BLOCK 1 THE SPECIAL THEORY OF RELATIVITY

Albert Einstein (1877–1955)

What is it that you think of when you hear or see the word **relativity?** The name of Albert Einstein? Or the equation $E = mc^2$? Or a vision of astronauts who return young from their trips to space lasting many years? This signifies the enormous intellectual impact (even after almost a hundred years) of what Einstein called his **special theory of relativity.** The development of this theory is rightly regarded as one of the greatest strides ever made in our way of understanding the physical world. And yet the basic concept of relativity is as old as the mechanics of Galileo and Newton. So what did Eistein do to make his name almost synonymous with relativity?

At the beginning of the twentieth century, two great and beautiful theories were known in the physical sciences – Newtonian mechanics and Maxwell's electrodynamics. Both of them gave a unified explanation of countless physical phenomena. These theories were expressed in concise mathematical language within a certain conceptual framework. You have studied both these theories in Block 1 of PHE-01 (Elementary Mechanics) and Block 4 of PHE-07 (Electric and Magnetic Phenomena), respectively. You have also learnt of the numerous applications of these theories. Both theories have been confirmed many a times; and they have been extraordinarily successful in their predictions. And yet these two theories were *conceptually* in contradiction with one another!

What was this contradiction? Unit 1 of this block explains it as well as the dilemma which occupied the best minds of the time in the physical sciences. How was this contradiction resolved? Its solution was provided by none other than Albert Einstein, He resolved the contradictions as he saw them and formulated a new theory based on two new principles, These two principles led Einstein to a new view of space and time about which you will study in Unit 2. Naturally, a radically new attitude to the notions of space and time resulted in changes in well-established areas of physics. In Unit 3, we shall discuss the new mechanics that replaced Newtonian mechanics as a result of these changes.

What we had, in effect, was an Einsteinian revolution. Its impact far exceeds that of the Copernican revolution and has rarely been equalled in the history of physics. We have tried our best to bring to you the beauty and logic of the special theory of relativity. We hope that you will appreciate and enjoy studying it as much as we did presenting it. We wish you good luck.

UNIT 1 EMERGENCE OF SPECIAL RELATIVITY

Structure

1.1 INTRODUCTION

You have studied Newtonian mechanics in your school science courses and in your first physics elective entitled 'Elementary Mechanics' (PHE-01). You are familiar with the concept of inertial frames of reference. You know that Newton's laws of motion are the same in all inertial frames of referenc- You must have recognized the validity of this statement in everyday life. An objecf moves in the same way in a uniformly moving train or an aeroplane as it does on earth. For instance, when you walk, drop a coin or throw a ball up in the air while riding in such a train or aeroplane, the bodies move just as they do on earth. Both Galileo and Newton were deeply aware of this principle that the laws of mechanics are the same in all inertial reference frames; this is the classical principle of relativity. So the classical notion of relativity is not new to you. However, you have not encountered this terminology before. Therefore, we shall begin this unit with a brief review of the classical notion of relativity as embodied in the works of Galileo and Newton.

You know that Newtonian Mechanics was highly successful in describing motion in the world of our everyday experiences. Then why did the need arise for re-examining Newtonian mechanics and the notion of relativity it contained? The need arose when the classical principle of relativity was applied to the propagation of electromagnetic waves and that led to certain inconsistencies. In Sec. 1.3, you will learn about some of these inconsistencies and find that the Newtonian relativistic world view could not easily incorporate the laws of electromagnetism. The question is: What replaced it? It was replaced by a radically different way of understanding the world when, in 1905, Albert Einstein proposed his special theory of relativity. In the last section (1.4) of this unit you will study the main features of this theory.

"What I see in Nature is a magnificent structure which we comprehend only very imperfectly, and that must fill a thinking person with a feeling of humility."

–Albert Einstein, 1944/5

In brief, what we intend to say in this unit is this: Einstein was not the first to introduce relativistic notions in physics. What he did was to generalise the classical notion of relativity (applicable only to mechanics) to all physical phenomena. Although we will go into some detail of the background in which Einstein's special relativity emerged, it will not necessarily be a historical description. We will simply bring out the factors which induced scientists to change their concepts in so radical a manner. In this process we hope that you will be able to appreciate and understand the special theory of relativity much better.

In the next unit, you will learn about the consequences of the special theory of relativity. In particular, you will understand in what way special relativity altered the established notions of space and time.

Objectives

After studying this unit you should be able to

- use the Galilean coordinate transformations to describe events in different inertial frames of reference
- explain the Galilean principle of relativity and state why it became necessary to generalise it
- state the postulates of special theory of relativity
- apply the principle of relativity to physical phenomena
- compare the nature of time in classical relativity and the special theory of relativity.

Study Guide

This unit presents the background out of which the special theory of relativity emerged. Therefore, we shall be using many concepts and ideas from our earlier physics courses. We strongly advise you to go through the Block 1 of PHE-01 (Elementary Mechanics), Block 4 of PHE-07 (Electric and Magnetic Phenomena) and Block 2 of PHE-09 (Optics) before studying this unit. It will help you in understanding the ideas presented in Sec. 1.2 and 1.3 better, and in less time. In our estimate, you should take about 6 to 7h to complete this unit.

1.2 CLASSICAL RELATIVITY

You may like to refer to Sec. 2.2.1 of Unit 2, PHE-01 where the concept of an inertial observer has been discussed in detail. What we have said there for inertial observer applies to an inertial frame of reference.

You have studied the concepts of inertial frames of reference and relative motion in Unit 1 of PHE-01. You are familiar with the relationship between the velocity and acceleration of an object measured with respect to two inertial frames in uniform relative motion. You have also studied the laws of Newtonian Mechanics in Unit 2 of PHE-01. Here we shall use these concepts to explain briefly the notion of classical or Galilean relativity.

Let us begin by considering a physical event. An idealised version of an event is that it is something that happens at a point in space and at an instant in time. While discussing the theory of relativity, Einstein often used this dramatic example of an event—lightning strikes the ground. A small explosion is an equally dramatic event. You can think of several other examples of an event. There are two basic questions that we can ask about any event:

Where did it take place?

When did it take place?

How do we answer these questions? As you are well aware, we specify an event by four measurements in a particular frame of reference — three for the position and one for the time t. We usually fix the position of the event by the Cartesian coordinates (x, y, z). You have used the Cartesian coordinate system quite often in your physics elective courses, For example, two particles may collide at $x = 1$m, $y = 2$m, $z = 3$m and at time $t = 4$s in one frame of reference such as a laboratory on the earth. Then the four numbers (1, 2, 3, 4) specify the event in that reference frame: the first three numbers specify its position and the fourth the time at which it occurred.

Thus, we must first establish a frame of reference to accurately describe where and when an event happens. You know that for describing an event we are free to use any frame of reference we wish. In this course we shall restrict our study to what are called inertial reference frames, Recall that

An inertial frame is a frame of reference in which Newton's first law holds true.

So in an inertial frame of reference, objects at rest remain at rest and objects moving uniformly in a straight line continue to do so, unless acted upon by a not external force. From this concept you can readily conclude that

Any frame that moves with constant velocity relative to an inertial frame is also an inertial frame.

Would you like to test whether you understand the concept of an inertial frame of reference before studying further? If so, try the following SAQ.

Spend 2 min

SAQ 1

Classify the following frames of reference as inertial and non-inertial (i.e., frames which are not inertial). The frames attached to

a) a car in circular motion

b) spaceships cruising uniformly

c) an electron accelerating in an electric field in space

d) a boat moving at a constant speed in a river flowing uniformly

e) an apple at rest on a fixed table in your room.

Suppose now that we have made space and time measurements describing an event in one inertial frame of reference. We want to describe the same event in another inertial frame of reference. For example, consider the following event. A boy throws a ball vertically upwards in a train moving at a uniform velocity with respect to the ground. In the frame of reference attached to the train, the ball goes straight up and comes down along the same path. Now how do we describe the ball's motion in another frame of reference attached to the ground?

We can use the Galilean coordinate transformations to describe an event in different inertial frames of reference. Let us briefly study the Galilean coordinate transformations.

1.2.1 Galilean Coordinate Transformations.

Consider an inertial frame S and another frame S' which moves at a constant velocity u with respect to S (Fig. 1.1). We define the x-axis (and the x'-axis) to be along the direction of motion. We assume the other two axes (y, z) and (y', z') to be parallel to each other — y parallel to y' and z parallel to z'. Further, we define the origin of time, $t = 0$, to be the instant when the origins of the two coordinate systems coincide, i.e., when point O' coincides with O.

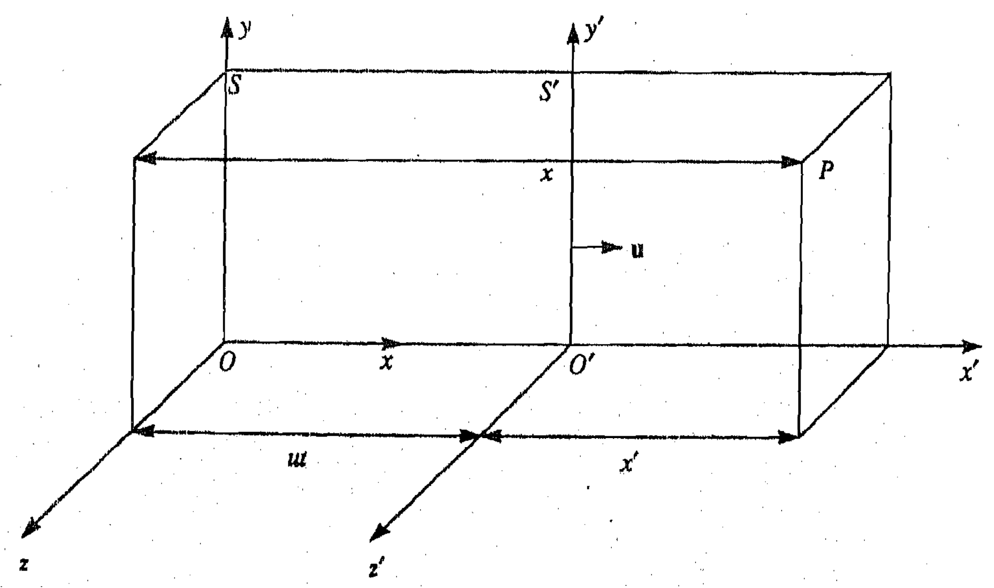


Fig.1.1: Two inertial frames of reference S and S'. S' moves with a constant velocity u ($= u\hat{i}$) with respect to S so that the x-x' axis is common and the y-y', z-z' axes are parallel. As seen from frame S', S moves with a velocity $-$u, i.e., at a speed u in the negative x direction. Point P represents an event whose space-time coordinates can be measured by observers in S and S'. The origins O and O' coincide at time $t=0$ and $t'=0$. You can see that $x = x' + ut$, $y' = y$ and $z' = z$.

Suppose that an event E occurs at point P. Let us assume that any measurement in the two frames of reference are being made by observers who have jointly calibrated their metre sticks and clocks. The observer attached to S ascribes the coordinates x, y, z, r to P and the observer attached to S' specifies the same event by x', y', z' and t'. The coordinates (x, y, z) give the position of P relative to O as measured by observer S and t is the time at which E occurs as recorded by the clock af S. The coordinates (x', y', z') give the position of P with respect to O' and t' is the time at which E occurs according to the clock of S'. For simplicity, the clocks of each observer read zero at the instant that the origins O and O' of the frames S and S' coincide.

What is the relationship between (x, y, z, t) and (x', y', z', t')? The Galilean coordinate transformation (see Fig. 1.1) relates these measurements as follows:

$$x' = x - ut$$

$$y' = y \tag{1.1}$$

$$z' = z$$

$$t' = t$$

We can write the generalized Galilean transformation in vector notation as fallows:

$$\mathbf{r}' = \mathbf{r} - \mathbf{u}t \tag{1.2a}$$

$$t' = t \tag{1.2b}$$

where, $\mathbf{r}$ is the position vector of P in S and $\mathbf{r}'$ in S'. Under our simplifying assumptions u = $u\mathbf{i}$, and Eqs. (1.2a, b) reduce to Eqs. (1.1).

Differentiating Eq. (1.2a) with respect to t gives

$$\frac{d\mathbf{r}'}{dt} = \frac{d\mathbf{r}}{dt} - \mathbf{u} = \mathbf{v} - \mathbf{U}$$

But since $t = t'$, $\frac{d\mathbf{r}'}{dt} = \frac{d\mathbf{r}'}{dt'} = \mathbf{v}'$. Hence, we get

$$\mathbf{v}' = \mathbf{v} - \mathbf{u} \tag{1.3a}$$

Differentiating Eq. (1.3a) with respect to t and using Eq. (1.2b) we get

$$\mathbf{a}' = \mathbf{a}, \text{ since u is constant.} \tag{1.3b}$$

You have encountered Eqs. (1.3a and b) in Sec. 1.5 of Unit 1, PHE-01. These are the Eqs. (1.37) and (1.38) given there

The equation of motion is then given as

$$m\,\mathbf{a}' = m\,\mathbf{a} = \mathbf{F} \tag{1.4}$$

This means that we obtain the same law of motion in the frame S' as in the frame S.

In relation to Eq. (1.4) we would like to ask another question, How does the force F transform when we go from one frame to another? You know ihat forces considered in mechanics depend either on distance (gravitational forces, elastic forces) or on relative velocity (friction forces) and on time interval. Sa let us find out how distance, relative velocity and a time interval change under the Galilean coordinate transformations,

Suppose we investigate two objects P and Q. Let the force of their interaction depend on the distance between them, their relative velocity and time. From Eqs. (1.1) we can at once see that the distance between P and Q, measured at the same instant is the same in S and S':

$$x'_P - x'_Q = x_P - x_Q, \quad y'_P - y'_Q = y_P - y_Q, \quad z'_P - z'_Q = z_P - z_Q$$

or in vector notation

$$\mathbf{r}'_P - \mathbf{r}'_Q = \mathbf{r}_P - \mathbf{r}_Q \tag{1.5a}$$

On differentiating Eq. (1.5a) with respect to time we find that the relative velocity of P with respect to Q remains the same in bath the frames of reference.

$$\mathbf{v}'_P - \mathbf{v}'_Q = \mathbf{v}_P - \mathbf{v}_Q \tag{1.5b}$$

Remember that in arriving at Eq. (1.5b) we have also used the fact that Galilean transformation does not change time and so also the time interval between any two events say A, and B:

$$t'_A - t'_B = t_A - t_B \qquad (1.6)$$

Hence, we can conclude that forces occurring in mechanics, that depend on time intervals, distance and relative velocity, do not change under the Galilean transformation. We say that forces remain invariant under Galilean **transformation.** Thus, all the quantities appearing in Eq. (1.4) do not change under the Galilean transformation. Therefore, the fundamental equation of classical mechanics – Newton's second law – has the same form in a stationary frame S as in a frame S' moving with constant velocity with respect to S. With this information at our command, we are now ready to present the classical principle of relativity, It is also called the **Galilean** principle of relativity, since it was Galileo who first enunciated it, although its mathematical basis given above was provided only later by Isaac Newton.

1.2.2 Galilean Principle of Relativity

Eqs. (1.5a and b) and (1.6) tell us that according to the Galilean transformations the time interval, space interval (distance) and relative velocity measurements and hence the force law in mechanics is the same in all inertial frames. The relative velocity of the frames can be arbitrary and does not affect these results. Implicit in Eq. (1.4) is the basic postulate of classical mechanics that the mass of a body is constant, **i.e.,** it is an invariant quantity.

So what do Newtonian mechanics and Galilean transformation put together imply? The length, mass and time – the three basic quantities in mechanics – as well as forces (which depend upon time interval, space interval and relative velocity) are independent of the relative motion of an inertial observer. The laws of mechanics hold good in all inertial frames of reference. Thus, we arrive at the classical or Galilean principle of **relativity:**

> The laws of mechanics can be written in the **same form** in all inertial frames. If they hold in one inertial frame, they will also hold in all other inertial frames.

It is a limited principle of relativity in that it applies to only the laws of mechanics. Let us understand with the help of a simple example what the Galilean principle of relativity means. Suppose you arc in a car which is moving at a constant speed and **you** cannot look out. Then to you, all mechanical experiments performed and all mechanical phenomena occurring in the car will appear the same as if the car were not moving. For instance, a ball thrown vertically upwards will always fall down along the same path.

No mechanical experiment performed in the car could help you determine whether it was moving uniformly or was at rest provided, of course, you did not look out. This is what we mean when we say that if the laws of mechanics are true in one inertial frame of reference, they will be true and of the same form in any other inertial frame as well. Thus, as far as mechanics is concerned there is no preferred inertial frame of reference in which alone the classical laws have the most basic form. Therefore, there is no absolute frame of reference.,

You may like to pause for a while and find out whether you have understood these ideas. Try the following SAQ.

SAQ 2

Spend 2 min

a) Does the fact that Eq. (1.4) is invariant under Galilean transformation mean that all inertial observers will measure the same values for the position, time; velocity, energy and momentum corresponding to an event?

b) Are the laws of conservation of linear momentum and energy invariant under Galilcan transformation?

An interesting aspect of the classical principle of relativity pertains to the nature of space and time. And we would like you to know about it.

Absolute Space and Absolute Time

You have just studied that according to Newtonian mechanics and Galilean relativity, the measurements of length (relative position), mass, time and their relationship are independent of the relative motion of an inertial observer. They do not depend on which inertial observer measures them. This implies that there exist absolute space intervals and absolute time intervals in Newtonian mechanics. In other words we may say that space and time exist in themselves and have properties that do not depend upon anything else.

In Newton's own words: "Absolute space, in its own nature, without relation to anything external, remains always similar and immovable." So according to Newton, space represents a giant empty box which contains material objects and various physical phenomena take place in it, and it does not get affected by these. Likewise time is thought to flow absolutely, uniformly and evenly without king affected by any actual events that happen as time passes. Once again we quote Newton: "Absolute, true and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and is otherwise called duration".

Thus, in Newton's world-view space and time are things external in relation to nature. Further, there is no relationship between space and time – the properties of space are determined independently of movements of objects with the passage of time; the flow of time is independent of the spatial properties of such objects.

In a nutshell, as per Newtonian ideas, space and time exist by themselves, independent of each other and do not depend on material bodies located in space or physical phenomena occurring therein.

We have briefly introduced these ideas here so that you are able to appreciate the difficulties that arose when the classical principle of relativity was applied to electricity, magnetism and optics. Let us now study how classical relativity proved to be inconsistent with the laws of electromagnetism.

1.3 ELECTROMAGNETISM AND CLASSICAL RELATIVITY

You have thus far studied that the Galilean principle of relativity applies to mechanical phenomena. The question that arises next is: Do other laws of physics (e.g., the laws of electromagnetism and optics) have the same form in all inertial frames? In other words, are they invariant under a Galilean transformation?

In fact, when the principle of relativity was applied to Maxwell's equations, certain problems arose immediately — they did not seem to obey it. Let us briefly outline some of these problems.

1.3.1 Problems of Relativity vis-a-vis Laws of Electromagnetism

Let us first consider a simple example of two equal, positive point charges carrying charge q, as shown in Fig. 1.2a. We will first examine the system as seen by an observer in the reference frame S'.

As you can see in Fig. 1.2a one charge rests at the origin of S' and the other rests at a distance y'_2 on the y' axis of S'. From Maxwell's equations we can determine the electromagnetic force that the charges at rest exert on each other in S': it is just the electrostatic Coulomb force of magnitude $F_C = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{y_2'^2}$.

Let us now consider the electromagnetic force from the point of view of S. This observer sees the charge q unchanged and $y_1 = y'_2$. So Coulomb's force law is unchanged. However, the observer in S also sees both charges moving to the right at a speed v. Now two positive charges moving to the right constitute two conventional parallel currents which attract each other, Therefore, the total force in S has two components – the electrostatic force of repulsion and the attractive force between parallel curents. We find that it is different from the force in S'. But according to Newtonian physics, these forces should be the same, This is an inconsistency (Fig. 1.2b).

Another problem arises when we try to transform Maxwell's equations according to Galilean coordinate transformations — they change their form. For example, the wave equation for electromagnetic fields, deduced from Maxwell's equations does not remain the same (See Unit 14 of PIE-07 entitled Electric and Magnetic Phenomena for

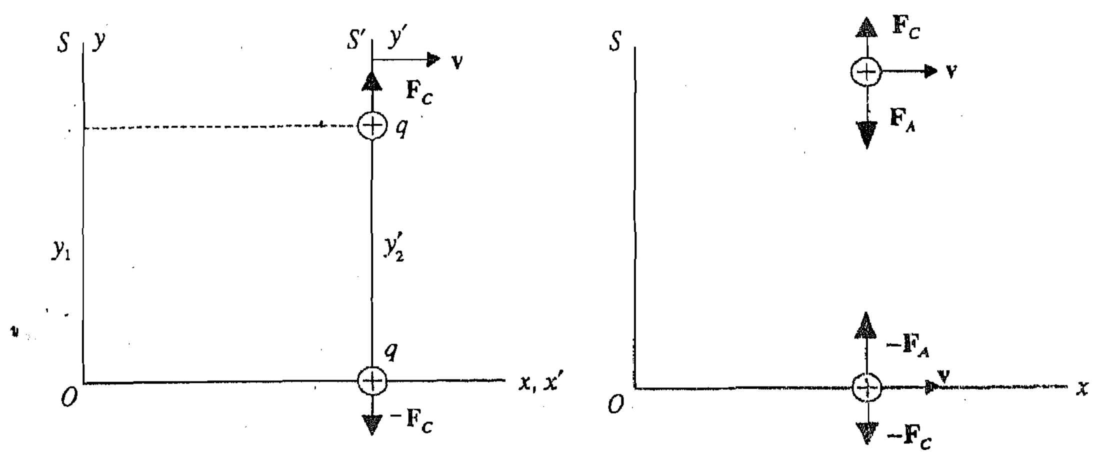


Fig. 1.2 : (a) Two equal positive point charges (carrying charge q) at rest on the y' axis of the frame of reference S'. In S' the charges repel each other with a force of magnitude F_C; (b) as seen in S the charges are moving to the right with a velocity v and attract each other with an additional force of magnitude F_A, giving a total force of magnitude $|F_C - F_A|$.

Maxwell's equations and the electromagnetic wave equation). It is a simple exercise and you could try it out yourself.

Spend 10 min

SAQ 3

Show that the electromagnetic wave equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = 0$$

does not retain its form (i.e., it is not invariant) under the Galilean transformations (Eq. 1.1).

Hint: Use the chain rule in which if x is a function of (x', y', z', t'), then for any function f,

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'}\cdot\frac{\partial x'}{\partial x} + \frac{\partial f}{\partial y'}\cdot\frac{\partial y'}{\partial x} + \frac{\partial f}{\partial z'}\cdot\frac{\partial z'}{\partial x} + \frac{\partial f}{\partial t'}\cdot\frac{\partial t'}{\partial x}$$

Thus, there seems to be a fundamental disagreement between Maxwell's theory of electromagnetic fields, Newtonian mechanics and Galilean principle of relativity. Historically, this disagreement centred around the 'problem of light'. We too would like to focus on this problem. However, we shall confine ourselves only to one aspect of light, namely its propagation, You know that one of the consequences of Maxwell's equations is that light is an electromagnetic wave which propagates in all directions at the same speed $c = 3 \times 10^8 \text{m s}^{-1}$. Another consequence of the equations is that if the source of light is moving, the light emitted still travels at the same speed c. This brings up an interesting problem when Galilean relativity is applied to the propagation of light. Let us examine it in some detail.

1.3.2 Galilean Relativity and the Speed of Light

The wave nature of light was recognised even before Maxwell described its electromagnetic nature (for example, in the works of Young, Huygens and Fresnel). A search was on to find the medium in which light propagated, Sound waves require air to propagate and ocean waves travel on water. So what was the medium for propagation of light? Nineteenth century physicists believed that light propagated through a rarefied, all-pervasive (space filling) elastic medium called **lumineferous ether.** It was assumed to be so fine that planets and other heavenly bodies passed through it without appreciable friction. When Maxwell described the electromagnetic nature of light waves, the associated electric and magnetic fields were visualised as stresses and strains in the ether. It was in his medium that light propagated with a speed $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$. When the Galilean principle of relativity was applied to the propagation of light in ether, it led to an inconsistency.

To understand the inconsistency, let us consider a frame S with respect to which light travels with velocity **c**. What is the velocity of light in a frame S' moving with a constant velocity **u** with respect to S? We can carry out the Galilean velocity transformation and obtain

$$\mathbf{v}' = \mathbf{c} - \mathbf{u}, \quad |\mathbf{v}'| = \left(c^2 + u^2 - 2\mathbf{c} \cdot \mathbf{u}\right)^{1/2} \tag{1.7}$$

where $|\mathbf{v}'|$ is the speed of light in S'. Clearly the speed of light in S' depends on the direction in which it is travelling. If **c** is in the direction of **u**, the speed of light in S' is c − u. In the direction opposite to **u**, the speed of light in S' is c + u. In any other direction it has a value between c − u and $c + u$ as given by Eq. (1.7). We can also see that according to Galilean relativity principle, the speed of light would be different in different inertial frames of reference. In other words, Maxwell's equation would have to be of different forms in different inertial frames of reference, to give different speeds of light in those frames. So it appears that the Galilean principle of relativity is incompatible with the laws of electromagnetism, which give a constant speed of light.

Now suppose we accept both the Galilean transformation and the laws of electromagnetism (or Maxwell's equations) as basically correct. Then it follows that there is one unique privileged inertial frame of reference (the absolute frame) in which Maxwell's equations are valid. In this unique frame the speed of light would be $c = 1/\sqrt{\mu_0 \varepsilon_0}$ whereas in other frames it would be different.

Let us now put all these developments in physics which led to the special theory of relativity in a perspective. The situation towards the end of nineteenth century seems to be as follows: The Galilean relativity principle does apply to Newton's laws of mechanics but not to Maxwell's laws of electromagnetism. This requires us to choose the correct consequences from among the following possible allematives.

1. **Retain the Relativity Principle for mechnnics but not for electrodynamics.**	2. **Retain the Relativity Principle** for **both mechanics and electrodynamics but** hold **the laws** of **electromagnetism** as **not** correct.	3. **Retain the Relativity Principle for both mechanics and electrodynamics; but hold that the Newtonian mechanics is nut correct.**
This would mean that **Newton's mechanics remains unchanged. But the** laws **of electromagnetism hold only in one** privileged **frame of** reference, **that is the ether frame. If this alternative were** correct we **should** be able **to locate the ether frame experimentally.**	**If this alternative were correct,** we **should be able to do experiments that show deviations from the electromagnetic theory. Then we would need to reformulate the** laws **of** electromagnetism so **that the Galilean transformations apply to the new Laws.** .	**If this alternative were correct then** we should **be able to do** experiments which show **deviations** from Newtonian **mechanics. Then we would need to reformulate** Newtons's **Laws. We would** also have **to** give **up** the **Galilean transformation because they do not give us an invariant** form **of** Maxwell's **equations. We** shall **need to look for some** other transformation **which** is **consistent** with classical **electromagnetism and the new laws of mechanics.**

Several investigations **were carried** out to decide which of **the three** alternatives **was** correct. Their net outcome was to provide an experimental basis for rejecting the **alternatives** 1 and **2. The** most famous of *these* experiments is the one **performed** by **Michelson** and Morley in 1887 to **locate the** absolute frame. You **have** studied **this** experiment in Unit 7, **Block 2 of** the **course PHE-09** on optics. However, let us **briefly** study **this celebrated** historic **experiment.**

1.3.3 Attempts to Locate the Absolute Frame – The Michelson-Motley Experiment

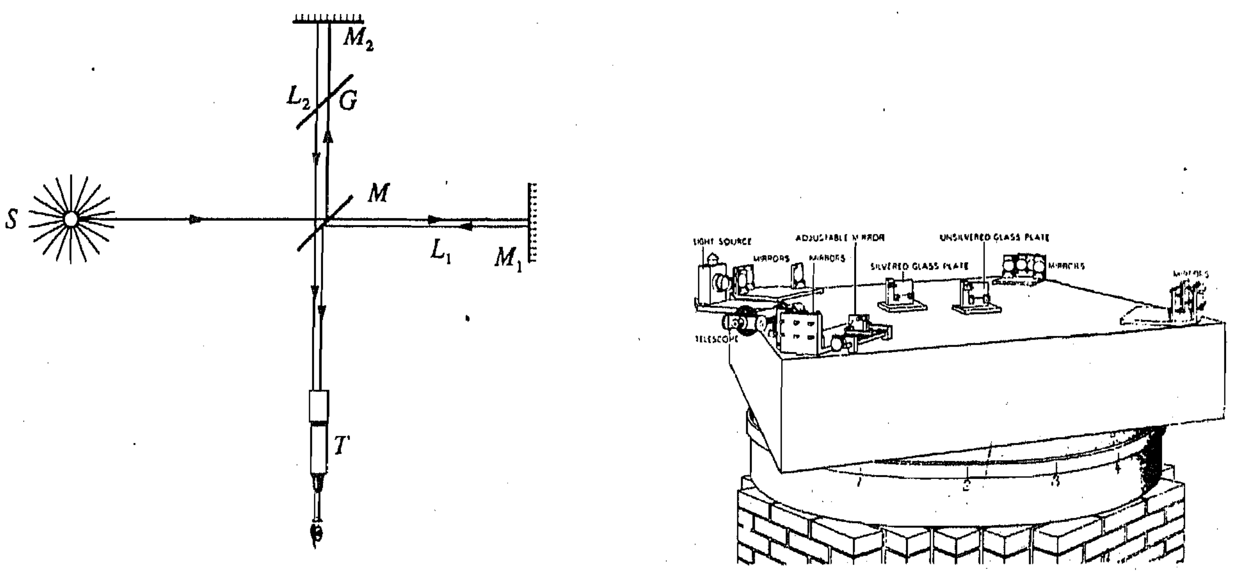
Let us first understand what was being investigated through this experiment. Consider a simple example. When we say that sound travels at 340 m s^{-1}, we are refemng to the speed of sound with respect to air through which it propagates. If we move through still air towards an oncoming sound wave at a speed of 30 m s^{-1} (relative to the air), we observe the speed of sound to be 310 m s^{-1}. Clearly, the speed of sound relative to us varies with our speed **relative** to air.

Now the ether hypothesis suggests that the earth is moving in the ether medium as it orbits the sun. Therefore. in analogy to the example above. we can say that the speed of light relative to an observer on the earth varies with the earth's speed relative to the ether. The speed at which the earth orbits the sun is 30 km s^{-1}, about 0.01% (10^{-4}) of the speed of light. This is the maximum change which we can observe in the speed of light on earth as it moves through ether. Michelson, in 1881, and then in collaboration with Morley, in 1887, performed an experiment designed to detect such a change in the speed of light.

The essential principle of the experiment was to send a light-signal from a source to a mirror and back, noting the total time taken. The experiment was to be done twice:

(i) in the direction of earth's motion in ether, and

(ii) at right-angles to it.



(a) (b)

Fig.1.3: (a) Schematic representation of Michelson- Morley experiment; (b) the apparatus.

In the experiment a beam of light from a source S (fixed with respect to the apparatus) is separated into two coherent beams by a partially silvered mirror M inclined at 45° to the beam direction (Fig. 1.3a). Two mirrors M, and M_2 are placed at nearly equal distances from M and at right angles to each other. These reflect the beams back to M. A part of each of the two beams reflected by M_1 and M_2, respectively, are reunited at M and the recombined beams are observed through a telescope T. A glass plate G is placed between M and M_2 to compensate for the extra distance travelled by light through M to M,. Now if you have studied Block 2 of the course PHE-09, you would realise that when the two parts of the split beam recombine, they will **interfere.** Now, suppose the time taken for light to travel from M to M_1 and back is t and the time required to travel from M to M_2 and back is t'. Then the interference will be constructive at a given point if at that point the difference

$$t - t' = nT, \qquad n = 1, 2, 3, ... \qquad (1.8a)$$

and destructive if

$$t - t' = n + \frac{1}{2}T, \qquad n = 1, 2, 3, ... \qquad (1.8b)$$

where T is the time period of the light wave. So the existence of a time difference between the two paths travelled by light influences the illumination of a given point (say A) viewed through the telescope. It is either bright (constructive interference) or dark (destructive interference). If M_1 and M_2 are very nearly at right angles, the fringe pattern consists of nearly parallel lines.

Now, suppose we rotated the entire apparatus by 90° in the plane of M, M_1 and M_2. Then the orientation of MM_1 and MM_2 relative to the direction of Earth's motion through ether would change. Such a rotation would alter the time taken along each path and thus change the illumination of the point A. Or we could say that the fringe pattern would shift. It was this change in illumination of a given point (or a shift in the fringe pattern), as a result of rotation, that Michelson actually tried to detect. The expected shift was of the order of four-tenths of a fringe.

Michelson and Morley took utmost care in eliminating all possible sources of error, such as stresses and temperature effects. And this shift should have been clearly observable. Nevertheless,

no fringe shift was observed.

If you with to go into the historical details you may read the first book listed as further readinp.

One could say that at the time when the experiment was being done, the earth was at rest relative to the ether, However, the result did not change when it was repeated after a gap of six monlhs. Indeed, this experiment was repeated many times by many workers over a 50-year period, in more sophisticated ways, at different times of the year. But the result was always the same. As far as Michelson was concerned, its implication was clear as he wrote at that time

'The result of the hypothesis of a stationary ether is shown to be incorrect.'

Needless to say, the ether hypothesis was not given up immediately. Several interpretations of the null result of this experiment to preserve the concept of ether were suggested, We will not go into the details of all these interpretations because, with time, (as evidence accumulated) it turned out that these were either inconsistent with observations and experiment or lacked sound conceptual bases.

Einstein was motivated by both the problem of the ether as a preferred reference frame and his thoughts on Maxwell's electrodynamics and, in particular, on Faraday's law of induction. Recall from Unit 13 of PHE-07 that Faraday's law of electromagnetic induction refers to the relative motion of a wire loop and a magnet. That motion together with the magnetic field of the magnet results in a current flowing in the loop. The detailed explanation of this effect as given at that time was not symmetrical. It was not the same when looked at from the reference frame of the magnet and from the reference frame of the loop. Einstein felt that this phenomenon should be exactly symmetrical since only relative motion is involved. He resolved both these problems by a fiat: by postulating the principle of relativity he insisted that all laws be valid for all inertial observers. The second postulate stated what he believed nature was trying to tell us all along. The constancy of the speed of light was for him not something that needed to be explained. It was a new law of nature that had to be accepted.

Various experiments performed to measure the speed of light over the years have confirmed this result. Indeed, the speed of light in free space has been found to be constant at all times. It is independent of the place where measurements were carried out, It does not depend on its frequency, nature and motion of its source. direction of its propagation. It is also constant with respect to all inertial frames of reference. Thus, experiments help us to accept, indisputably, the following principle.

The speed of light in free space is a universal constant.

This result obviously contradicts the Galilean principle of relativity. At the same time the laws of electromagnetism are upheld by experiment. Moreover in certain other experiments performed in the early twentieth century departures from Newtonian mechanics were observed. In 1902, the motion of electrons (emitted by radioactive sources) in electric and magnetic fields was investigated experimentally. It was found that Newton's second law did not correctly describe the motion of these electrons which moved with velocities close to that of light. To sum up, we have found that the classical principle of relativity is incompatible with the laws of electromagnetism. Michelson-Morley experiment fails to detect ether (i.e., an absolute frame of reference). Thus, the ether hypothesis is untenable. It is experimentally established that the speed of light, in free space, is a constant.

What is more, experiments done on electrons moving at speeds close to that of light in electric and magnetic fields show a breakdown of Newton's laws of motion. Hence we can see that a relativity principle. applicable to both mechanics and to electromagnetism, is operating. Clearly it is not the Galilean principle, since that requires the speed of light to depend on the frame of reference in which it is measured. We conclude that the Galilean transformations should be replaced. Hence, the laws of mechanics, which are consistent with these transformations, need to be modified.

The discussion so far gives you an idea of the background in which Einstein's special theory of relativity emerged. Let us now study the special theory of relativity.

1.4 THE SPECIAL THEORY OF RELATIVITY

You have studied in the previous section that the constancy of the speed of light in all inertial frames stands in contradiction with the Galilean transformations. In 1905, Albert Einstein (1879-1955) presented a revolutionary proposal which resolved this contradiction. Rather than modifying electromagnetic theory, he rejected the ether hypothesis and generalised Galilean principle of relativity. In his paper "On the Electrodynamics of Moving Bodies", Einstein formulated the two postulates of the special theory of relativity, which we rephrase here:

> Postulates of the Special Theory of Relativity
>
> Postulate 1 — The Principle of **Relativity**
>
> The laws of physics are the same in all inertial frames of reference.
>
> Postulate 2 — The Principle of Constancy of Speed of Light
>
> The speed of light (in vacuum) has the same constant value in all inertial reference frames.

These two assumptipns led Einstein to a new theory of physics which is now known as the special theory of relativity. It is special because it only deals with observations made in inertial frames. For example, it does not say anything about the relationship between two frames undergoing relative acceleration. Non-inertial frames are the subject matter of another of Einstein's theories — the general theory of relativity.

Let us now understand the meaning of these postulates.

1.4.1 The Principle of Relativity

You have briefly studied the Galilean principle of relativity which applied to the Newtonian laws of mechanics. This limited principle has now been generalised to all laws of physics — any law of physics that is true in one inertial frame will also be true in all other inertial frames. Let us consider an example to understand this statement.

Suppose a positive electric charge q was fixed at a point (X, 0, 0) in a stationary inertial frame S (Fig.1.4). If another positive charge q' was released at some point on the x-axis. it would accelerate away from the fixed charge at $x = X$. We could experimentally determine the x component of the acceleration of the moving charge as a function of its distance from the fixed charge. The relationship would be of the following form:

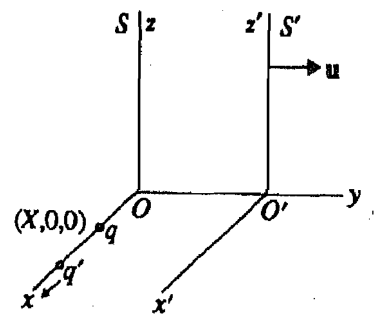


Fig. 1.4: A positive charge q is fixed at $x = X$, $y = 0$, $z = 0$ in frame S. Another positively charged particle q' released from a point on the x-axis experiences an acceleration awry from the charge.

For a charged particle movingaway from a fixed charge

$$\frac{d^2 x}{dt^2} = \frac{k}{(x - X)^2} \qquad (1.9a)$$

where k is a constant.

Now suppose that another observer is stationed in an inertial frame of reference S' moving with respect to S with a constant velocity **u**. The principle of relativity tells us that if Eq. (1.9a) is really a law of physics, then the observer in the S' frame should find that

$$\frac{d^2 x'}{dt'^2} = \frac{k'}{(x' - X')^2} \qquad (1.9b)$$

where X' is the location of the fixed charge q on the x'-axis of the S' frame. Thus, even though the values of x', t', X' and the constant k' are different from x, t, X, and k, the relationship between them is of the same form as Eq. (1.9a). Conversely, we can also say that any equation that cannot be written in the same form in all inertial frames cannot be a law of physics. So the principle of relativity also allows us to determine which relationships (or equations) can or cannot be laws of physics,

Incidentally, using this principle Einstein discovered that many of the equations which were thought to be 'laws of physics' in his day could not be laws at all, evtn though they agreed with all existing experiments. He proposed modifications or alternatives to many of these 'laws' — alternatives that could be written in the same form in all inertial frames, in accordance with tha principle of relativity. Experiments done later have shown that the old 'laws' though adequate in their dwn time, do not describe the results that am now available.

In summary, the principle of relativity implies that the laws of nature do not depend upon the choice of an inertial frame of reference or the position or motion of an observer – they will always retain their form in any inertial frame of reference. Indeed, the measurements of various quantities, like the positions, times, velocities, energies, momenta, electric and magnetic fields may be different in different inertial frames. However, the relationships between these quantities governed by various laws would remain the same in all inertial frames.

In philosophical terms, we can say that the principle of relativity underscores the objective character of the laws of nature and not the relativity of knowledge.

The principle of relativity is also stated in the following form which you will often encounter: The laws of physics do not allow us to distinguish between different inertial **frames.**

In other words, **you** will not **be** able **to distinguish through any experiment whether you are at rest or in a state of uniform motion. For, if** there were such an experiment, it would mean that the laws of physics depended in some way on your velocity and were different from the laws of physics when you were at rest.

You should understand that the principle of relativity does **not** claim that all inertial frames are the same in **all** respects To appreciate this point, consider two different spacecrafts, each travelling with a different constant velocity with respect to S. The principle of **relativity** tells us that the two frames cannot be distinguished as far as the laws of physics are concerned. However, if one could look outside each spacecraft through a window, it would be easy to know that they are moving at different velocities, relative to S. Does this contradict the principle of relativity? No, because the velocity of the spacecraft relative to S is not determined by a law of physics. Besides, in formulating this form of the principle of relativity the essential condition was that the spacecrafts were completely isolated.

The ideas presented here are worth careful thought and you may' need to go through them more than once. You may now like to attempt the following SAQ to know whether you have grasped the principle of relativity or not.

Spend 10 min

SAQ 4

(a) Suppose the observer in S' frame finds that Eq.(1.9b) is supported by experiment. Does it automatically follow that Eq. (1.9b) is a law of physics?

(b) Suppose you observe the motion of a particle in an inertial frame. You find that the x-component of the acceleration of the particle is described by the following equation:

$$\frac{d^2x}{dt^2} = k_1 \frac{dx}{dt} + k_2 [(x-X)^2 + (y-Y)^2 + (z-Z)^2]$$

where k_1 and k_2 are constants and (X, Y, Z) are the coordinates of a second particle in your frame. If this equation is to be regarded as a possible law in physics, what relationship must an observer in a different inertial frame find to be experimentally valid?

Let us now study the second postulate of the special theory of relativity.

1.4.2 The Principle of Constancy of Speed of Light

We have chosen to highlight this aspect because of its historical significance. When asked how long he had worked on the Special Theory, Einstein replied that he had started at age 16 and worked for ten years. He had to abandon many fruitless attempts 'until at last it came to him that time was suspect' — in particular, the assumption that there exists a universal time which is the same. In this section, we shall briefly study this aspect.

The second postulate about the constancy of speed of light is crucial in leading to the theory of relativity. It is very important because it radically alters the classical notions of absolute space and time. Here we shall briefly discuss the implications of the second postulate of special relativity theory, particularly regaiding the notion of time.

The Nature of Time in Special Relativity

The basic premise of Newtonian mechanics was that the same time scale applied to all inertial frames of reference (recall the equation $t' = t$ in Galilean transformation). Using this universal time scale, we must be able to give meaning to statements such as "Events A and B occurred at the same time", without referring to any inertial frame of reference. To use the exarnple'given by Einstein, when we say that a train arrives at 7 o'clock, what we mean is this: The pointing of the clock hand to 7 and the arrival of the train are simultaneous events.

Thus, the assigning of time to events involves judging whether they are simultaneous or not. So, if all observers, independent of their position and velocity, agreed that any two events (e.g., the arrival of the train at the station and the pointing of the clock hand to 7) are simultaneous, we could certainly say that the absolute Newtonian time scale existed.

We shall certainly not have an absolute time scale if different inertial observers disagree about two events being simultaneous, i.e., one inertial observer says that two events occur at the same time and another inertial observer says that they do not. This is precisely what happens if we uphold the constancy of the speed of light. Let us understand this idea with the help of an imaginary experiment.

Consider a train compartment travelling at a very high constant velocity V to the right of an observer S at rest on the earth (Fig. 1.5). A highspeed flashbulb is situated at the exact centre of the compartment. It sends out light pulses to the right and left when it flashes. There are photocells at each end of the compartment, so that an observer S', in the compartment can detect when the light pulses strike its ends. Now, suppose by some ingenious device, the observer S on the earth is also able to measure the progress of the two pulses. Let the positions of S and S' coincide with that of the bulb when it flashes (Fig. 1.5a).

The flashbulb is at rest relative to the observer S' in the compartment. Since it is at the centre, when the bulb flashes, two light pulses travel equal distances to the two ends of the compartment in equal times. Hence, S' observes that the light pulses hit the two ends of the compartment at **the same time.**

Is the same conclusion drawn by S, who is stationary on the earth? Refer to Figs. 1.5b and 1.5c. The light pulses travel equal distances to the right and left in equal time, But in the frame of S the compartment is moving to the right. So in the frame of S, the distance

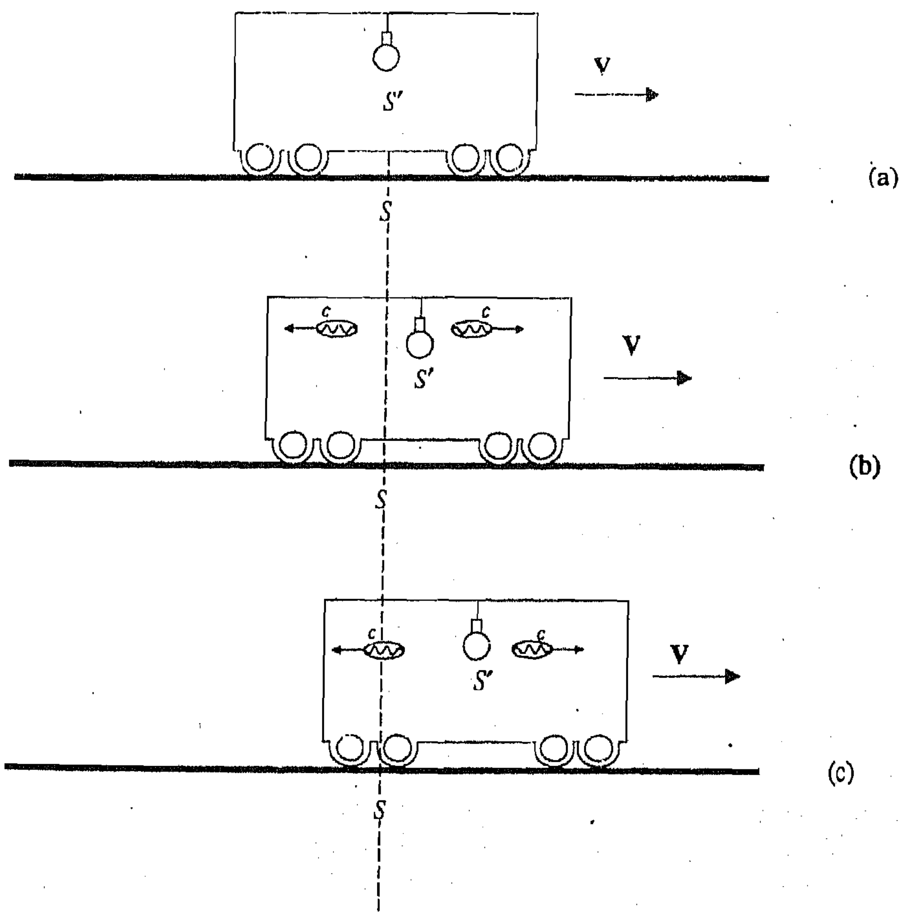


Fig.1.5: Unlike the inertial observer S' in the moving train compartment, the stationary observer S on the earth observes that the light pulses do not strike the ends of the car simultaneously. The figures are drawn with respect to the inertial observer S.

between the point at which S observes the bulb flashing and the left end of the compartment is shorter compared to its distance from the right end. As a result, S measures that the light pulse moving to the left strikes the end of the compartment before the other pulse strikes the apposite end. In the frame of reference of S, the light pulses do not hit the two ends of the compartment at the same time.

Now, if Newtonian mechanics were valid, this difference in the distance travelled in the frame of S would be compensated by the different speeds of light measured by S. The observer S would assign a lower speed $(c - V)$ to the pulse travelling to the left (opposite to the direction of the train's motion); the pulse travelling to the right would travel a greater distance but at a greater speed of $(c + V)$. Thus, S would measure both the times to be equal and conclude that the two pulses hit the ends of the compartment at the same time,

But the speed of light is constant. Therefore, in the frame of S, the two events (the light, pulses hitting the two ends of the compartment) do not occur simultaneously.

This signifies a major break with the older ideas of absolute time, because different observers do not agree on what is the same time. Of course, remember that this result is arrived at for events occurring at different locations (the two ends of the compartment for instance). In the next unit we shall come back to this discussion and also consider events occurring simultaneously at the same point in space.

To sum up, we can conclude that the notion of absolute time is contradicted by the second postulate because

> Events (occurring at different points in space) that are simultaneous in one inertial system may not be simultaneous in another.

This conclusion is termed the relativity of simultaneity, It is the fundamental difference between Newtonian relativity and special relativity. In Newtonian relativity, observers in S and S' always agree everywhere about events occurring at the same time. This is also the origin of other features of space and time that follow from the special theory of relativity, namely, the phenomena of length contraction, time dilation, twin paradox, etc. In this brief discussion we have given you a flavour of what follows in the next unit. We end this section with an SAQ for you.

Spend 10 min

SAQ 5

a) The speed of light is 3.0×10^8 m s^{-1}. You travel towards a light source in a spaceship at a constant speed of 2.0×10^8 m s^{-1} (both speeds are measured with respect to an inertial frame at rest), What is the speed of light relative to you?

b) One observer moves toward a star at a speed V, another moves away from it at the same speed. Considering the speeds, frequency n and wavelength λ of the star light ($c = n\lambda$), what do the observers measure to be the same and what do they measure to be different?

c) Suppose the compartment of the train in Fig. 1.5 would shrink so that the distance between its two ends approaches zero. Can you give a simple argument to show that two events not separated in space are simultaneous for all inertial observers?

Let us now summarise what you have studied in this unit.

1.5 SUMMARY

- An event is an occurrence that happens at a point in space and at an instant in time. The Galilean coordinate transformations from an inertial frame of reference S to another inertial frame S' moving at a velocity $\mathbf{u} = u\hat{i}$ relative to S are given by

$$x' = x - ut$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Here (x, y, z) are the coordinates of an event and t is the time at which it occurs, as measured in the frame S. The coordinates (x', y', z') and the time t' are measured in S'. S' moves with respect to S so that the x-x' axes are common and the y-y', z-z' axes are parallel.

- The Galilean or classical principle of relativity states that the laws of mechanics can be written in the same form in all inertial frames of reference. If they hold in one inertial frame, they also hold in all other inertial frames.
- Galilean coordinate transformations predict that the velocity of light should be different in different inertial frames and do not preserve the form of Maxwell's equations. Thus, the Galilean principle of relativity does not apply to the laws of electromagnetism.
- Experiments, especially the Michelson-Morley experiment, indicate that the speed of light is a universal constant and is independent of the relative uniform motion of the observer, the transmitting medium, and the source. The laws of electromagnetism are also upheld by experiments. Newtonian mechanics is experimentally observed to break down for particles moving at speeds close to that of light.
- In his special theory of relativity Einstein affirms the classical principle of relativity and generalises it to include all laws of physics. This also means that the speed of light should be the same in all uniformly moving systems.
- The postulates of special relativity are as follows:

  Postulate 1 — The Principle **of** Relativity

  The laws of physics are the same in all inertial frames of reference.

  Postulate 2 — The Principle **of** Constancy of Speed of Light

  The speed of light (in vacuum) has the same constant value in all inertial frames of reference.

1.6 TERMINAL QUESTIONS

Spend 30 *min*

1. Linear momentum and kinetic energy are conserved in an elastic collision. Use the Galilean transformation equations to show that if a collision is elastic in one inertial frame it is elastic in all inertial frames.
2. a) Does the Michelson-Morley experiment indicate that ether is an unnecessary concept, or does it prove that there is no such thing?

   b) One of Maxwell's equations is $\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial \Phi_B}{\partial t}$ in an inertial frame S.
   According to the principle of relativity, what will its form be in another inertial frame S'?
3. How is the nature of time as enunciated in classical relativity different from that in the special theory of relativity?

1.7 SOLUTIONS AND ANSWERS

SAQs (Self-Assessment Questions)

1. a) Non-inertial since the car is accelerating
   b) Inertial
   c) Non-inertial since the electron is accelerating
   d) Inertial
   e) Inertial
2. a) No. Different inertial observers can measure different values of physical quantities but the relationship between them remains the same,

   b) Yes, since these laws follow from Newtonian mechanics,

3. Using the chain rule we get

$$\frac{\partial\phi}{\partial x} = \frac{\partial\phi}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial\phi}{\partial y'}\frac{\partial y'}{\partial x} + \frac{\partial\phi}{\partial z'}\frac{\partial z'}{\partial x} + \frac{\partial\phi}{\partial t'}\frac{\partial t'}{\partial x}$$

From Galilean transformations given by Eq. (1.1)

we have $$\frac{\partial x'}{\partial x} = 1,\ \frac{\partial y'}{\partial x} = 0,\ \frac{\partial z'}{\partial x} = 0,\ \frac{\partial t'}{\partial x} = 0.$$

Thus, $$\frac{\partial\phi}{\partial x} = \frac{\partial\phi}{\partial x'} \text{ and } \frac{\partial^2\phi}{\partial x^2} = \frac{\partial^2\phi}{\partial x'^2}.$$

You can also show that, $$\frac{\partial^2\phi}{\partial y^2} = \frac{\partial^2\phi}{\partial y'^2} \text{ and } \frac{\partial^2\phi}{\partial z^2} = \frac{\partial^2\phi}{\partial z'^2}.$$

Now $$\frac{\partial\phi}{\partial t} = \frac{\partial\phi}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial\phi}{\partial y'}\frac{\partial y'}{\partial t} + \frac{\partial\phi}{\partial z'}\frac{\partial z'}{\partial t} + \frac{\partial\phi}{\partial t'}\frac{\partial t'}{\partial t}.$$

From Eqs. (1.1) $$\frac{\partial x'}{\partial t} = -u,\ \frac{\partial y'}{\partial t} = 0,\ \frac{\partial z'}{\partial t} = 0,\ \frac{\partial t'}{\partial t} = 1.$$

Thus $$\frac{\partial\phi}{\partial t} = -u\frac{\partial\phi}{\partial x'} + \frac{\partial\phi}{\partial t'}$$

and $$\frac{\partial^2\phi}{\partial t^2} = u^2\frac{\partial^2\phi}{\partial x'^2} - 2u\frac{\partial^2\phi}{\partial x'\partial t'} + \frac{\partial^2\phi}{\partial t'^2}$$

Thus there are two **extra terms** in the expression of $\partial^2\phi/\partial t^2$ in the S' frame. Hence the wave equation does not retain its form in the S' **frame.**

4. a) No, the Eq. (1.9b) has to satisfy the principle of relativity as well, **i.e.,** it has to hold in all inertial frames.

   b) The relationship in another inertial frame S' should be of **the form**

$$\frac{\partial^2 x'}{\partial t'^2} = k'_1\frac{\partial X'}{\partial t'} + k'_2\left[(x'-X')^2 + (y'-Y')^2 + (z'-Z')^2\right]$$

   where k'_1 and k'_2 **are** constants, (X', Y', Z') are the coordinates of the second particle in S' frame, and (x', y', z', t') are the space-time coordinates of the particle in S' frame.

5. a) It is $3.0 \times 10^8\ \mathrm{m\ s^{-1}}$ since it is a universal constant.

   b) The speed of light is **measured** to be the same, its frequency and wavelength are measured to be different **by** the two observers.

   c) The loss of simultaneity in S is due to the finite length of the compartment, which **makes** the observer S measure the distance to the left to be smaller. If the compartment's length **shrunk** to zero, the two pulses would strike its ends simultaneously for all inertial frames. Thus, two events **occurring** at the same position would occur at the **same** time for all inertial observers.

Terminal **Questions**

1. Let an object of mass m and velocity $\mathbf{v}_1$ collide elastically with an object of mass M and velocity $\mathbf{V}_1$ in frame S. Let their velocities after collision be $\mathbf{v}_2$ and $\mathbf{V}_2$, respectively in S. Then since linear momentum and kinetic energies are conserved in an elastic collision, we **have**

$$m\mathbf{v}_1 + M\mathbf{V}_1 = m\mathbf{v}_2 + M\mathbf{V}_2 \qquad \text{(A)}$$

and $$\frac{1}{2}mv_1^2 + \frac{1}{2}MV_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}MV_2^2 \qquad \text{(B)}$$

Now let S' **frame** move with a velocity v **with** respect to S. Then the Galilean velocity **transformation** gives us **the** velocities of m **and** M before and after collision as

$$\mathbf{v}_1' = \mathbf{v}_1 - \mathbf{v},\ \mathbf{V}_1' = \mathbf{V}_1 - \mathbf{v},\ \mathbf{v}_2' = \mathbf{v}_2 - \mathbf{v},\ \mathbf{V}_2' = \mathbf{V}_2 - \mathbf{v}, \qquad \text{(C)}$$

Substituting for $\mathbf{v}_1$, $\mathbf{V}_1$, $\mathbf{v}_2$, $\mathbf{V}_2$ from **(C) in** (A) **and** (B) **we** get

$$m(\mathbf{v}_1' + \mathbf{v}) + M(\mathbf{V}_1' + \mathbf{v}) = m(\mathbf{v}_2' + \mathbf{v}) + M(\mathbf{V}_2' + \mathbf{v})$$

or $$m\mathbf{v}_1' + M\mathbf{V}_1' = m\mathbf{v}_2' + M\mathbf{V}_2' \qquad \text{(D)}$$

Thus linear momentum is conserved in the S' frame. For the **conservation of kinetic** energy, we proceed as follows:

$$m\,|\mathbf{v}_1' + \mathbf{v}|^2 + M\,|\mathbf{V}_1' + \mathbf{v}|^2 = m|\mathbf{v}_2' + \mathbf{v}|^2 + M\,|\mathbf{V}_2' + \mathbf{v}|^2$$

or $$m(v_1'^2 + v^2 + 2\mathbf{v}_1'.\mathbf{v}) + M(V_1'^2 + v^2 + 2\mathbf{V}_1'.\mathbf{v}) = m(v_2'^2 + v^2 + 2\mathbf{v}_2'.\mathbf{v}) + M(V_2'^2 + v^2 + 2\mathbf{V}_2'.\mathbf{v})$$

or $$mv_1'^2 + 2m\mathbf{v}_1' . \mathbf{v} + MV_1'^2 + 2M\mathbf{V}_1' . \mathbf{v} = mv_2'^2 + 2m\mathbf{v}_2' . \mathbf{v} + MV_2'^2 + 2M\mathbf{V}_2' .. \mathbf{v}$$

or $$mv_1'^2 + MV_1'^2 + 2\,(m\mathbf{v}_1' + M\mathbf{V}_1') . \mathbf{v} = mv_2'^2 + MV_2'^2 + 2\,(m\mathbf{v}_2' + M\mathbf{V}_2') . \mathbf{v}$$

Using (D) we get

$$mv_1'^2 + MV_1'^2 = mv_2'^2 + MV_2'^2,$$

or $$\frac{1}{2}(mv_1'^2 + MV_1'^2) = \frac{1}{2}(mv_2'^2 + MV_2'^2)$$

Thus, K.E. is conserved in S'.

2. a) The Michelson-Morley experiment indicates only that the concept of ether is unnecessary.

   b) The **form** in frame S' will be

$$\oint_C \mathbf{E}' . d\mathbf{l}' = -\frac{\partial \Phi_B'}{\partial t'}$$

3. In classical relativity observers in different inertial frames will always agree about the time at which an event occurs. If two events occur simultaneously for one inertial observer, then according to classical relativity, these two events will be simultaneous for all other inertial observers. This need not be so according to special theory of relativity. Thus, in special relativity two inertial observers need not measure the time at which an event occurs to be the same, if the events occur at different positions in space.