UNIT 4 WAVE-PARTICLE DUALITY

Structure

4.1 INTRODUCTION

You have already taken several courses in physics, from elementary mechanics to electric and magnetic phenomena which constitute classical physics. However, that is not enough if you wish to go deep into physics. You may ask: Why? The answer is that classical physics on its own cannot explain many a **natural/observed** phenomena. Hence, there seems to be a need of a new physics. In this **unit** we will dwell briefly on some **phenomena** and experimental results which defy classical analysis. We will introduce you to the ***quantum postulate of Planck,*** which was given to explain the experimental results of black body radiation.

It was further extended by Einstein and Bohr to explain the **phenomena** of photoelectric emission and line spectra of atoms, respectively. The sequence of events is chosen to give you an idea of how quantum physics came into being. Then we discuss one of the basic concepts which laid down the foundations of quantum mechanics, namely, the 'wave-particle duality.

Objectives

After studying this unit you should be able to

- discuss how quantum physics emerged,
- calculate the de Broglie wavelength of a particle in motion,
- **explain** the concept of wave-particle duality.

4.2 THE BIRTH OF QUANTUM PHYSICS

You already know that a black body absorbs all radiations which fall on it. **(Since** it does not reflect light and appears black, hence the name — black body.) Usually in the laboratory experiments, a hollow body (cavity) with blackened walls and having a small hole, as shown in Fig. **4.1,** acts as a black **body. The** radiation contained in the body and emitted from the hole produces a black body spectrum. In the last century, a number of experiments were carried out to measure the energy per unit volume contained by a black body, denoted by $\rho_T(\lambda)$, at different temperatures. Some of the representative curves showing the variation of $\rho_T(\lambda)$ as a function of λ (black body spectrum) at different temperatures are given in Fig. 4.2. Various investigators tried to explain the nature of these curves using well established laws of classical physics, including thermodynamics. You have studied in Block 4 of the course **PHE-06** on Thermodynamics and Statistical Mechanics that it **was** Planck who came up with a theoretical explanation of the black body radiation curve. You know that till 1900, most of the measurements of the energy spectrum of black body radiation were made at smaller wavelengths. These could be satisfactorily explained by **Wein's formula** given by

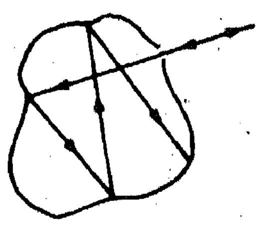


Fig. 4.1 : A black body.

$$\rho_T(\lambda)\, d\lambda = a\lambda^{-5} \exp\,(-b/\lambda kT)\, d\lambda \qquad (4.1)$$

where a and b were adjustable parameters.

of the development of quantum mechanics because it is based on mathematics more familiar to you. We introduce **wave-particle duality** in Unit **4** itself. Unit 5 **deals** with the concepts of matter **waves** and the **uncertainty principle.** The wave equation of matter waves, called the **Schrodinger equation** is the subject matter of Unit 6. Finally in Unit 7 we discuss the basic concepts of the matrix version of quantum mechanics without going into detailed matrix algebra and briefly present the basic features of the unified version of wave mechanics and matrix mechanics formulated by **Dirac.** Contentwise, the units are more or less evenly balanced and will take about the same time (5 to 6 hours each) to study.

Now there are two aspects of learning quantum mechanics. The first and foremost is, as Richard Feynman used to say, to learn to calculate. However, the quantum mechanical way of calculating is quite different from the classical ways; you will find that you have to get used to a radically new way of thinking. So you will also have to learn to think quantum mechanically. This will involve a certain effort towards exploring the meaning of quantum mechanics, but it will certainly be worthwhile. And if the exploration of the meaning shacks you at times, do not worry. You can take consolation from a comment made by Neils Bohr, "Those who are not shocked when they first come across quantum theory cannot possibly have understood it."

We hope you enjoy studying the block and we wish you success.

Acknowledgement

We are thankful to Shri Gopal Krishan Arora for his invaluable contribution in word-processing of the entire course, and for secretarial assistance, . -

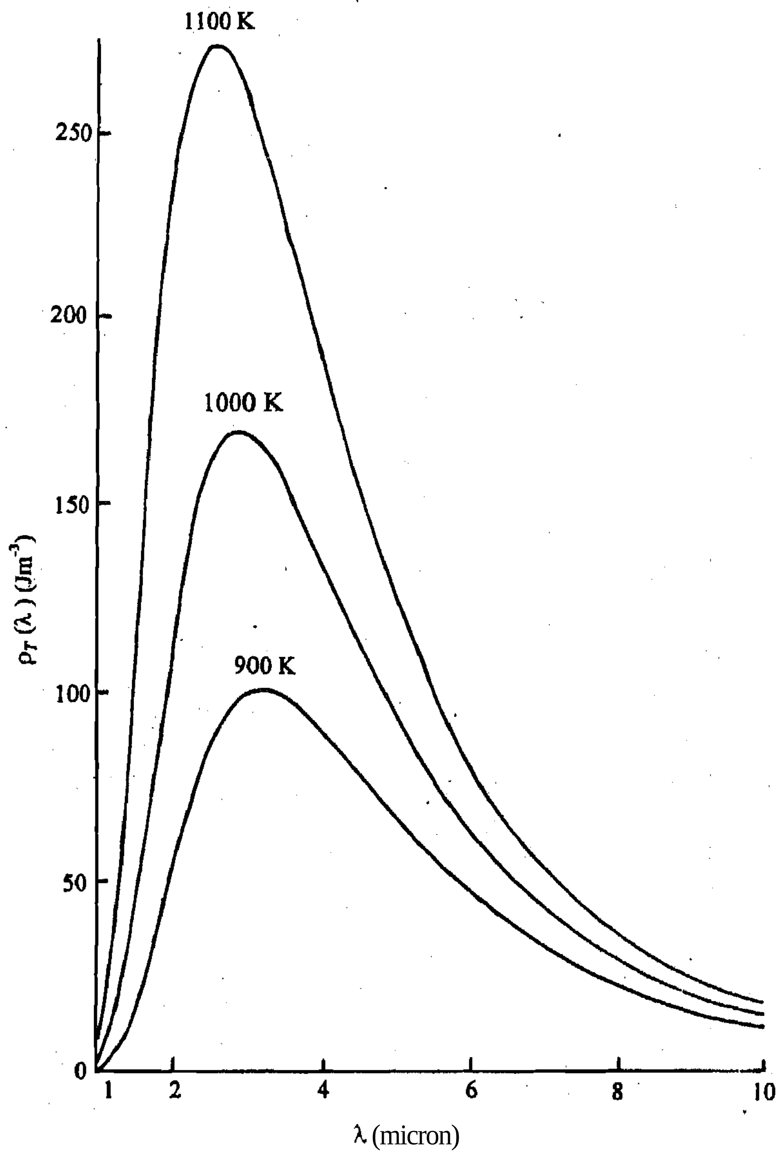


Fig. 4.2 : **Black body radiation.**

Planck found empirically that by the replacement of $\exp\left(-\frac{b}{\lambda kt}\right)$ by $\exp\left(\frac{b}{\lambda kt}-1\right)^{-1}$; a set of parameters a and b could be obtained which would fit Eq. (4.1,) beautifully with the experimental data over all values of λ. However, he found it extremely difficult to give a theoretical justification for the above mentioned replacement. **Ultimately,** out of desperation, on 18th December, 1900, Planck declared that the only way to derive the correct black body **radiation formula** was to postulate that the **exchange of energy between matter** (walls) **and radiation (cavity) could take place only in bundles of a certain** size.

To **realise** the significance of the above postulate let us consider the following simple example. Suppose two **litres** of milk is to be **distributed between** two persons, In how **many** ways can you distribute the milk? Milk is an infinitely divisible quantity. Hence, you can divide milk between two persons in an infinite number of ways. Now **suppose** you are restricted to distribute milk only in the units of a **litre.** Now the number of distribution reduces just to 3 (both persons can receive **only** 0, 1 or 2 litres of milk). What a drastic change! The number of ways will increase to 5 if you reduce the unit **to** half a **litre.**

In, classical physics. energy is regarded as an infinitely divisible quantity. Hence the exchange of energy between the walls and the cavity can take place in an infinite number of ways. However, through his postulate Planck reduced the number of ways **to** be finite. In his model, the exchange must take place in the units **of** U_0. Thus he introduced the idea of discreteness in the division of energy (which was thought to be infinitely divisible). If the energy $U(\lambda)$ is to be exchanged, $U(\lambda)/U_0$ must be an integer. If it is not, Planck suggested that it should be an integer close to $U(\lambda)/U_0$.

Planck further postulated that the unit or the *quantum* of energy U_0 is directly proportional to its frequency, i.e.,

$$U_0 = h\nu \quad (4.2)$$

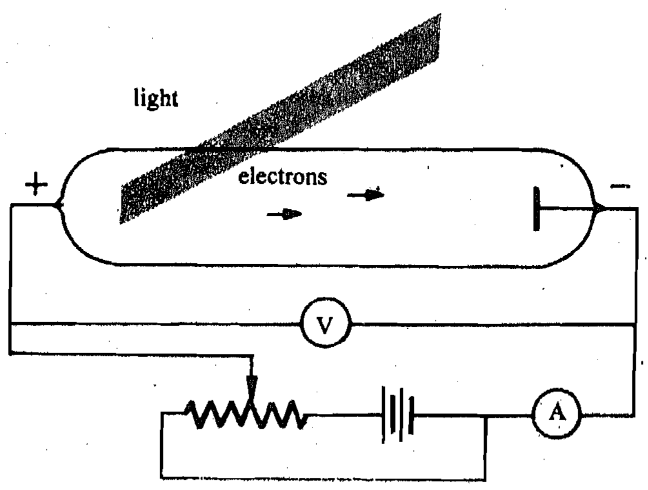
The constant of proportionality h is now known as **Planck's constant** in his honour. Its value is 6.62618×10^{-34} Js. Planck was awarded the Nobel prize for physics in 1918 for his work on black body radiation. [You should note that greater the value of ν, higher will be the value of the quantum of energy U_0 and consequently, lesser will be the number of ways in which energy U can be exchanged.] This new concept of Planck's gave birth to a new physics, known as *quantum* physics. Hence, it is appropriate to take 18th December, 1900 as the date of birth of quantum physics which later on developed into quantum mechanics.

In a further development, Einstein used Planck's quantum hypothesis to successfully explain the **photoelectric effect.**

The Photoelectric Effect

In 1887, Hertz, while working on electromagnetic waves, discovered that the air in a spark gap became a better conductor when it was illuminated by ultraviolet rays. Further investigations by him showed that zinc acquired a positive charge when it was irradiated with ultraviolet rays, i.e., it lost negative charges. In 1900, Leonard showed that the ejected particles were electrons. A series of such experiments revealed that electrons are emitted from a metal surface when light of sufficiently high frequency falls upon it This phenomenon is known as the **photoelectric effect.**

Fig. 4.3 shows a schematic diagram of the apparatus that was employed in some of these experiments. An evacuated tube contains two electrodes connected to an external circuit like that shown schematically. The anode is made up of the metal plate whose surface is to be irradiated. Some of the photoelectrons that emerge from the irradiated surface have sufficient energy to reach the cathode despite its negative polarity, and they constitute the current that is measured by the ammeter in the circuit.



Rig. 4 3 : Schematic diagram of the apparatus for photoelectric effect.

As the collecting: voltage V, which retards the electrons, is increased, fewer and fewer electrons get to the cathode and the current drops. Ultimately, when V equals or exceeds a certain value V_0, of the order of a few volts, no further electrons strike the cathode and the current ceases. Figs. 4.4a and b show the experimental curves corresponding to this effect when the intensity of light and collecting voltage V are kept constant.

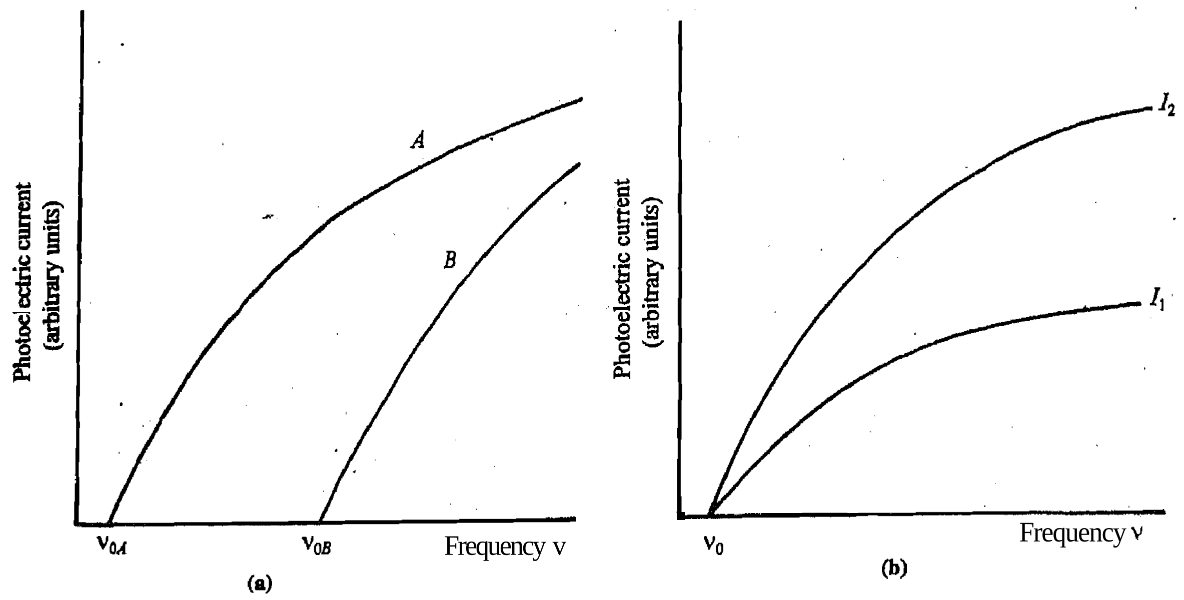


Fig. 4.4 : (a) Variation of the photoelectric current (in arbitrary. units) as a function of frequency for two materials A and B. The intensity of light and the collecting voltage are kept constant; (b) variation of the photoelectric current (in arbitrary units) as a function of frequency for a single material at two values of the intensity; of the intensities, I_2 is greater than I_1.

Note that photoelectron emission occurs only when the frequency of the falling radiation is higher than some threshold frequency ν_0. It was found that for $\nu < \nu_0$, no emission takes place, no matter how intense the radiation is, The value of ν_0 depends upon the material of the surface irradiated. It was also determined that for a given frequency ν ($> \nu_0$), the kinetic energy of the emitted photoelectrons has values between zero and a definite maximum value E_{max}. For any given metal, E_{max} is proportional to $(\nu - \nu_0)$ and is independent of the intensity of the falling light. Further, when electromagnetic waves fall on the material, emission of photoelectrons starts instantaneously (within 10^{-9}s), no matter how weak or strong the falling light is. All these features of the photoelectric effect could not be explained by the classical electromagnetic theory of light on the basis of its wave nature.

You are familiar with the dual nature of light about which you have studied in Unit 1 of the physics elective PHE-09 entitled Optics.

In 1905, Einstein proposed a simple but revolutionary explanation for the photoelectric effect. Einstein extended Planck's postulate of the quanta of energy to the quanta of energy of the electromagnetic field. He viewed the photoelectric phenomenon as a collision between a photon (a quantum of the energy of an electromagnetic field) and a bound electron. In the collision, the photon is completely absorbed and the energy of the bound electron is increased by h ν. Since the electrons are bound in the metal their initial energy E is negative and the largest value of E is –W, where W is the work function of the metal. Hence, to escape from the metal, the electron has to use at least an energy equal to W. Thus, the maximum kinetic energy of the photoelectrons will be

$$E_{max} = \frac{1}{2} m v^2_{max} = h\nu - W \tag{4.3a}$$

If we take $W = h\nu_0$, Eq. (4.3a) may be written as

$$\frac{1}{2} m v^2_{max} = h(\nu - \nu_0) \tag{4.3b}$$

The implications of Eqs. (4.3 a and b) are:

(1) Since v_{max} has to be positive, no emission can take place for $\nu < \nu_0$.

(2) E_{max} is proportional to $(\nu - \nu_0)$.

(3) An increase in the radiation intensity of frequency ν corresponds to an increase in the number of photons. Since each one of them has the same energy $h\nu$, there is no increase in the energy of the photoelectrons. Only the number of emitted electrons and hence the photoelectric current increases (see Fig. 4.5).

(4) Since the effect is produced by mechanical collisions between electrons and photons, the energy transfer from photons to the electrons is instantaneous. Consequently the time lag is very small.

(5) Since work function $W(=h\nu_0)$ is a characteristic property of the emitting surface, ν_0 is independent of the intensity of incident radiation.

You thus see that Einstein's quantum theory explained each and every aspect of the photoelectric effect with brilliant success, and so the absorption of light in the form of packets or quanta was firmly established.

The next important step in the development of quantum physics was the explanation by Bohr of the stability of the atom as well as the line spectrum emitted by hydrogen atoms. You must be familiar with the Bohr model of the hydrogen atom from your +2 physics courses. However, we have included the details here for completeness.

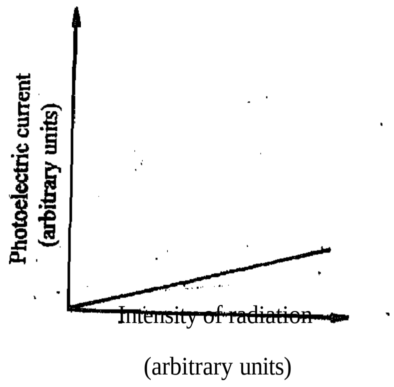


Fig. 4.5 : Variation of the photoelectric current with the intensity of radiation of frequency ν ($> \nu_0$) at a constant collecting voltage.

Bohr's postulates for atomic model

The classical crisis with the model of the atom was not dissimilar to the case of black-body radiation. Ernest Rutherford had proposed the nuclear model of an atom based on his discovery of the atomic nucleus. The electrons, in this model, were supposed to revolve around the nucleus. But then they must radiate and lose energy and eventually spiral into the nucleus. A classical nuclear atom turned out to be unstable!

In 1913, Neils Bohr proposed an atomic model which accounted for the stability of the atom, by injecting quantum ideas into Rutherford's theory. The model also proved highly successful for explaining the spectrum of the hydrogen atom. Bohr's atomic model was based on the following four postulates, three of which were radically different from the earlier models.

(1) Electron in an atom moves in **circular orbits** about the nucleus with the centripetal force supplied by the Coulomb attraction between the electron and the nucleus.

(2) Of **the infinite** number of possible circular orbits, only those are allowed for which the value of the orbital angular momentum $|L|$ of the electron is an integral multiple of $h/2\pi$.

You should note that Bohr preferred quantization of angular momentum instead of energy as was done by Planck, in order to introduce h (the quantum of action) in his theory.

(3) An electron moving in an allowed orbit does not radiate any energy. These states of constant energies are called **stationary states.**

Note that the electron is not stationary in a stationary state.

(4) Energy is emitted (or absorbed) from an atom only when its electron jumps from one allowed orbit of energy E_i to another allowed orbit of energy E_f. The frequency of the emitted (or absorbed) radiation is, given by Einstein's, frequency condition

$$h\nu = E_i - E_f \qquad \text{(emission } E_i > E_f\text{)}$$

$$= E_f - E_i \qquad \text{(absorption } E_i < E_f\text{)}$$

You should appreciate that the above four postulates are a **hybrid** of classical and non-classical physics. For example, the **first** postulate is in accordance with classical physics while the fourth postulate uses **quantum** ideas. The postulates of the quantization of angular momentum and stationary states are also non-classical.

The first postulate yields the following result for the nth allowed orbit:

$$\frac{mv_n^2}{r_n} = \frac{Ze^2}{4\pi\varepsilon_0 r_n^2} \qquad (4.4)$$

where m is the mass of the electron. Ze is the charge of the nucleus and the electron is moving with a speed v_n in the nth allowed orbit of radius r_n. The second postulate yields

$$\mathbf{L}_n = \mathbf{r}_n \times \mathbf{p}_n = \frac{nh}{2\pi}\hat{\mathbf{L}}\,; \qquad n = 1, 2, 3, \ldots \tag{4.5}$$

and

$$L_n = m\,v_n\,r_n \qquad \text{(since } \mathbf{v}_n \perp \mathbf{r}_n \text{ for a circular orbit)}$$

where $\hat{\mathbf{L}}$ is a unit vector perpendicular to the plane of the orbit. Thus, we have

$$m\,v_n\,r_n = n\hbar, \text{ where } \hbar = \frac{h}{2\pi}, \qquad n = 1, 2, \ldots$$

or

$$v_n = \frac{Ze^2}{2\,\varepsilon_0}\,\frac{1}{nh} \tag{4.6}$$

and

$$r_n = \frac{n^2 h^2 \varepsilon_0}{Z e^2 m \pi} \tag{4.7}$$

The total energy of the electron is the sum of the kinetic energy T_n and the potential energy U_n. Hence for the nth stationary orbit

$$E_n = \frac{1}{2}mv_n^2 - \frac{Ze^2}{4\pi\,\varepsilon_0\,r_n} \tag{4.8a}$$

Putting the values of v_n and r_n from Eqs. (4.6) and (4.7), we may write

$$E_n = -\frac{Z^2 e^4 m}{8\,\varepsilon_0^2\,h^2}\,\frac{1}{n^2} \tag{4.8b}$$

or

$$E_n = -\frac{R_\infty Z^2}{n^2} \tag{4.8c}$$

where

$$R_\infty = \frac{me^4}{8\,\varepsilon_0^2\,h^2} \tag{4.8d}$$

Thus $E_n \propto n^{-2}$. The suffix ∞ on R appears because the mass of the proton has been assumed to be infinity. Putting the standard values of m, e, ε_0 and h in Eq. (4.8d), we obtain $R = 2.18 \times 10^{-18}$ J (or 13.6eV).

According to Bohr's fourth postulate, the frequency ν_{mn} of the emitted (absorbed) radiation when the electron jumps from the nth state to mth state is given by

$$\nu_{mn} = \frac{R_\infty Z^2}{h}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \tag{4.9}$$

This agrees remarkably well with the frequency spectrum of the hydrogen atom (see Fig. 4.6). Immediately after Bohr's atomic theory was published, Franck and Hertz performed experiments which demonstrated the existence of discrete energy states.

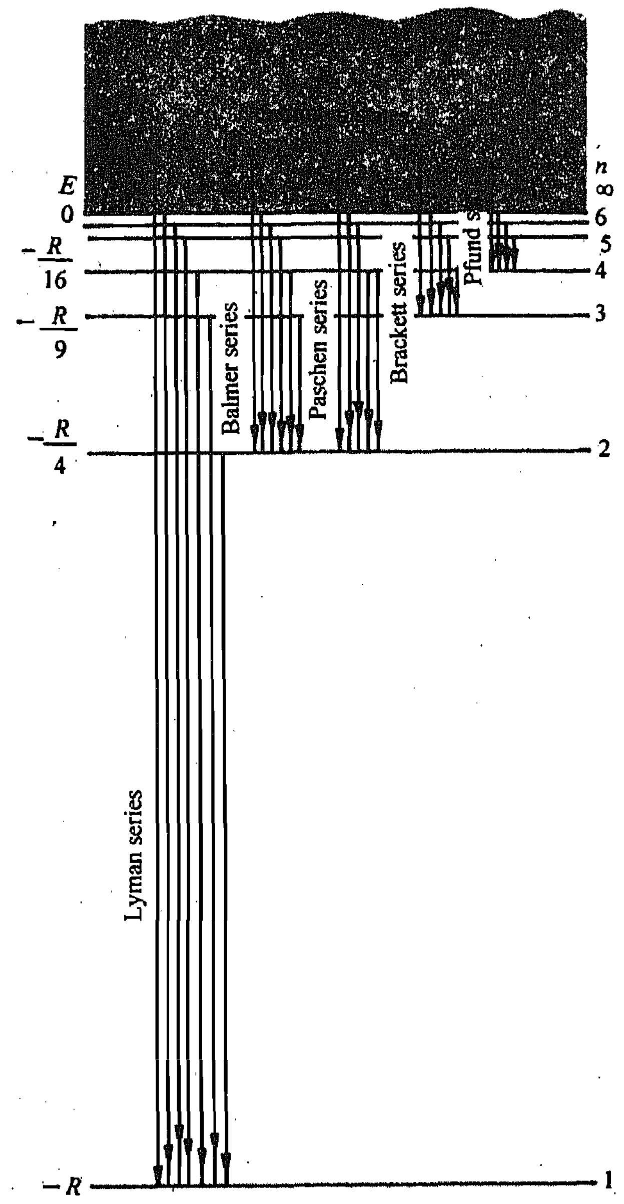


Fig. 4.6 : Energy levels of the hydrogen atom according to the Bohr model. Some of the transitions leading to Lyman series, Balmer series, Paschen series, Brackett series and Pfund series are also shown. All negative energy states represent bound states while positive energy states are continuous states.

However radical these ideas were, you can notice that there was still an attempt to retain a link with the classical physics. You know that at high temperatures or at low frequencies, the black body radiation formula given by Planck reduces to the classical Rayleigh-Jeans formula. Based on these ideas Bohr, in 1923, gave the Correspondence Principle.

Bohr's Correspondence Principle

The idea underlying this principle is as follows: *The principles of quantum physics must yield the same results as those of classical physics in situations for which classical physics is valid.* According to the correspondence principle

(a) Quantum theory should give the same results for the behaviour of any physical system as classical physics, in the limit in which the quantum numbers specifying the state of the system become very large.

(b) A selection rule holds true over the entire range of the quantum number concerned. Hence, at large quantum numbers-if any selection rule is required to obtain correspondence between classical and quantum physics then the same selection rule holds at low quantum numbers also, To sum-up the discussion so far, recall that Planck introduced a non-classical postulate to explain the black body spectrum which was further extended by Einstein. According to this postulate, the energy states of a simple harmonic oscillator of frequency ν are discrete and the energy of the nth discrete state is equal to $nh\nu$ where n is a positive integer and h is a universal constant (called the Planck constant). Einstein regarded a' quantum of energy as a particle. The quantum of electromagnetic wave is known as photon. As you know, this is a particle of zero rest mass which always travels with the velocity of light and carries a momentum $h\nu/c$. Einstein explained the photoelectric effect by regarding the phenomenon as a collision between a photon of energy $h\nu$ and the weakly bound metallic electron in which a photon is completely absorbed and gives its whole energy to the electron which may escape from the metal.

You have also read about Bohr's atomic model according to which electrons in an atom move around the nucleus in certain allowed orbits. In these orbits, energy is conserved and the angular momentum of the electron is an integral multiple of $h/2\pi$. An atom emits or absorbs radiations only when its electron jumps from one allowed orbit to another. His theory proved highly successful in explaining the discrete frequency spectrum of the hydrogen atom. Thus a new physics was born, which is now known as "old quantum theory".

Fig.4.7 : Louis Victor, prince de Broglie (pronounced de Broy), 1892-1987, French theoretical physicist. For his discovery of wave properties of matter he was awarded the Nobel Prize in 1929, after his hypothesis was experimentally confirmed.

This was further developed by de Broglie, Heisenberg and Schrödinger into a new mechanics, now known as quantum mechanics.

Let us now study one of the basic concepts which forms the foundation of quantum mechanics, namely, the de Broglie hypbthesis, which led to wave-particle duality.

4.3 THE DE BROGLIE HYPOTHESIS

Louis de Broglie (Fig. 4.7) must have been a lover of music, for he realised that Bohr's stationary orbits of electrons confined in atoms must have something in common with stationary waves on guitar strings. Could the discreteness of atomic orbits be due to the discreteness of electron waves in captivity? On a guitar string, stationary waves form a discrete pattern of harmonics just like the discrete Bohr orbits. De Broglie asked: Could atomic electrons be confined waves and therefore produce a discrete stationary wave pattern? For example, the lowest atomic orbit is one in which one electron wavelength fits the circumference of the orbit and the higher orbits fit two or more electron wavelengths (Fig. 4.8).

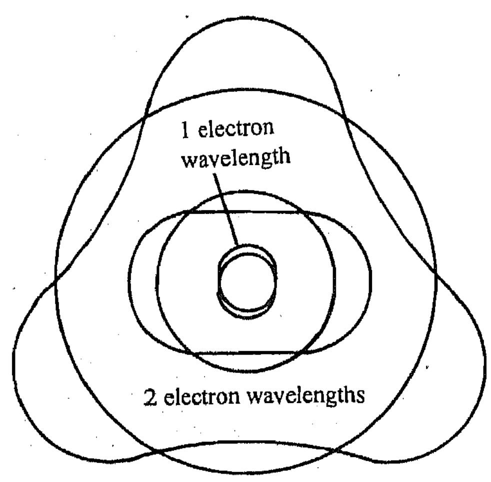


Fig. 4.8 : Stationary waves of electrons confined in an atom. The electron wave fits an integral number of wavelengths in each of the successive Bohr orbits.

It was established by then that electromagnetic waves exhibit both wave and particle properties. But electromagnetic waves and material particles are two important classes of entities which appear to be basic to the structure of matter. If electromagnetic waves have a dual wave-particle nature then why shouldn't electrons; and in fact, all matter, too have a dual nature?

As a young French graduate student, de Broglie, in 1924, argued with a great amount of insight that since nature loves symmetry and simplicity in physical phenomena, all material particles should exhibit both wave and particle nature. He further argued that the wave description of light in geometrical optics is an approximation of the more general wave analysis. Similarly, the description of particle motion in terms of line trajectories is an approximation of a more general description of the particle, containing its wave aspect. De Broglie further proposed that the wavelength and frequency of the matter waves should be determined by the momentum and energy of the particle in exactly the same way as for photons. Recall that for a photon, energy and momentum are related as follows:

$$E = pc$$

and from **Eq.** (4.2), $E = h\nu = \hbar\omega$, $(\omega = 2\pi\nu)$. From wave theory, the angular frequency ω is related to the wave number k:

$$\omega = ck$$

where c is the wave speed.

Combining these equations, we get

$$\begin{aligned} pc &= \hbar\omega \\ &= \hbar\, c\, k \end{aligned}$$

or

$$p = \hbar\, k \tag{4.10}$$

Since $k = \dfrac{2\pi}{\lambda}$, Eq. (4.10) can also-be rewritten to get an expression for the **de Broglie wavelength** of matter waves associated with a particle having *momentum* p:

$$\textbf{de Broglie wavelength } h = \frac{h}{p} \tag{4.11}$$

Thus, the wavelength of the wave associated with a particle of matter in motion is inversely proportional to the particle's momentum, and the constant of proportionality is just the Planck constant. We can recast Eq. (4.11) into another form by using the relation $E = p^2/2m_0$ for a free particle:

$$\lambda = \frac{h}{p} = \frac{h}{(2m_0E)^{1/2}} \tag{4.12}$$

where m_0 is the rest mass of the particle, and E its energy.

Now the **phase velocity** v_p of **a** wave is given by. $v_p = \nu h$. Hence, we may write

$$v_p = \frac{E}{h}\frac{h}{p} = \frac{E}{p} \tag{4.13a}$$

Putting $E = mc^2$ and $p = mv$ in Eq. (4.13a), we get

$$v_p = c^2/v \tag{4.13b}$$

Since $v < c$, the phase velocity of waves associated with matter turns out to be greater than the velocity of light, Does this disturb you? Do not worry because no physical quantity like energy, signal or information etc. associated with the wave, travels with its phase velocity.

Let us now understand what Eq. (4.11) means.

Eq. (4.11) is a complete statement of the wave-particle duality. It clearly shows that a particle with a momentum p can exhibit wave-like properties and the wavelength of the associated matter waves is h/p. The converse is also true, **i.e.,** a wave of wavelength λ can exhibit particle-like properties and the momentum of the wave-matter is h/λ. However, you should clearly understand the difference between electromagnetic waves and matter waves. For matter waves, phase velocity is always greater than the velocity of light but, since energy is carried by the particle, the velocity with which energy is transported by matter waves is equal to the velocity of the particle. On the other hand, the phase velocity of electromagnetic waves and the velocity with which energy is transported by them are both equal to the velocity of light.

By now, are you not wondering that if matter can exhibit wave-like properties, why don't macroscopic objects appear like waves to us?

To understand this, make the following simple calculation.

Spend 5 min

SAQ 1

A ball of mass 10^{-3} kg moves with a velocity of 10^{-2} m s^{-1}. What is the de Broglie wavelength of the ball?

Clearly, this wavelength is too small to be detected experimentally. Hence, we can say that matter waves are associated with macroscopic objects. However, their wave character is not observable. Thus, they can safely be described as particles under all circumstances. Now do the following exercise.

Spend 5 min

SAQ 2

Calculate the de Broglie wavelength of a 100 eV electron and a 1 MeV neutron, using the formula given by Eq. (4.12).

You have calculated the wavelength to be about 1.2×10^{-10} **m** for the 100 **eV** electron and 2.9×10^{-14} **m** for the 1 MeV neutron, respectively. Clearly the wavelength of the matter wave associated with 100 **eV** electron lies in the X-ray region and is of the same order as the atomic spacing in a crystal. The electron waves, like X-rays, are, therefore, expected to undergo diffraction by crystals. On the other hand, the wavelength of a 1 MeV neutron is too small for observing the diffraction by a diffracting grating, However, low energy neutrons, say 100 **eV** neutrons, would have wavelength in the X-ray region and then their diffraction pattern can be obtained.

We thus see that for the motion of macroscopic objects, the de Broglie hypothesis does not change the classical description as developed by Newton. But for microscopic objects the wavelengths of matter waves are long enough to undergo observable diffraction. The diffraction of matter waves was observed experimentally as early as 1927.

4.3.1 Experimental Evidence for the Existence of Matter Waves

The first experimental demonstration of matter waves came three years after de Broglie advanced his hypothesis and was accidentally obtained by two American physicists Clinton J. Davisson and his assistant L.H. Germer. They were studying the scattering of electrons by crystals using the apparatus shown in Fig. 4.9.

Electrons from an electron gun were accelerated by a positive electrode maintained at V volts as compared to the filament. The accelerated beam of electrons was incident on a strip of nickel containing many crystals and the number of electrons scattered in various directions was then measured. A smooth variation in the intensity of the scattered electron with the angle was observed. Midway through the experiment, an accident occurred which permitted air to enter into the vacuum tube containing the strip.

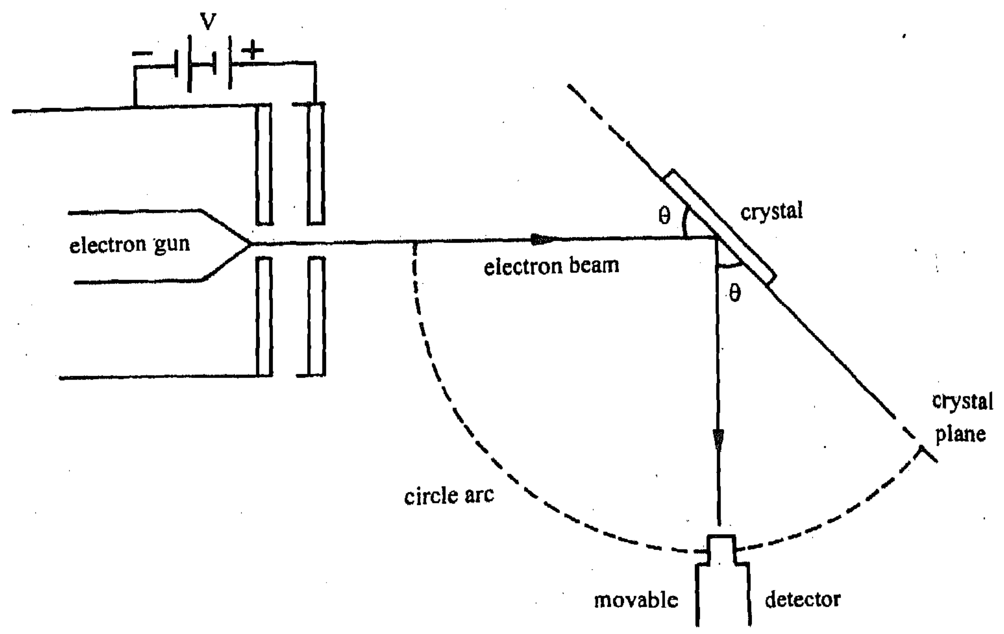


Fig. 4.9 : Schematic diagram of Davisson-Germer Experiment.

This resulted in the formation of an oxide film on the surface. Davisson and Germer were then forced to heat the strip to a very high temperature in order to reduce the oxide. This heating, and slow cooling had the effect of turning the polycrystalline nickel sample into a large single crystal. After this forced heat treatment of the sample, the experiment was repeated. The experimental results this time were quite different from those obtained before the accident. The intensity of scattered electrons showed some sharp maxima and minima at certain angles which were found to depend upon the electron energy and hence upon the accelerating voltage. This pattern was similar to a wave diffraction pattern. Some typical results from these experiments are shown in Fig. 4.10.

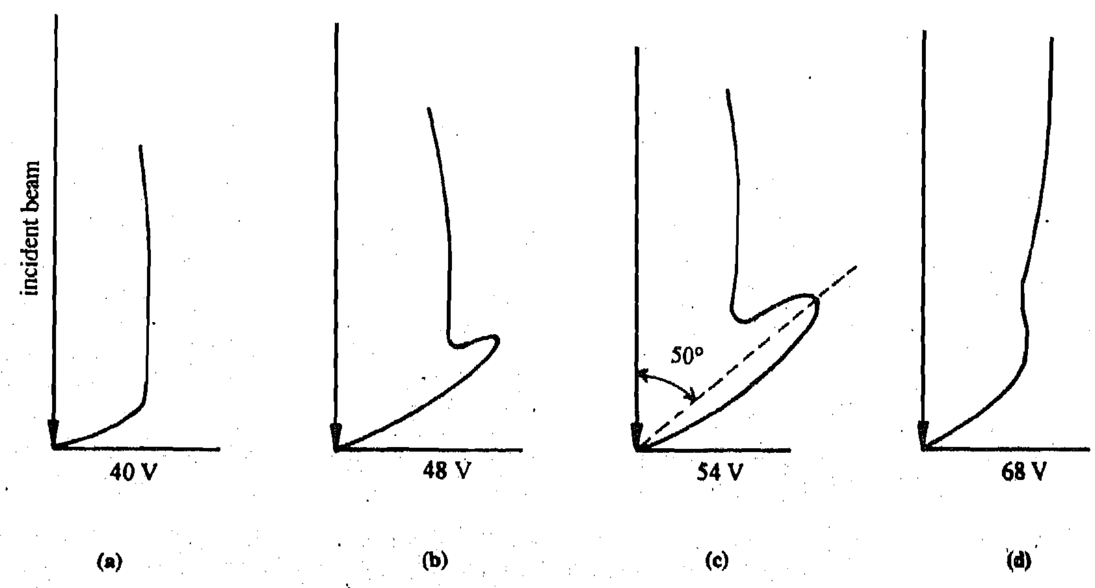


Fig. 4.10 : Polar plots of some typical results of Davisson-Germer experiment.

Davisson and Germer found that the angular peaks of scattered electrons could be explained as diffraction of electrons by atoms if the regular arrays of atoms in the nickel crystal are regarded as a diffraction grating.

Diffraction of Electrons

Using Bragg's analysis developed for X-ray scattering, and the angles of scattering at which the pronounced peaks are produced, they calculated the wavelength of electron waves. They finally found very good agreement between their values of the wavelength and those predicted by de Broglie hypothesis. Thus the validity of the de Broglie hypothesis was established.

In one particular experiment, the electron beam was accelerated to a potential of 54 volts and the maximum intensity was observed at an angle of 65° between a particular family of crystal planes and the incident (or the scattered) beam. The spacing between the crystal planes, as measured by X-ray diffraction technique was found to be 0.91 Å. Let us now calculate the wavelength of the electron-waves from these electron diffraction data.

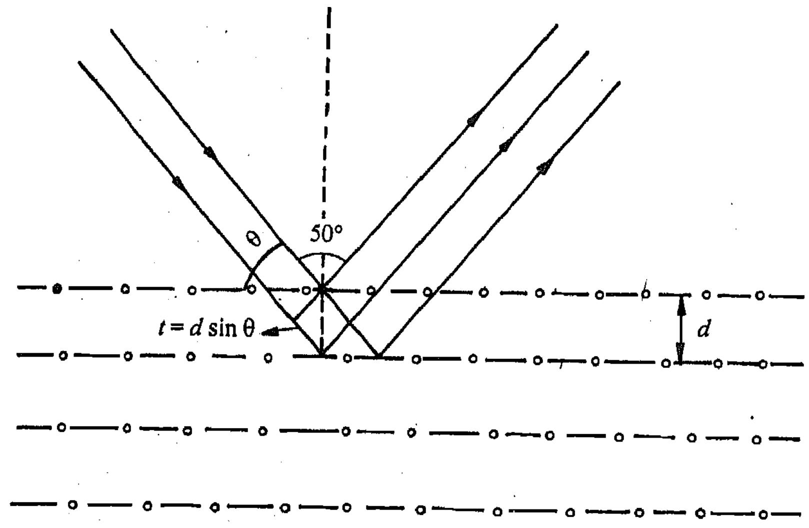


Fig. 4.11 : Bragg's analysis of scattering by crystal planes.

Fig. 4.11 clearly shows that the phase difference between waves corning from adjacent scattering planes of crystals is given by $(2\pi/\lambda) \times 2d \sin\theta$. For constructive interference we, therefore, have

$$(2\pi/\lambda)\, 2d \sin\theta = 2\pi n$$

i.e.,
$$\lambda = 2d \sin\theta / n, \qquad n = 1, 2, 3, \ldots \tag{4.14}$$

Putting the given data in Eq. (4.14) along with $n = 1$, we get

$$\lambda = 2\,(0.91\ \text{Å}) \sin 65° = 1.65\ \text{Å} \tag{4.15a}$$

Let us now compare this value of λ with the corresponding value predicted by de Broglie's hypothesis. The de Broglie wavelength of an electron accelerated by a potential of V volts is given by

$$\lambda = \frac{h}{p} = \frac{h}{(2em_0V)^{1/2}} = \frac{12.264}{(V(\text{volt}))^{1/2}}\ \text{Å} \tag{4.15b}$$

In deriving Eq. (4.15b) we have used Eq. (4.12). Hence the wavelength of a 54 eV electron is

$$\lambda = \frac{12.264}{(54)^{1/2}}\ \text{Å} = 1.67\ \text{Å} \tag{4.15c}$$

You will notice that the agreement between the two values of the wavelength of electron, given by Eqs. (4.15a) and (4.15c) is remarkable.

Within months of the discovery by Davisson and Germer, a British physicist G.P. Thomson also discovered diffraction effects with a beam of highly energetic electrons. In 1928, Thomson repeated his experiment by using platinum rather than the celluloid film. The diffraction rings observed with the polycrystalline metallic foil were found to be exactly similar to those observed for X-rays of the same wavelength (as that of electron). This experiment of Thomson had provided even more convincing evidence in support of the de Broglie hypothesis. De Broglie was rewarded with a Nobel Prize in Physics in 1929 and the same was awarded to Davisson and Thomson in 1937. It is of interest to note that Sir J.J. Thomson was awarded Nobel Physics Prize in 1906 for discovering the electron as a particle carrying negative electrical charge and in 1937 his son G.P. Thomson got the same prize for establishing the wave nature of electron. With suitably designed experiments, the wave-like behaviour of particles such as a-particles, protons, neutrons etc. has also been established.

You may now like to work out an SAQ.

SAQ 3

Spend 10 min

Electrons of 400 eV are diffracted through a crystal and a second order maximum is observed where the angle between the diffracted beam and incident beam is 30°. Calculate

(i) the wavelength of the electron matter wave,

(ii) the interplanar distance of those lattice planes which are responsible for this maximum.

Has the wave behaviour of electrons 'come as a surprise to you or shocked your sensibilities? After all, all along we have been accustomed to regarding an electron as a particle and now it exhibits wave properties. What is this duality in an electron's behaviour? What is wave-particle duality? Let us further explore the meaning of wave-particle duality.

4.3.2 Wave-Particle Duality

Let us go to the very basics. What do we mean by a particle? We define it as an entity possessing a definite position, size, mass, velocity, momentum, energy, etc. Its motion is described by Newton's laws of motion. It must remain in a state of uniform motion or a state of rest when no external force acts on it. It should accelerate under the influence of a force field and move along a particular trajectory with a well defined position and time relationship. This, in a nutshell, is a picture of a particle that we have acquired from our studies in physics so far. If there exists any entity which does not conform to this description of a pnrticle, we should not call it a particle.

Now, what do we understand by a wave? A wave is characterised by its properties of periodicity in space and periodicity in time; it possesses a wavelength, amplitude, frequency and propagates at a certain wave velocity. It can transport energy without transport of matter. It cannot be localized and extends in space, These are the basic ideas associated with a wave. If we find anything which does not conform to all of these ideas, we should not call it a wave.

Having thus conceptualised a particle and a wave, the next step is fairly easy. If there exists something in nature which has neither purely particle properties nor purely wave properties but has properties of both, e.g., mass, momentum, wavelength, amplitude, frequency and is neither localized at a point nor extends to infinity, we should call-it neither a particle nor a wave. For want of a better name, we simply call it a wave-particle,

The fact that there are no particles and no waves in this universe but only wave-particle dualities should not unduly bother you. If that's the way nature works, that's the way we accept it. The definitions of a wave and a particle are still very useful and serve as good approximations as was amply demonstrated in SAQ 1, To sum up, the concept of wave-particle duality applies universally to all objects. However, since this duality involves Planck's constant, which has a very small value, the effect is appreciable only in the microscopic world. In the macroscopic world of our experience, objects obey the classical laws of motion.

You should also realise that wave-particle duality arises because of the finite value of Planck's constant. In classical physics it is assumed that h = 0, i.e., energy quanta do not exist. Hence it cannot explain wave-particle duality. However, by now you have seen that there are compelling reasons to take energy in the quantized form such that each quanta of energy is equal to $h\nu$ or its integer multiple.

One last word on the concept of wave-particle duality; its acceptance is not merely a question of belief or faith — it is a question of experimental observation and accepting a model for explaining them.

Let us now summarise what you have studied in this unit.

4.4 SUMMARY

- To explain black body radiation Planck proposed the idea of a quantum of energy for a harmonic oscillator. According to Planck's quantum postulate, a quantum of energy E for a wave is given by

  $$E = h\nu$$

  where ν is the frequency of the wave and h is a universal constant, known as Planck's constant.
- This idea was extended by Einstein to light to explain the photoelectric effect. Einstein regarded the quantum of light, the photon, to be a particle of energy $E = h\nu$.
- Bohr used the quantum postulate and certain other postulates in his atomic model and successfully explained the stability of atom and the line spectra of hydrogen atoms. He further gave the correspondence principle which establishes a correspondence between classical and quantum physics.
- Just as electromagnetic fields exhibit both wave-like and particle-like properties, de Broglie proposed that matter too bad wave-like properties, giving rise to wave-particle duality. The de Broglie wavelength of a particle of momentum p is

  $$\lambda = \frac{h}{p}$$

  For a free particle of mass m and energy E

  $$\lambda = \frac{h}{(2mE)^{1/2}}$$

4.5 TERMINAL QUESTIONS

Spend 30 min

1. Derive Bohr's angular momentum quantization condition for the Bohr atom from de Broglie's relation.
2. High energy protons of 200 GeV (1 GeV = 10^9 eV) are diffracted by a hydrogen target at an angle θ given by

   $$p \sin\theta = \frac{1.2}{c} \text{ GeV}$$

   Note that the protons are moving at relativistic energies. Estimate the radius of the proton.
3. A 150 eV increase in an electron's energy changes its de Broglie wavelength by a factor of two. Calculate the initial de Broglie wavelength of the electron.

4. Calculate the de Broglie wavelength and the kinetic energy of electrons which undergo first-order Bragg diffraction by a nickel crystal at an angle of 30°. For nickel, d = 2.15 Å.

4.6 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \text{Js}}{10^{-3}\ \text{kg} \times 10^{-2}\ \text{m s}^{-1}} = 6.626 \times 10^{-29}\ \text{m}$$

2. $$\lambda_e = \frac{h}{(2m_0 E)^{1/2}} = \frac{6.626 \times 10^{-34}\ \text{Js}}{(2 \times 9.109 \times 10^{-31}\ \text{kg} \times 100 \times 1.6 \times 10^{-19}\ \text{J})^{1/2}}$$

$$= 1.227 \times 10^{-10}\ \text{m}$$

$$\lambda_n = \frac{6.626 \times 10^{-34}\ \text{Js}}{(2 \times 1.675 \times 10^{-27}\ \text{kg} \times 10^{6} \times 1.6 \times 10^{-19}\ \text{J})^{1/2}}$$

$$= 2.862 \times 10^{-14}\ \text{m}$$

3. $$\lambda_e = \frac{h}{(2m_0 E)^{1/2}} = \frac{6.626 \times 10^{-34}\ \text{Js}}{(2 \times 9.109 \times 10^{-31}\ \text{kg} \times 400 \times 1.6 \times 10^{-19}\ \text{J})^{1/2}}$$

$$= 0.61 \times 10^{-10}\ \text{m} = 0.61\ \text{Å}$$

$$n\lambda_e = 2d \sin\theta$$

$$n = 2,\ \theta = 30°,\ \lambda_e = 0.61 \times 10^{-10}\ \text{m}$$

$$\therefore d = \frac{\lambda_e}{\sin\theta} = 1.22\ \text{Å}$$

Terminal Questions

1. De Broglie visualised that atomic electrons were confined waves and, therefore, produced a discrete stationary wave pattern. Then only those orbits would be allowed in which an integral number of electron wavelengths could fit the circumference (see Fig. 4.8). For example, one wavelength would fit the circumference of the lowest atomic orbit and two or more electron wavelengths would fit into higher orbits. Thus if de Broglie waves of wavelength λ fit a Bohr orbit of radius r to satisfy the stationary condition, we must have

$$2\pi r = n\lambda, \qquad n = 1, 2 \ldots..$$

Since $\lambda = \frac{h}{p} = \frac{h}{mv}$, we get

$$\frac{2\pi r\, mv}{h} = n$$

or $$mvr = \frac{nh}{2\pi}$$

which is the Bohr angular momentum quantization.

2. $1\ \text{GeV} = 10^9\ \text{eV}$

Let R be the radius of the proton. The dimension of the 'slit' which scatters protons is $2R$. Therefore

$$\lambda = 2 \times 2R \sin\theta$$

or $$\sin\theta = \frac{\lambda}{4R}$$

For protons $p = \frac{200 \text{ GeV}}{c}$ $(\because E = pc)$

$$\therefore \sin\theta = \frac{1.2 \text{ GeV}}{pc}$$

$$= \frac{1.2}{200} = 0.006$$

From de **Broglie** relation

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ Js} \times 3 \times 10^{8} \text{ ms}^{-1}}{200 \times 10^{9} \times 1.6 \times 10^{-19} \text{ J}}$$

$$= 6.212 \times 10^{-18} \text{ m}$$

$$R = \frac{\lambda}{4 \sin\theta} = \frac{6.212 \times 10^{-18} \text{ m}}{4 \times 6 \times 10^{-3}}$$

$$= 2.5 \times 10^{-16} \text{ m.}$$

The radius of the proton is of the order of 10^{-16} **m.**

3. Since the energy of the electron increases, its wavelength will decrease. If λ be the initial electron wavelength, E its initial energy and ΔE, the increase in energy, we can write using Eq. **(4.12):**

$$\lambda = \frac{h}{(2mE)^{1/2}}$$

and $$\frac{\lambda}{2} = \frac{h}{[2m(E+\Delta E)]^{1/2}}$$

Simple algebra yields the relation

$$\lambda^2 = \frac{3h^2}{2m\,\Delta E}$$

or $$\lambda = h\left(\frac{3}{2m\Delta E}\right)^{1/2}$$

Substituting the values of h, m and AE = **150 eV,** we get

$$\lambda = 6.626 \times 10^{-34} \text{ Js}\left[\frac{1.5}{9.1 \times 10^{-31} \text{ kg} \times 150 \times 1.6 \times 10^{-19} \text{ J}}\right]^{1/2} = 1.73 \text{ Å}$$

4. For the first-order Bragg diffraction, n = 1 and

$$\lambda = 2 \mathbf{d} \sin 0$$

$$= 2 \times 2.15 \text{ Å} \sin 30°$$

$$= 2.15 \text{ Å}$$

The kinetic energy of the electron is obtained from Eq. **(4.12):**

$$E = \frac{h^2}{2m\lambda^2}$$

or $$E = \frac{(6.626 \times 10^{-34} \text{ Js})^2}{2 \times 9.1 \times 10^{-31} \text{ kg} \times (2.15 \times 10^{-10})^2 \text{ m}^2} = 52 \times 10^{-19} \text{ J} = 32.5 \text{ eV.}$$