series has been traced, known as the neptunium series; this starts with plutonium and terminates in a stable isotope of bismuth.
- If the parent atom has a half-life very long compared to any of its decay products, we get a long- term equilibrium, known as secular equilibrium, between the parent and the daughter atoms, when each member decays at the same rate as they are produced, i.e. we have $\lambda_A N_A = \lambda_B N_B = h3\, N_B$. However, if the parent A is longer lived than the daughter B, but $T_{1/2}$ is not very long, we obtain transient equilibrium, in which the ratio of the members of A and B atoms at any instant remains a constant.

12.7 TERMINAL QUESTIONS

1. Given that the half-lives of Radium and Radon are 1620 yr and 3.82 d respectively. Calculate the volume of Radon gas at N.T.P. equivalent to one curie.
2. A sample containing 0.1 mg of ^{230}Th undergoes 4.32×10^6 disintegrations per minute. What is the half- life of this nuclide?

12.8 SOLUTIONS AND ANSWERS

SAQs

1. Here $\tau = 14.43$ months

$$T_{1/2} = \tau \ln 2$$
$$= (14.43 \text{ months}) \times 0.693$$
$$= 10 \text{ months}$$

Since 75% of the substance decays, only 25% remains.

$$N = \frac{N_0}{4} = \left(\frac{1}{2}\right)^2 N_0$$

Using Eq. (12.2) we can write

$$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

so that

$$\frac{t}{T_{1/2}} = 2$$

or

$$t = 10 \times 2 \text{ months}$$
$$= 20 \text{ months}$$

2. Half-life $T_{1/2} = 4.51 \times 10^9 \text{ yr} = 4.51 \times 10^9 \times 365 \times 86400 \text{ s}$

Disintegration constant $\lambda = \frac{0.693}{T_{1/2}}$

$$= \frac{0.693}{4.51 \times 10^9 \times 365 \times 86400} \text{ s}^{-1} = 4.87 \times 10^{-18} \text{ s}^{-1}$$

Number of atoms per g of uranium is

$$N = \frac{6.03 \times 10^{23}}{238}$$

. Rate of disintegration $\left|\frac{dN}{dt}\right| = \lambda N$

$$= \frac{6.03 \times 10^{23}}{238} \times 4.87 \times 10^{-18} \text{s}^{-1}$$
$$= 1.234 \times 10^4 \text{s}^{-1}$$

3. From Eq. (12.17) we know that

$$\frac{dN_B}{dt} + \lambda_B N_B = \lambda_A N_0 \exp(-\lambda_A t)$$

Multiplying throughout by exp ($\lambda_B t$) and re-arranging terms, you will obtain

$$\frac{d}{dt}[N_B . \exp(\lambda_B t)] = \lambda_A N_0 \exp[(\lambda_B - \lambda_A)t]$$

This can readily be integrated to give

$$N_B e^{\lambda_B t} = \frac{\lambda_A}{\lambda_B - \lambda_A} N_0 \exp[(\lambda_B - \lambda_A)t] + K$$

where K is constant of integration. To evaluate it, we use the condition $N_B = 0$ at $t = 0$. This gives

$$K = \frac{\lambda_A}{\lambda_B - \lambda_A} N_0 = \frac{\lambda_A}{\lambda_A - \lambda_B} N_0$$

On inserting this value of K in the above expression we obtain the required result:

$$N_B = \frac{\lambda_A}{\lambda_B - \lambda_A} N_0 [\exp(-\lambda_A t) - \exp(-\lambda_B t)]$$

4. We know from Eq. (12.18) that the number of RaB atoms is given by

$$N_B = \frac{\lambda_A}{\lambda_B - \lambda_A} N_0 [e^{-\lambda_A t} - e^{-\lambda_B t}]$$

If N_B attains maximum value at $t = t_m$, we must have $\frac{dN_B}{dt} = 0$ at $t = t_m$ and $\frac{d^2 N_B}{dt^2} < 0$.

Therefore, on differentiating the expression for N_B once with respect to t and equating it to zero, we obtain

$$-\lambda_A e^{-\lambda_A t_m} + \lambda_B e^{-\lambda_B t_m} = 0$$

or

$$\lambda_B e^{-\lambda_B t_m} = \lambda_A e^{-\lambda_A t_m}$$

$$e^{(\lambda_A - \lambda_B)a} = \frac{\lambda_A}{\lambda_B}$$

and

$$t_m = \frac{\ln(\lambda_A/\lambda_B)}{\lambda_A - \lambda_B}$$

Substituting $\lambda_A = 3.8 \times 10^{-3} \text{s}^{-1}$ and $\lambda_B = 4.3 \times 10^{-4} \text{s}^{-1}$, we get

$$t_m = \frac{\ln\left(\frac{3.8 \times 10^{-3}\ s^{-1}}{4.3 \times 10^{-4}\ s^{-1}}\right)}{(3.8 \times 10^{-3} - 4.3 \times 10^{-4})\ s^{-1}} = \frac{\ln (8.837)}{33.7 \times 10^{-4}\ s^{-1}} = \frac{2.179\ s}{33.7 \times 10^{-4}}$$

$$= 647s = 10m\,47s$$

5. Since uranium is in secular equilibrium with radium, we have

$$\frac{N_1}{\tau_1} = \frac{N_2}{\tau_2}$$

or $$\tau_1 = \frac{N_1}{N_2}\tau_2 = 2.8 \times 10^6 \times 1620\ yr = 4.5 \times 10^9 yr$$

6. For elements A and B in transient equilibrium, we have from **Eqn.** (12.27),

$$\frac{N_B}{N_A} - \frac{\lambda_A}{\lambda_B - \lambda_A}$$

Hence the ratio of the measured activities at equilibrium is given by

$$\frac{A_B}{A_A} = \frac{\lambda_B N_B}{\lambda_A N_A} = \frac{\lambda_B}{\lambda_A} \cdot \frac{\lambda_A}{\lambda_B - \lambda_A} = \frac{\lambda_B}{\lambda_B - \lambda_A}$$

$$A_B = \frac{\lambda_B}{\lambda_B - \lambda_A} A_A$$

TQs

1. One curie is equivalent to the amount of radon in equilibrium with 1 g of radium. Hence if N_{Rn} is the ***number*** of Rn atoms in equilibrium with 1 g of Ra and N_{Ra} Lhe number of atoms in 1 g of radium, then we have $\lambda_{Rn} N_{Rn} = \lambda_{Ra} N_{Ra}$.

$$\therefore\ N_{Rn} = N_{Ra}\frac{\lambda_{Ra}}{\lambda_{Rn}} = N_{Ra}\frac{\tau_{Rn}}{\tau_{Ra}} = \frac{3.82\ d}{1620\ yr} \times N_{Ra}$$

$$= \frac{3.82}{1620 \times 365} \times \frac{N}{226}$$

where N is **Avogadro's** number.

$\therefore$ Volume occupied by N_{Rn} atoms at STP

$$= \frac{N_{Rn}}{N} \times 22.4 \times 10^3\ cm^3$$

$$= \frac{3.82}{1620 \times 365 \times 226} \times 22.4 \times 10^3\ cm^3$$

$$= 6.4 \times 10^{-4}\ cm^3$$

2, No. of **atoms** in 0.1 **mg** of ^{230}Th

$$N = \frac{6.02 \times 10^{23}}{230\ g} \times (10^{-4}\ g) = 2.62 \times 10^{17}$$

Rate of disintegration $\frac{dN}{dt} = 4.32 \times 10^6\ m^{-1}$

$$= 7.2 \times 10^4\ s^{-1}$$

. Decay constant $\lambda = \frac{1}{N}\frac{dN}{dt}$

$$= \frac{7.2 \times 10^4\ s^{-1}}{2.62 \times 10^{17}} = 2.75 \times 10^{-13}\ s^{-1}$$

$$\therefore\ T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{2.75 \times 10^{-13}}\ s$$

$$= 7.99 \times 10^4\ yr$$

We have from Eq. (12.13 b)

$$\frac{N_2(t)}{N_2(\infty)} = 1 - e^{-\lambda_2 t} = \frac{90}{100} \text{ (given)}$$

$$\therefore \quad e^{-\lambda_2 t} = \frac{1}{10} \text{ or } \quad t = \frac{\log_e 10}{\lambda_2} = \tau_2 \frac{\log_e 10}{\log_e 2}$$

Hence, $\frac{t}{\tau_2} = \frac{\log_e 10}{\log_e 2} = 3.32$