UNIT 13 ELECTROMAGNETIC INDUCTION

Structure

13.1 INTRODUCTION

When you step inside a dark room and illuminate it by a mere flick of a switch, do you ever pause and wonder for a minute: What makes this possible? To find the answer you could follow the copper wires (connected to the switches) which bring electric power to your homes and are spread out across the countryside. You will most certainly find an **electric generator** which would most likely be situated in a hydel or thermal power plant. Somewhere between your homes and these generators, there will be several **transformers.** Both these devices are essential for large-scale generation and distribution of electrical energy today. Both are based on momentous discoveries made independently by Michael Faraday and Joseph Henry more than 250 years ago. Indeed, it can be said that their discoveries form the basis of modem electrical technology.

In 1831, Faraday and Henry discovered that electric currents were **induced** in circuits subjected to **changing magnetic fields.** Let us, in this unit, study these phenomena associated with changing magnetic fields, explored by Faraday in his experiments and understand the principles underlying them. We shall also study some applications of these principles, particularly the ones that form the cornerstone of electrical technology, namely, the generator and the transformer.

Objectives

After studying this unit you should be able to

- apply Faraday's law of electromagnetic induction and Lenz's law
- compute the self-inductance and the back emf of an inductor possessing a simple geometry
- compute the mutual inductance of circuits in simple configurations

- determine the energy stored in any given magnetic field.

'13.2 INDUCED CURRENTS

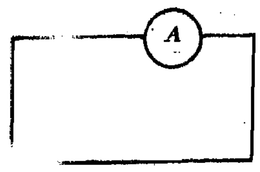
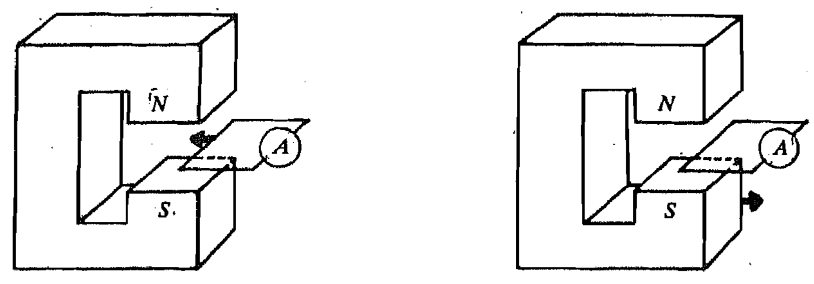


Fig.13.1: Can current be made to now in this circuit?

Consider the simple circuit of Fig. 13.1, connecting a loop of wire to an ammeter. Is there a way by which you can induce an electric current in this circuit? Now study Fig. 13.2, in which we show two simple experiments similar to Faraday's which demonstrate the existence of induced currents.



(a) (b)

Fig.13.2: The ammeter registers a current both when **(a) the circuit** is **moved** in (he **magnetic field of a magnet, and (b)** (he **circuit is** hdd **fixed and** (he magnet is **moved. As long as both the circuit** and **the magnet** a n **held still, there** is no **current. This current, termed induced current, flows only when the magnet and the loop move relative to each other.**

There is another way of inducing an electrical current in a circuit. This **experiment** is shown in Fig. 13.3. Study the figure before proceeding further.

What do all these experiments reveal? Can you explain these observations on the basis of what you have studied so far? In fact, it is possible to explain the results of the first experiment **(Fig.13.2a)** on the basis of **Lorentz** force. Would you like to try giving the explanation?

Spend 5 min

SAQ 1

Explain qualitatively how an induced current results in the circuit of **Fig. 13.2a?**

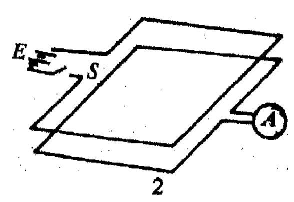


Fig. 13. 3: When the switch *S* in the first circuit (1) is closed, (he **ammeter in the second circuit (2) shows a deflection for a brief time; after a short time the current** in **the second circuit returns to zero. There is again a brief response in** the circuit 2 **when the switch** in **the circuit 1 is opened again. This suggests that** the a d **of opening and closing the switches gives rise to induced currents.**

What about the other two experiments shown **in** Fig. **13.2b** and Fig. **13.3?** In both these cases, the charges are stationary (ignoring the thermal and drift effects) and no **Lorentz** force acts on them. So how do we explain the origin of induced currenls? The answer is: **we** are observing a new phenomenon and we have to look for new explanations. Now, did you notice that there **was** a common element in all these' experiments? The magnetic field was changing in all cases. An induced current appeared in a circuit subjected to this changing magnetic field. This leads us to introduce a new basic physical principle:

A changing magnetic field produces an electric field.

This new principle explains the origin of **the** induced current. It is the electric field produced by the changing magnetic field that **sets** the charges in the wire of the **circuits** into motion,

This explanation is, of course, qualitative. **We should also** be able to give a quantitative relationship **between** the induced current and the changing magnetic field: This is precisely what Faraday's law **of** electromagnetic induction does.

13.2.1 Faraday's Law of Electromagnetic Induction

What is it that gives rise to a current in a circuit? As you may **know,** we need a source of electromotive force (emf) a device, like a battery that supplies energy to the circuit. **Similarly,** when an induced current flows in a circuit, an induced emf must be present. It is along these lines that Faraday developed a general theoretical **formulation** of his experimental results. He formulated the law that the emf induced in **any circuit** depends only on the time rate of change of flux of the magnetic field going through the circuit. Mathematically, this law can be expressed as follows:

$$\varepsilon = -\frac{d\Phi}{dt} \qquad (13.1)$$

where ε is the induced emf and Φ the magnetic flux linked by that circuit.

We can rewrite Faraday's law **given** by Eq. (13.1) so that we can omit any reference to circuits. The induced emf is simply the work per unit charge done on a test charge that is moved around a circuit. Since work is the line integral of force over distance, and electric field is the force per unit charge, we can write the emf as the line integral of the electric field along the circuit C.

$$\varepsilon = \oint_C \mathbf{E} \cdot d\mathbf{l}$$

In Unit 9 you have already come across the integral representation of magnetic flux

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

where S is any surface whose boundary is the circuit C.
Faraday's law can then be expressed as

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S} \qquad (13.2a)$$

In this form of Faraday's law, there is no need to have a physical circuit. C can just represent a loop in space and S a surface bounded by C. It simply describes induced electric fields, which arise whenever there are changing magnetic fields. If electric circuits are present, induced currents arise as well. Using Stokes' theorem (see Unit 9), we can also express Eq. **(13.2a) in** the differential form:

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = \int_S \text{curl}\,\mathbf{E} \cdot d\mathbf{S} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

Since the surface S is arbitrary, we must **have**

$$\text{curl}\,\mathbf{E} = -\frac{d\mathbf{B}}{dt} \qquad (13.2b)$$

Since B may depend on position as well as time, we write $\frac{\partial \mathbf{B}}{\partial t}$ instead of $\frac{d\mathbf{B}}{dt}$ to account for only the time variation of **B**. We thus have two entirely equivalent **statements** of Faraday's law:

$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$	(integral form)	(13.2a)
$\text{curl}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	(differential form)	(13.2b)

Remember that in **explaining** the experiments discussed in the beginning **of** the section, we have used two different explanations in terms of Faraday's law and Lorentz force law. When we move the circuit, it is just the **Lorentz** force that **gives** rise to induced current. But when we move the magnet, the changing magnetic field induces an electric field and it is the electrical force due to this field that induces the current, However, the law that the induced emf in a circuit is equal to the rate of change of the magnetic flux through the circuit applies to both the cases. Viewed in this light, does it not seem surprising that the two processes yield the same emfs? In fact, consideration of this symmetry between the two cases led Einstein to his special theory of relativity. But, that is a different story altogether.

So you have found that electric fields are produced by static charges as well as changing magnetic fields. But is the nature of these fields the same? Try to find out yourself!

Spend 3 min

SAQ 2

What is the basic difference in the nature of the electric fields produced by static charges and those induced by changing magnetic fields?

So far we have not said anything about the negative sign in Faraday's law. The negative sign has an important purpose in the law: it tells us the direction of the induced current; and it is nothing but the consequence of conservation of energy. Let us explore this connection a bit further. In doing so, we will arrive at **Lenz's** law.

13.2.2 Induction and Conservation of Energy: Lenz's Law

When an induced current flows through a coil, electrical energy is dissipated as heat, because the coil possesses some resistance. Where does this energy come from? The principle of conservation of energy tells us that energy cannot be created from nothing. Now, the induced current is caused by a changing magnetic **flux.** So the agent that changes the flux ***must do work*** to supply the energy.

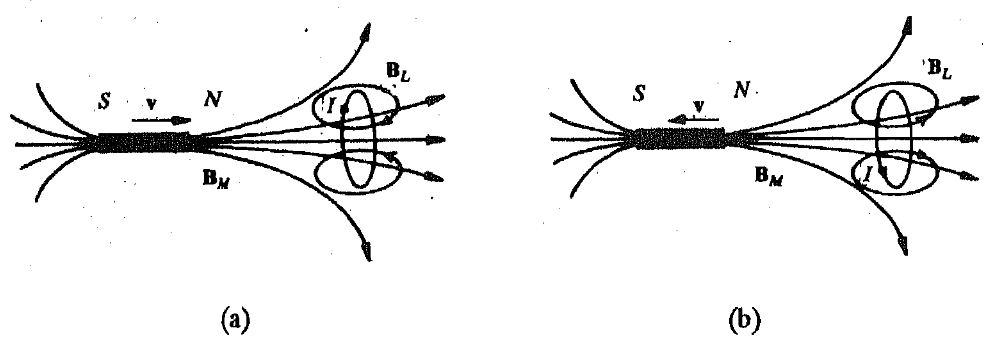


Fig. 13.4: (a) As the bar magnet moves toward the loop, the changing magnetic flux induces an emf that drives a current in the loop. Conservation of energy requires that the magnetic field due to the induced current oppose the motion of the magnet. Thus the agent moving the magnet must do work that ends up as heat in the conducting loop; (b) when we pull away the magnet from the loop, the direction of the induced current is such that the magnetic force due to the loop opposes the withdrawal of the magnet. B_M is the magnetic field due to the magnet and B_L is due to the loop

Let us consider a simple example of a bar magnet which is moved near a loop of wire. As we move the magnet towards the loop, a current is induced in the loop (Fig. **13.4a).** Like any other current this induced current gives rise to a magnetic field. The magnet experiences a force in the field. ***This*** force ***must be repulsive so*** that we have to do ***work*** to push the magnet toward the coil rather than having work done on us. Suppose we are moving the north pole of the magnet towards the loop. With its induced current, the loop behaves like a magnetic dipole **giving** rise to a field similar to the bar magnet's. For a repulsive force on the magnet, the north pole of the loop dipole must be toward the approaching magnet. Applying the right-hand rule to the current loop, we see that the induced current **must** flow in the direction shown in Fig. **13.4(a).**

What happens when we pull the magnet away from the loop? Conservation of energy requires that we work against a force to do this. The loop must then present a south pole to the magnet. This would result in an attractive force which opposes the withdrawal of the magnet. **As** a result, the loop current is in the opposite direction (Fig. **13.4b**).

Both these results stemming from the principle of conservation of energy are presented in the form of Lenz's law:

> The direction of the induced current (emf) is such as to oppose **the** change giving rise to it.

Lenz's law is reflected mathematically in the minus sign on the right-hand side of Faraday's law given by Eq. (13.2).

To concretise this idea, think of what would happen if the direction of the induced current aided the change in the flux. For instance, when you pushed the magnet toward the loop in Fig. **13.4a,** a south pole appeared towards the magnet. Then you would need to push the magnet only slightly to get it moving and the action would carry on forever. The magnet would accelerate toward the loop, gaining kinetic energy in the process. At the same time thermal energy would appear in the loop. Thus we would have created energy from practically nothing. Needless to say, this violates energy **conservation** and does not happen.

While applying Lenz's law you should keep in mind that the magnetic field of the induced current does not oppose the **magnetic field,** but the **change** in this field. For example, if the magnetic flux through a loop decreases, the **induced** current flows so that its magnetic field adds to the original flux; if the flux is increasing, the current will flow in the opposite direction. This is a sort of an "inertial" pheiiomenon: A conducting loop 'likes' to keep a constant flux through it; if we try to change the flux, the **loop** responds by sending a **current** in such a direction **as** to counter our efforts.

You may now like to apply Lenz's law to certain simple situations and **determine** the direction of the induced current.

SAQ 3

Spend 5 min

a) In Fig. **13.5(a)** what is the direction of the induced current in the loop when the area **of** the loop is decreased by pulling on it with the forces labelled F? B is directed into the page and perpendicular to it.

b) What is the direction of the induced current in the smaller loop of Fig. **13.5(b)** when a clockwise current as seen from the left is suddenly established in the larger loop, by a battery not shown?

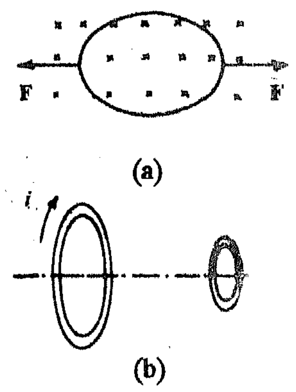


Fig. 13.5

Let us now consider an application of these laws, which is, perhaps, the 'most important technological application in use today.

Example 1: The ac generator

Do you know'that at present the world uses electrical energy at the rate of about 10^{13} watts daily, and virtually all of this power comes from electric generators? A generator is nothing but a system of conductors in a magnetic field. **Fig.13.6** shows a simple version of the generator.

Mechanical energy is supplied to rotate the coil placed between the-gales of a magnet. In the power stations the source of mechanical energy is **either** falling **water** (**hydroelectric** power plants), or **steam** from burning fossil fuels (**thermal** power **plants**) or **from** nuclear **fission** (nuclear power plants). The rotation **causes** a **change** in the magnetic flux through the coil which induces an emf **and** a **current** flows through the coil.

Let us determine the magnitudes of the induced emf and the induced current, Let S be the area of the coil and θ the angle between the magnetic field and the normal to the plane of the coil: The flux through the coil is

$$\Phi = BS\cos\theta \qquad (13.3a)$$

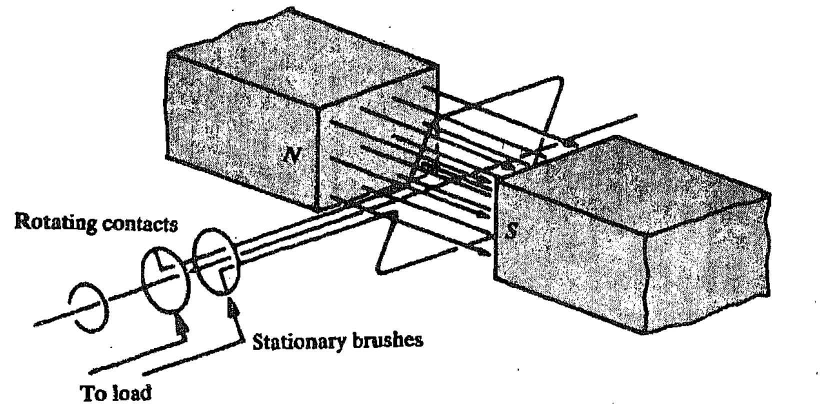


Fig.13.6: A simple diagram of an ac electric generator. As the loop rotates in the magnetic field, an induced emf is produced in it by the changing magnetic flux. Current flows through (he rotating contacts and stationary brushes to an electrical load

If the coil is rotating with a uniform angular speed ω, θ varies with time as $\theta = \omega t$. The emf in the coil is then

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d}{dt}(BS\cos\omega t)$$

or

$$\varepsilon = BS\,\omega\sin\omega t \qquad (13.3b)$$

If we bring the wires of the coil to a point P, quite some distance from the rotating coil, where the magnetic field (due to the magnet) does not vary with time, then from Eq. (13.2b) the curl of E in this region will be zero, **i.e.,** E is conservative. Then we can define an electric potential associated with this field. Let the two ends of the coil be at a potential difference V. If no current is being drawn from the generator, the potential difference between the two wires will be equal to the emf in the rotating coil, **i.e.,**

$$V = BS\,w\sin wt = V_0\sin\omega t \qquad (13.4a)$$

where $V_0 = BS$ w, is the peak output voltage of the generator. As given by Eq. **(13.4a),** V is an **alternating** voltage. If we now attach a load R to these wires, we can generate an alternating current given by

$$I = \frac{V}{R} = \frac{V_0}{R}\sin\omega t \qquad (13.4b)$$

Would you now like to work out a numerical problem on the design of an ac generator?

Spend 10 min

SAQ 4

An electric generator like the one in **Fig.13.6** consists of a 10 turn square wire loop of side 50 cm. The loop is turned at 50 revolutions per second, to produce the standard 50 Hz ac produced in our country. How strong must the magnetic field be for the peak output voltage of the generator to be **300V?**

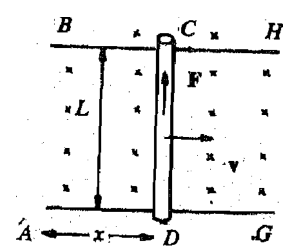


Fig. 13.7: The induced emf in the loop can be explained in terms of the forces on the charges in the moving wire.

Recall that for the first experiment shown in **Fig.13.2a,** you had solved **SAQ 1** to show qualitatively that the emf was **induced** in a constant magnetic field due to **the Lorentz** force acting on a moving wire. Later while formulating **Faraday's** law we had said that Eq. (13.2) was valid for this **case** as well. The emf generated in this manner is **termed** the motional **emf.** Let us **determine** the motional emf quantitatively.

13.23 Motional Electromotive Force

Let us consider the simple situation shown in Fig. 13.7. **A** wire CD of length L is pulled to the right with constant velocity v. It is in contact with the wire GABH at points C and D so as to form a wire loop $ABCD$. We measure the position of the wire by the distance x shown in the figure. The loop is immersed in a constant magnetic field B which is directed perpendicular to the page and into it. Each charge in the wire experiences a force

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

Here we have ignored the thermal velocities of the electrons in the wire. As you know, thermal velocities of electrons in the wire are very large. The reason we ignore them here is that they are randomly oriented and if we add up the magnetic force on all the eletrons of the wire CD, the net contribution of thermal velocities to Lorentz force is zero. Now since $v \perp B$, $F = q\,v\,B$ and $\mathbf{F}$ points in the direction shown in Fig. 13.7. To calculate the effective emf around the loop, we must find the net work per unit charge; which is

$$\frac{W}{q} = \frac{1}{q}\oint_C \mathbf{F} \cdot d\mathbf{l} \qquad (13.5a)$$

where dl is an infinitesimal element of length directed downward along CD. Thus, the induced emf is

$$\varepsilon = \frac{W}{q} = \frac{1}{q}\oint_C \mathbf{F} \cdot d\mathbf{l}$$ (Here $\mathbf{F} \cdot d\mathbf{l} = -Fdl$ since $\mathbf{F}$ and $d\mathbf{l}$ are in the opposite directions, and $F = qvB$.)

$$= -vB\int dl$$

or

$$\varepsilon = -vLB \qquad (13.5b)$$

But the quantity vLB is just the change in the flux through the loop, since the magnetic flux is

$\Phi = BLx$, where Lx is the area of the loop

and

$$\frac{d\Phi}{dt} = BL\frac{dx}{dt} = BLv \qquad (13.5c)$$

Therefore, comparing Eqs. (13.5b) and (13.5c) we once again find that the net motional emf resulting from the motion of a loop in a constant magnetic field is given by

$$\varepsilon = -\frac{d\Phi}{dt}$$

This is in perfect agreement with Faraday's law. Thus Faraday's law is a new principle which is consistent with the law of magnetic force on a moving charge. In summary, we can say that the physics of induction is governed by the following two basic laws:

$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	Lorentz force law
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	Faraday's Law

So far, you have studied Faraday's law which relates the changing magnetic flux through a circuit to the emf induced in it. Let us now consider a situation when the changing flux through an electrical circuit is itself mused by a changing current in it or in a circuit nearby. We then speak of inductance of the circuit. Let us now study this phenomenon.

13.3 INDUCTANCE

When a current changes in a circuit, there is a consequent changing magnetic field around it. If a part of this field passes through the circuit itself then an emf is induced in it. If there is another circuit in the neighbourhood, then the magnetic flux through that circuit changes, causing an induced emf in that circuit. Thus, induction in circuits can occur in two ways:

i) a coil of wire can induce an emf in itself,

ii) for a pair of coils situated near enough, so that the flux associated with one coil passes through the other, a changing current in one coil induces an emf in the other.

In the first case we associate a property called self-inductance with the coil, while in the second we speak of **mutual inductance.** Let us consider these effects separately.

13.3.1 Self-Inductance

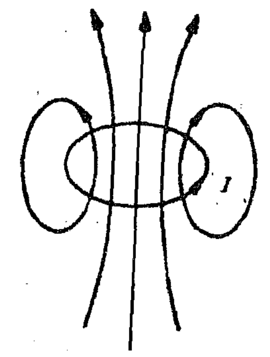


Fig. 13.8 : Self-inductance of a coil.

Consider a circular loop carrying a current I (Fig. 13.8). A magnetic field is set up by the current in this loop, so there is a magnetic flux through it. As long as the current is steady, the magnetic flux does not change and there is no induced current. But if we change the current in the loop, the flux changes and an emf is induced. The direction of the induced current is determined by Lenz's law, i.e., it opposes the change in the loop current. The more rapidly we change the current in the loop, the greater is the rate of change of flux, and the induced emf which opposes the change in current.

To discuss this property quantitatively, we associate a quantity L, called **self-inductance**, with every circuit. The self-inductance is defined as follows:

$$L = \frac{\Phi}{I} \tag{13.6a}$$

where Φ is the flux passing through the circuit when a current I flows in the circuit. The unit of self-inductance is the henry (H), named after Joseph Henry. It is defined as

$$1 \text{ henry} = 1\text{H} = 1 \text{ tesla} - \text{m}^2/\text{ampere}$$

For a coil having N turns

$$L = \frac{N\Phi}{I} \tag{13.6b}$$

All loops whether in the form of straight wires or coiled ones, possess self-inductance. However, the effect of self-inductance is important only when the magnetic flux through the circuit is large or when current changes very rapidly. For example, a 1 cm length of straight wire has an inductance of about 5×10^{-9} H and it exhibits very little opposition to current changes in the 50 Hz ac. But in TV sets, high speed computers or in high frequency communications, such as satellite communication, current changes on time scales of the order of 10^{-9} s. Then the self-inductance of the wires themselves must be taken into account. There are devices, called inductors, designed specifically to exhibit self-inductance.

From Faraday's law, the emf induced in an inductor is

$$\varepsilon = -\frac{d\Phi}{dt} = -L\frac{dI}{dt} \tag{13.7}$$

This induced emf is also called the back emf. Eq. (13.7) tells us that the back emf in an inductor depends on the rate of change of the inductor current and acts to oppose that change in current. Since an infinite emf is impossible, so from Eq. (13.7), an instantaneous change in the inductor current cannot occur. Thus, we can say that

> The current through an inductor cannot change instantaneously

You have just studied that the self-inductance of an inductor is a measure of the opposition to the change in current through it. How do we determine the magnitude of the self- inductance of an inductor?

The inductance of an inductor depends on its geometry. In principle, we can calculate the self-inductance of any circuit, but in practice it is difficult unless the geometry is pretty simple. A solenoid is a device with a simple geometry and is widely used in electrical circuits. Let us, therefore, determine the self-inductance of a solenoid.

The self-inductance of a solenoid

Consider a long solenoid of cross-sectional area A and length L, which consists of N turns of wire. To find its inductance we must relate the current in the solenoid to the magnetic flux through it. In Unit 9, you have used Ampére's law to determine the magnetic field of a long solenoid, which is given as

$$B = \mu_o n I$$

where n is the number of turns per unit length and I is the current through the solenoid. For our problem $n = N/l$, which gives

$$B = \frac{\mu_o N I}{l}$$

The total flux through the N turns of the solenoid is

$$\Phi = N \int_{1\ \text{turn}} \mathbf{B} \cdot d\mathbf{S} = NB \int_{1\ \text{turn}} dS = NBA = \frac{\mu_o N^2 A I}{l}$$

Here we have written $\mathbf{B} \cdot d\mathbf{S} = B\,dS$, since the magnetic field of the solenoid is uniform and perpendicular to the cross-section of the individual turns. The self-inductance of the solenoid is

$$L = \frac{\Phi}{I} = \frac{\mu_o N^2 A}{l} \qquad (13.8)$$

Given this information you may like to determine the self-inductance and the back emf for a typical solenoid to get an idea of their magnitudes.

SAQ 5

Spend 10 min

A solenoid 1m long and 20 cm in diameter contains 10,000 turns of wire. A current of 2.5 A flowing in it is reduced steadily to zero in 1.0 ms. What is the magnitude of the back emf of the inductor while the current is being switched off? Take $\mu_o = 1.26 \times 10^{-6}\ \text{Hm}^{-1}$

As we have said earlier, the back emf in an inductor opposes the change in current and its magnitude depends on how rapidly the current changes. If we try to stop current in a very short time, $\frac{dI}{dt}$ is very large and a very large back emf appears. This is why switching off inductive devices, such as solenoids, can result in the destruction of delicate electronic devices by induced currents. Having worked out SAQ 5 you would realise that you have to be extremely cautious in closing switches in circuits containing large inductors. Even in your day-to-day experience, you may have seen that you often draw a spark when you unplug an iron. Why does this happen? This is due to electromagnetic induction which tries to keep the current going, even if it has to jump the gap in the circuit. Now, you may wonder what happens when you plug in the iron and put on the switch? This brings us to the role of inductors in circuits, Let us consider the example of an LIP circuit to understand this role.

Example 2: *LR* circuit

Consider the circuit in Fig. 13.9a which has a battery (a source of constant emf ε_0) connected to a resistance R and an inductance L. What current flows in the circuit when we close the switch?

through that circuit changes, causing an induced emf in that circuit. Thus, induction in circuit can occur in two ways:

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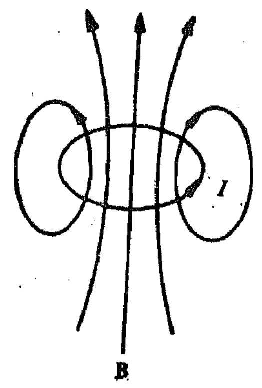
13.3.1 Self-Inductance



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From Faraday's law, the emf induced in an inductor is

$$\varepsilon = -\frac{d\Phi}{dt} = -L\frac{dI}{dt} \tag{13.7}$$

This induced emf is also called the backemf. Eq. (13.7) tells us that the back emf in an inductor depends on the rate of change of the inductor current and acts to oppose that change in current. Since an infinite emf is impossible, so from Eq. (13.7), an instantaneous change in the inductor current cannot occur. Thus, we can say that

The current through an inductor cannot change instantaneously.

You have just studied that the self-inductance of an inductor is a measure of the opposition to the change in current through it. How do we determine the magnitude of the self- inductance of an inductor?

The inductance of an inductor depends on its geometry. In principle, we can calculate the self-inductance of any circuit, but in practice it is difficult unless the geometry is pretty simple. A solenoid is a device with a simple geometry and is widely used in electrical circuits. Let us, therefore, determine the self-inductance of a solenoid.

The self-inductance of a solenoid

Consider a long solenoid of cross-sectional area A and length L, which consists of N turns of wire. To find its inductance we must relate the current in the solenoid to the magnetic flux through it. In Unit 9, you have used Ampere's law to determine the magnetic field of a long solenoid, which is given as

$$B = \mu_o n I$$

where n is the number of turns per unit length and I is the current through the solenoid. For our problem $n = N/l$, which gives

$$B = \frac{\mu_o NI}{l}$$

The total flux through the N turns of the solenoid is

$$\Phi = N \int_{1\ \text{turn}} \mathbf{B} \cdot d\mathbf{S} = NB \int_{1\ \text{turn}} dS = NBA = \frac{\mu_o N^2 AI}{l}$$

Here we have written $\mathbf{B} \cdot d\mathbf{S} = BdS$, since the magnetic field of the solenoid is uniform and perpendicular to the cross-section of the individual turns. The self-inductance of the solenoid is

$$L = \frac{\Phi}{I} = \frac{\mu_o N^2 A}{l} \qquad (13.8)$$

Given this information you may like to determine the self-inductance and the back emf for a typical solenoid to get an idea of their magnitudes.

SAQ 5

Spend 10 min'

A solenoid 1m long and 20 cm in diameter contains 10,000 turns of wire. A current of 2.5A flowing in it is reduced steadily to zero in 1.0 ms. What is the magnitude of the back emf of the inductor while the current is being switched off? Take $\mu_o = 1.26 \times 10^{-6}$ Hm^{-1}

As we have said earlier, the back emf in an inductor opposes the change in current and its magnitude depends on how rapidly the current changes. If we try to stop current in a very short time, $\frac{dI}{dt}$ is very large and a very large back emf appears. This is why switching off inductive devices, such as solenoids, can result in the destruction of delicate electronic devices by induced currents. Having worked out SAQ 5 you would realise that you have to be extremely cautious in closing switches in circuits containing large inductors. Even in your day-to-day experience, you may have seen that you often draw a spark when you unplug an iron. Why does this happen? This is due to electromagnetic induction which tries to keep the current going, even if it has to jump the gap in the circuit. Now, you may wonder what happens when you plug in the iron and put on the switch? This brings us to the role of inductors in circuits, Let us consider the example of an LR circuit to understand this role.

Example 2: *LR* circuit

Consider the circuit in Fig. 13.9a which has a battery (a source of constant emf ε_0) connected to a resistance R and an inductance L. What current flows in the circuit when we close the switch?

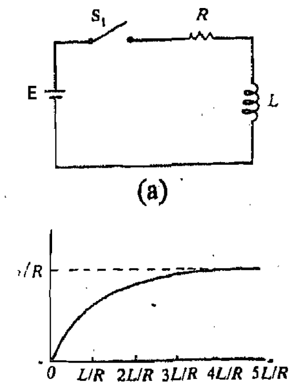

Fig. 13.9: *(a) LR* **circuit; (b) current in an** ***LR*** **circuit**

Solution

Let us analyse this current quantitatively. The total emf in the circuit is the sum of the emf provided by the battery and the back emf of the inductor. Therefore, Ohm's law gives

$$\varepsilon_0 - L\frac{dI}{dt} = IR \tag{13.9}$$

We can solve this equation to obtain $I(t)$:

$$\frac{dI}{I - \frac{\varepsilon_0}{R}} = -\frac{R}{L}dt$$

or
$$\int \frac{dI}{(I - \varepsilon_0/R)} = -\frac{R}{L}\int dt + C_1$$

or
$$\ln(I - \varepsilon_0/R) = -\frac{Rt}{L} + C_1$$

or
$$I - \varepsilon_0/R = C\exp(-Rt/L)$$

This gives

$$I = \frac{\varepsilon_0}{R} + C\exp(-Rt/L) \tag{13.10a}$$

The constant C can be determined from initial conditions.

Before we close the switch the current in the circuit is zero, giving us the condition that at $t = 0, i = 0$. This initial condition yields

$$C = -\varepsilon_0/R$$

or
$$I(t) = \frac{\varepsilon_0}{R}(1 - e^{-Rt/L}) \tag{13.10b}$$

Had there been no inductance in the circuit, the current would have jumped immediately to ε_0/R. With an inductance in the circuit, the current rises gradually and reaches a steady state value of ε_0/R as $t \to \infty$. The time it takes the current to reach about two-thirds of its steady state value is given by L/R, which is called the **inductive time constant of** the circuit. Significant changes in current in an LR circuit cannot occur on time scales much shorter than L/R. The plot of the current with time is shown in Fig. 13.9b.

You can see that the greater the L is, the larger is the back emf, and the longer it takes the current to build up. Thus, the role of an inductance in electric circuits is somewhat similar to that of mas. in mechanical systems. You know that the larger the mass of an object is, the harder it is to change its velocity. In the same way, the greater L is in a circuit, the harder it is to change the current in the circuit.

You may like to get an idea of the time scales involved in actual circuits. Work out the following SAQ.

Spend 10 min

SAQ 6

A 15-V battery, 2000 Ω resistor and 10 mH inductor are connected in series. What is the steady state current in the circuit after a sufficiently long time interval (~100 L/R)? How long does it take the current to reach half of its steady state value?

Let us now consider the second situation wherein the changing current in a circuit induces a current in an adjacent circuit, i.e., the phenomenon of mutual induction.

13.3.2 Mutual Inductance

Consider two circuits situated close together and at rest. If we pass a current I_1 through circuit 1, it will produce a magnetic field $\mathbf{B}_1$ (Fig. 13.10).

Let Φ_2 be the flux of $\mathbf{B}_1$, through 2. If we change I_1, Φ_2 will vary and an induced emf will appear in the circuit 2. This induced emf will drive an induced current in 2. Thus, every time we vary the current in circuit 1, an induced current will flow in circuit 2. Since from Biot-Savart's law (Unit 9), the magnetic field $\mathbf{B}_1$ is proportional to the current I_1, the flux of $\mathbf{B}_1$ through loop 2 is also proportional to I_1:

$$\mathbf{B}_1 = \frac{\mu_o I_1}{4\pi} \oint \frac{d\mathbf{l}_1 \times \hat{\mathbf{r}}}{r^2}$$

and

$$\Phi_2 = \int_S \mathbf{B}_1 \cdot d\mathbf{S}_2 \propto I_1 \qquad (13.11a)$$

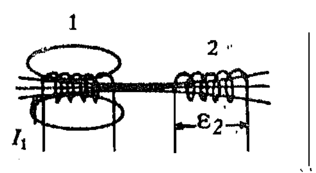

Fig. 13.10: Mutual inductance of two circuits.

Thus, we can write

$$\Phi_2 = M I_1 \qquad (13.11b)$$

or

$$\frac{d\Phi_2}{dt} = M \frac{dI_1}{dt} \qquad (13.12a)$$

From Faraday's law, the induced emf in coil 2 is

$$\varepsilon_2 = -\frac{d\Phi_2}{dt} = -M \frac{dI_1}{dt} \qquad (13.12b)$$

The proportionality constant M is known as the **mutual inductance** of the two circuits. It is a purely geometrical quantity which depends on the sizes, shapes and the relative arrangement of the circuits.

In arriving at **Eq.(13.12b)** we could as well have changed the current in the second coil to induce an emf in the first. **We** would have got a similar result fox the induced emf in coil 1:

$$\varepsilon_1 = -M \frac{dI_2}{dt} \qquad (13.13)$$

The unit of mutual inductance is also henry (H). Mutual inductances found in common electronic circuits range from microhenrys (pH) to several henrys.

Let us consider an **example** to determine the mutual inductance of coupled solenoids, Such an arrangement is also used in the ignition coil of vehicles,

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Example 3: Mutual inductance of coupled solenoids

A short solenoid of length L_2 is wound around a much longer solenoid of length L_1 and **area** A, as shown in **Fig.13.11.** Both solenoids have N turns per unit length. What is the mutual inductance of this arrangement?

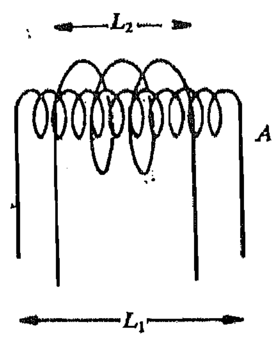


Fig. 13. 11: Mutual inductance of coupled solenoids.

Solution

The magnetic field produced by the long solenoid is

$$B = \mu_0 N I$$

This field is constant, and at right angles to the **plane** of the solenoid coils. It is **confined** to the interiors of the longer solenoid. So the flux through a single loop of the short solenoid is the product of the field strength and the area of the longer solenoid. Since the total number of turns in the shorter solenoid is NL_2, the total flux through it is

$$\Phi_2 = NL_2 \int_S \mathbf{B} \cdot d\mathbf{S} = NL_2 BA$$

and the mutual inductance is

$$M = \frac{\Phi_2}{I} = \frac{NL_2 BA}{I} = \frac{NL_2 \mu_0 NIA}{I} = \mu_0 N^2 L_2 A \qquad (13.14)$$

The reverse calculation, **i.e.,** computing the flux from the shorter one that was linked by the longer one, is pretty difficult, because of the diverging field lines at the ends of the short solenoid. However, such a calculation done by a computer gives the same result.

You may like to apply these ideas to a practical situation.

Spend 10 min

SAQ 7

Petrol in a vehicle's engine is ignited when a high voltage applied to a spark plug causes a spark to **jump** between two conductors of the plug. **This** high voltage is provided by an ignition coil, which is an arrangement of two **coils** wound tightly one on top of the other. Current from the vehicle's battery flows through the coil with fewer turns. This current is interrupted periodically by a switch. The sudden change in current induces a large emf in the coil with more turns, and this emf drives the spark. A typical ignition coil draws a current of 3.0 A and supplies an emf of 24 **kV** to the spark plugs. If the current in the coil is interrupted every 0.10 ms, what is the mutual inductance of the ignition coil?,

An **extremely** important application of the phenomenon of mutual inductance is found in the transformer. Let us study it in some detail.

13.3.3 The Transformer

You have just studied that a changing current in one coil induces an emf in another coil. And the emf induced in the second coil is given by the same law: that it is equal to the rate of change of the magnetic flux through the coil. Suppose we take two coils and connect one of them to an ac generator. The continuously **changing** current produces a changing magnetic flux in the second coil. This varying flux generates an alternating emf in the second coil, which has the same frequency as the **generating** current in the first coil. The induced emf in the second coil can, for example, produce enough power to light a bulb **(Fig.13.12).**

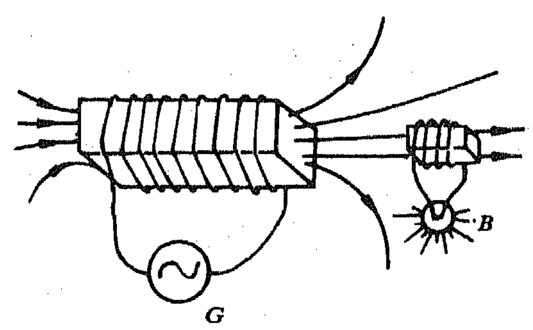


Fig. 13.12: Two coils allow a generator *G* with no direct connection, to light a bulb *B* — an application of mutual induction.

Now, the induced emf in the second coil can be made much larger than that in the first coil **if** we increase the number of turns in the second coil. This is because, in a given magnetic field, the flux through the coil is proportional to the number of **turns.** In the same way, the emf in the second coil can be made much smaller, if the number of turns in it is much less than the first coil.

Let us compute the magnitude of the voltage in the second coil (also known as the secondary coil) **vis-a-vis** the voltage in the first coil (known as the **primary** coil).

Voltages In the primary and secondary coils

Fig. (13.13b) shows a schematic diagram of a transformer shown in Fig. (13.13a). It has a primary coil (P) with N_1 turns. When the switch S_1 is closed, a current starts flowing in the primary coil. As the current increases, it generates an increasing magnetic flux in the circuit, which induces a back emf in the primary. The back emf exactly balances the applied voltage E if the resistance of the coil can be neglected. According to Faraday's law (Eq.13.1) the magnitude of the back emf ε_1 is given as

$$\varepsilon_1 = N_1 \frac{d\Phi}{dt} = E \qquad (13.15a)$$

Now the changing flux in the primary cuts the secondary S and so generates an emf ε_2 whose magnitude is given by

$$\varepsilon_2 = N_2 \frac{d\Phi}{dt} \qquad (13.15b)$$

where N_2 is the number of turns in the secondary. Eliminating $\frac{d\Phi}{dt}$ from Eqs. (13.15 a and b) we get

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{N_1}{N_2} \qquad (13.16)$$

Thus, the instantaneous voltages in the two coils are in the ratio of the number of turns on the coils. By setting the ratio of turns in the primary and secondary coils, we can make a step-up or step-down transformer that transforms a given ac voltage to any level we want. For example, if we want a step-down transformer to convert the high grid voltage of 22,000 V, to the low mains voltage of 220 V we must have

$$\frac{N_1}{N_2} = \frac{22{,}000\ V}{220\ V} = \frac{1000}{1}$$

Thus for every turn on the secondary there must be 1000 turns on the primary. In practice, the high grid voltage is reduced to the low mains voltage via a series of substations rather than via a single transformer. As you can see from Fig. 13.14, transformers are used throughout the power distribution network, to transform high voltages to low voltages.

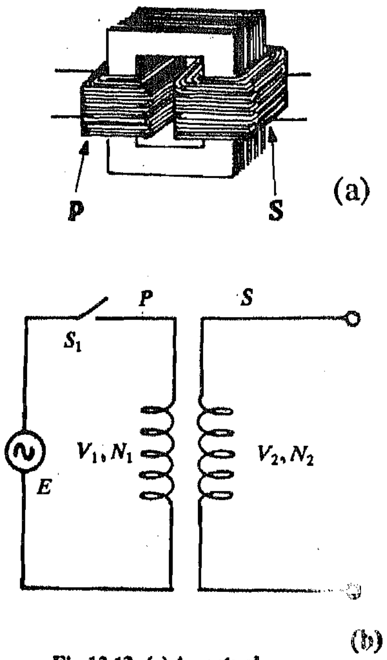


Fig.13.13: (a) An actual transformer; (b) schematic diagram of a transformer. The flux Φ is same in both the circuits because of their being identical geometrically.

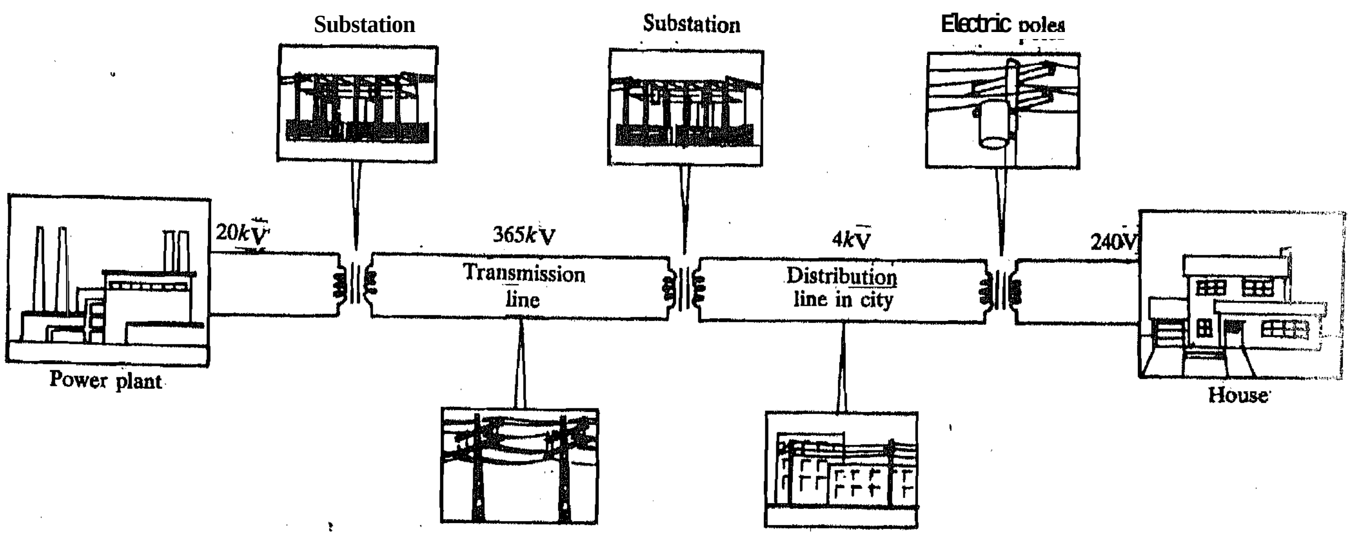


Fig. 13.14 : Transformers in a power distribution network

So far you have studied the phenomenon of electromagnetic induction and learnt about Faraday's Law, **Lenz's** law, self-inductance, mutual inductance and some of their applications. We will now turn our attention to another important aspect associated with this phenomenon, **viz.** the storage of energy in magnetic field.

13.4 ENERGY STORED IN A MAGNETIC FIELD

While discussing self-inductance, we had encountered the concept of back emf of an inductor. You know that we must do work against the back-emf to get the current going in a circuit. So it takes a certain amount of energy to start a current flowing in the circuit. This energy can be regarded as energy stored in the magnetic field of the current. In this section of the unit, we will **determine** the magnitude of the energy stored in a current-carrying circuit, and then the energy stored in a magnetic field.

Let us first determine the energy in a loop of inductance L. This is equal to the work required to build up a current I in it.

13.4.1 Energy Stored in an Inductor

When the current is built up in a circuit there is an induced back emf which opposes the flow of current. Suppose the back emf at some instant is ε. Then the **work** done on a unit charge againt the back emf ε, in one trip around the circuit is $-\varepsilon$. If the current at that instant is **I,** the charge passing through the wire in a small interval of time dt is **I** dt. Thus the work ***dW*** done in the interval dt is

Remember that here we have used the result $dI = \left(\frac{dI}{dt}\right) dt$

$$dW = -\varepsilon I\, dt = L\left(\frac{dI}{dt}\right) I\, dt$$

$$= L\, I\, dI$$

So, the total work done in building the current from a zero value to a value I_0 is

$$dW = L\int_0^{I_0} I\, dI$$

or

$$W = \frac{1}{2} L I_0^2 \qquad (13.17)$$

This was a specific example of storage of energy in an inductor. We can generalise the equation (13.17) to surface and volume **currents,** and show how this energy can be regarded as being the energy of the magnetic field produced by the steady current.

13.4.2 Magnetic Field Energy

You know that the flux through a single loop, is equal to L I where L is its inductance and **I** the current through the loop:

a? = L I **(13.18a)**

You also know that

$$\Phi = \int_S \mathbf{B} \,.\, d\mathbf{S} \qquad (13.18b)$$

Since the divergence of B is zero (Eq. 9.20 of Unit 9), we can use the vector identity V . (**V** x **A**) = 0 , where **A** is a vector field, to express B, in terms of **A:**

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (13.18c)$$

Here A is termed the vector potential associated with the magnetic field B.

Therefore, from Eq. **(13.18b)**

$$\Phi = \int_S (\nabla \times \mathbf{A}) \,.\, d\mathbf{S} \qquad (13.18d)$$

Using Stokes' theorem we get

$$\Phi = \oint_C \mathbf{A} \cdot d\mathbf{l} \qquad (13.18e)$$

Thus, from Eqs. (13.18a and 13.18e) we get

$$LI = \oint_C \mathbf{A} \cdot d\mathbf{l} \qquad (13.19)$$

Therefore, the energy of this loop is

$$U = \frac{L}{2} I^2 = \frac{1}{2} I \oint_C \mathbf{A} \cdot d\mathbf{l} \qquad (13.20)$$

Now, to generalise this expression, let us suppose that we do not have a current circuit defined by a wire. Instead, let the 'circuit' be a closed path that follows a line of current density. Then U given by Eq. (13.20) can approximate this situation very closely if we replace

$$I\, d\mathbf{l} \text{ by } \mathbf{J}\, dV$$

and

$$\oint_C \text{ by } \int_V$$

where V is the volume occupied by the current.

Hence we can write

$$U = \frac{1}{2} \int_V \mathbf{J} \cdot \mathbf{A}\, dV \qquad (13.21)$$

Using Ampére's law, ($\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$) we get

$$U = \frac{1}{2\mu_0} \int_V \mathbf{A} \cdot (\nabla \times \mathbf{B})\, dV \qquad (13.22)$$

We now use the following relation

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

to write

$$\mathbf{A} \cdot (\nabla \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \nabla \cdot (\mathbf{A} \times \mathbf{B})$$
$$= \mathbf{B} \cdot \mathbf{B} - \nabla \cdot (\mathbf{A} \times \mathbf{B})$$

$$(\because \mathbf{B} = \nabla \times \mathbf{A})$$

As a result we get

$$U = \frac{1}{2\mu_0} \left[\int_V \mathbf{B} \cdot \mathbf{B}\, dV - \int_V \nabla \cdot (\mathbf{A} \times \mathbf{B})\, dV \right]$$

or

$$U = \frac{1}{2\mu_0} \left[\int_V \mathbf{B} \cdot \mathbf{B}\, dV - \int_S (\mathbf{A} \times \mathbf{B}) \cdot d\mathbf{S} \right] \qquad (13.23)$$

where we have used Gauss' divergence theorem in the second term and S is the surface that bounds V. The integration is to be taken over the entire volume occupied by the current. However, we can even choose a larger region for integration without altering the result, since J will be zero beyond the volume occupied by the current. Let us extend the volume integral to include all space. In such an event, the contribution from the surface integral goes to zero, since the farther the surface is from the current, the smaller **B** and **A** are. Thus, we are left with

$$U = \frac{1}{2\mu_0}\int_V \mathbf{B}\cdot\mathbf{B}\, dV \qquad (13.24)$$

In view of this result we say that energy of the current-carrying circuits can be **regarded** as stored in the magnetic field produced by these currents, in the amount $B^2/2\mu_0$ per unit volume. Thus there are two ways to think about the energy stored in circuits which are entirely equivalent: **i.e.,** either $\frac{1}{2\mu_0}(\mathbf{A}\cdot\mathbf{J})$ or $B^2/2\mu_0$ of energy per unit volume.

Does it appear strange to you that it takes work to set up a magnetic field? The point is that setting up a magnetic field where there was none **requires** a changing magnetic field. And, as you know, a changing magnetic **field** induces an electric field. The electric field can do work. Thus, in the beginning **and** at the end there is no electric field. But, in between, when the magnetic **field** is building up, there is an electric field, and it is against this that the work is done. This work done appears as the energy stored in the magnetic field.

You may like to evaluate the magnitude of the energy thus stored for a specific situation.

Spend 10 min

SAQ 8

A long coaxial cable carries current I **which** flows down the surface of the inner cylinder of radius a and back **along the** outer cylinder of radius b (Fig. **13.15).** Find the energy stored in a section of length l of the cable. It is given that the magnitude of the magnetic field between the cylinder is

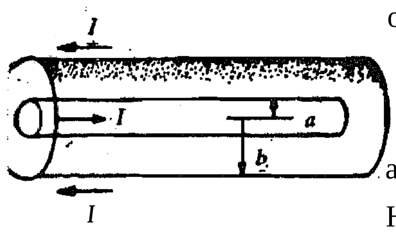


Fig. 13.15

$$B = \frac{\mu_0 I}{2\pi r}$$

and zero elsewhere.

Hence find the self-inductance per unit length of the cable.

We will now **summarise** what you have studied in this unit.

13.5 SUMMARY

- In this unit we have introduced you to two important phenomena: **electromagnetic** induction due to a changing magnetic field and **motional induction.**

- Motional induction can be explained by the **Lorentz** force. However, the explanation of electromagnetic induction requires the introduction of a new fundamental principle: A ***changing magnetic field gives rise to an induced electric field.***

- Electromagnetic induction is described by Faraday's **law** which gives the emf induced in a circuit when the flux through the circuit is changing with time as

$$\varepsilon = -\frac{d\Phi}{dt}.$$

Faraday's law applies to either of the **two kinds** of induction.

- Faraday's law can be rewritten in both integral and differential forms, to relate the **induced** electric field and the changing magnetic **field**

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

The **induced electric** field **is nonconservative** unlike the conservative electrostatic field of a stationary charge. Thus it **can** do work **on** charges as they move around a closed loop.

- The direction of an induced current is specified by **Lenz's law**: ***The direction in which induced current flows is such as to oppose the change that produced it.*** This law is reflected mathematically in the minus sign on the right hand side of Faraday's law. **Lenz's** law is a consequence of the conservation of energy principle.

- A changing current in a coil or circuit gives rise to a changing magnetic flux through the same circuit, which induces a **back** emf in it. The back emf opposes the original change in the current. This property of the circuit or the coil is called its **self-inductance.** Special devices that exhibit the property of **self-** inductance are called inductors. The self-inductance L of an inductor is the ratio of the magnetic flux to the current through it:

$$L = \frac{\Phi}{I}$$

An inductor opposes **instantaneous** change in current. Faraday's law **relates** the emf in an inductor to the rate of change of current:

$$\varepsilon = -L\frac{dI}{dt}$$

- When a pair of coils or conductors is placed so that the magnetic flux of one coil links the other, a changing current in one coil induces an emf in the other. This electromagnetic interaction of coils **is called** mutual induction. The mutual inductance of a pair of coils is defined as the ratio of the total flux in **the second** coil to the current in the first:

$$M = \frac{\Phi_2}{I_1}$$

Faraday's law relates the emf in the second **coil** to the rate of change of current in the first

$$\varepsilon_2 = -M\frac{dI_1}{dt}$$

The same mutual inductance M describes the emf induced in the first coil as a result of changing current in the second coil.

- Work needs to be done to build up **current** and, therefore, magnetic field in an inductor. This work **ends** up as stored energy in **the** inductor, given by

$$U = \frac{1}{2}LI^2$$

where L is the self-inductance of the inductor carrying current **I**.

This energy can also be regarded as the energy stored in the magnetic field produced by the current **I** and can be written as

$$U = \frac{1}{2\mu_0}\int_V \mathbf{B} \cdot \mathbf{B}\, dV$$

This expression is very general and applies to a single inductor, coupled inductors, and surface and volume distributions of currents.

13.6 TERMINAL QUESTIONS

Spend 45 min

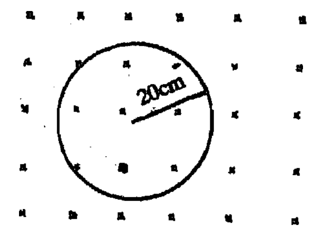
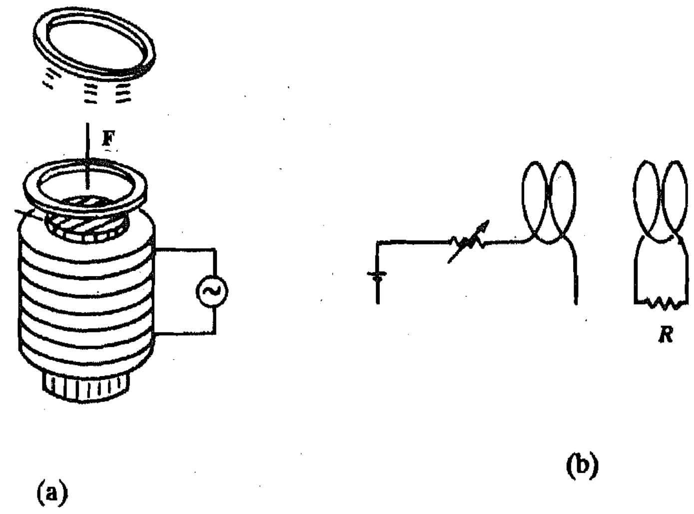


Fig. 13.16: The magnetic field B points into the page.

1. A wire loop of radius 20 cm having a resistance of 5.0 Ω is immersed in a uniform magnetic field B at right angles to it (Fig. 13.16). The field strength is increasing at the rate of 0.10 **tesla** per second. Find the magnitude and direction of the induced current in the loop.

2.a) A **metal** ring placed on top of a solenoid jumps when **current** through the solenoid is switched on (Fig. **13.17a**). Why?



(a) (b)

Fig. 13.17

b) Two coils are arranged as shown in **Fig.13.17b.** If the resistance of the variable resistor is baing increased, what is the direction of the induced current in the fixed resistor R?

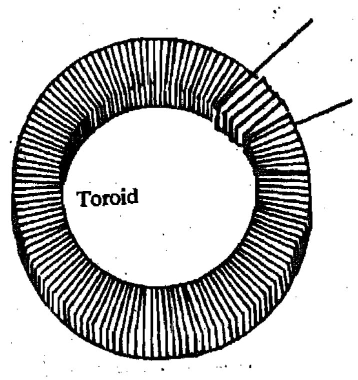


Fig. 13.18

3.a) Determine the self-inductance of a toroidal coil of rectangular cross-section, having N_1 turns and inner radius a, outer radius b and height h. If the current through the coil is i_1, what is the total magnetic energy stored in the coil?

b) Suppose a coil C of N_2 **turns** is wound over the toroidal coil of part (a) as **shown** in Fig. 13.18. Show that the mutual inductance for this arrangement is

$$M = \frac{\mu_0}{2\pi} N_1 N_2 h \ln \frac{b}{a}$$

4. The rim of a horizontally suspended wheel of radius R carries charge q. The wheel (with **wooden** spokes) is free to rotate. In the central region of the wheel **upto** a radius a, there exists a uniform magnetic field B pointing up (Fig.13.19). What happens when the magnetic field is turned off? How much angular momentum will **be** added to the wheel?

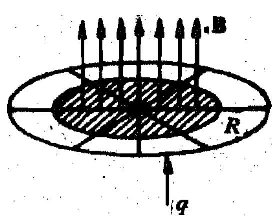


Fig. 13.19

5.a) A sheet of **copper** is placed in a magnetic field as **shown** in Fig.13.20. If we **try** to pull it out of the field or push it in, a resisting force appears; Explain **its** origin.

b) A superconducting solenoid designed for imaging the,human body by nuclear magnetic resonance is 0.9 m in diameter and 2.2 m long. The field at its centre is 0.4 **tesla.** Estimate the energy stored in the **field** of .this coil.

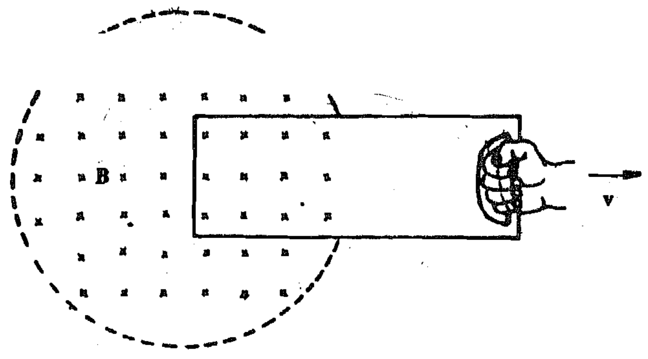


Fig. 13.20

13.7 SOLUTIONS AND ANSWERS

SAQs (Self-assessment Questions)

1. The circuit wire is made of metal and contains electrons which are relatively free to move. When the wire is moved in the magnetic field, the electrons in it also move with it. When these electrons move in the magnetic field, they experience the force $\mathbf{F} = -e\,\mathbf{v} \times \mathbf{B}$ that tends to push them along the wire. The relatively free electrons are set into motion by the force and form an electric current as they move along the wire. It is this current that is detected by the ammeter. This is termed the **induced current.**

2. Recall from Unit 4 that the curl of the electric field due to static charges is zero and the force field corresponding to it is conservative. From Eq. (13.2b) you will note that the curl of the electric field induced by a changing magnetic field is non- zero. Hence, the electric field given by Eq. (13.2b) is nonconservative. This is the basic difference between these two kinds of electric fields. The force corresponding to electric field of Eq. (13.2b) can do work on charges as they move around a closed loop. Moreover, we cannot associate scalar potential with this field.

3.a) As we stretch the loop, its area and hence the magnetic flux through it decreases. The direction of the induced current is such as to oppose this deaease, i.e., its magnetic field should add to the existing magnetic field. Thus the induced current should flow in the clockwise direction as we view it from top.

b) When a clockwise current is established in the bigger loop, it sets up a magnetic field similar to a bar magnet's with its north' pole facing toward the smaller loop. The induced current in the smaller loop should be such as to dppose this change in the magnetic field, i.e., it should offer a north pole to the magnetic field of the bigger loop. This will happen if the current in the smaller loop is in the counterclockwise direction as seen from the left.

4. The induced emf is given by

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

From Eq. (13.3a), for a uniform magnetic field the flux through one turn of the loop is

$$\Phi_B = BS\cos\omega t$$

$\omega = 2\pi f$, where f is the frequency at which the loop rotates. S is the area of the loop given by s^2, where s is the length of the side loop. Thus, the induced emf for an N turn coil is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -NBs^2[-2\pi f\sin(2\pi ft)] = 2\pi NBs^2 f\sin(2\pi ft)$$

Thus the peak emf $\varepsilon_0 = 2\pi NBs^2 f = 300V$

$$\therefore B = \frac{\varepsilon_0}{2\pi Ns^2 f} = \frac{300\text{ V}}{(2\pi)\,10\,(50\text{ Hz})\,(0.50\text{ m})^2} = 0.38\text{ T}$$

This is the typical field strength near the poles of a strong permanent magnet.

5. We will first have to **determine** the self-inductance of the solenoid which is

$$L = \frac{\mu_0 N^2 A}{l}$$

For the **parameters** given in the problem

$$L = \frac{(1.26\times10^{-6}\text{ Hm}^{-1})\times(10{,}000)^2\times\pi\,(0.1\text{m})^2}{1\text{ m}}$$

$$= 3.96\text{ H}$$

Now since the current changes steadily, the magnitude of its rate of change is

$$\frac{dI}{dt} = \frac{2.5\text{ A}}{1.0\text{ ms}} = 2500\text{ As}^{-1}$$

The magnitude of the back emf is

$$|\varepsilon| = L\frac{dI}{dt} = (3.96\text{ H})(2500\text{ As}^{-1}) = 9900\text{V}$$

The voltage is high enough to produce a lethal shock. Note that this voltage is unrelated to the voltage of the source supplying the inductor current. We could have a 6V battery and still be electrocuted trying to switch off a circuit rapidly when a large inductance is present. So be careful if you have to handle such circuits.

6. In the steady state the current is not changing and there is no emf in the inductor. So the current in the circuit is simply given by

$$I = \frac{\varepsilon_0}{R} = \frac{15\text{ V}}{2000\Omega} = 7.5\text{ mA}$$

When the **current** has half its final value, ε_0/R, **Eq.(13.10b)** gives

$$\frac{I}{(\varepsilon_0/R)} = \frac{1}{2} = 1-\exp\left(-\frac{Rt}{L}\right)$$

or

$$\exp\left(-\frac{Rt}{L}\right) = \frac{1}{2}$$

Taking the natural logarithm of both sides

$$\frac{Rt}{L} = -\ln\left(\frac{1}{2}\right) = \ln(2)\;[\because \ln(e^x) = x]$$

$$\therefore\; t = \frac{L}{R}\ln 2$$

or

$$t = \frac{10\times10^{-3}\text{H}\times(0.69)}{2000\Omega}\text{s} = 3.5\,\mu\text{s}$$

Although short, this time would be significant in a TV, computer or **electronic** communication involving high frequency signals,.

7. The rate of change of current is

$$\frac{dI}{dt} = \frac{3.0\text{A}}{1.0 \times 10^{-4}\text{s}} = 3.0 \times 10^4 \text{As}^{-1}$$

The mutual inductance is

$$M = \frac{|\varepsilon|}{dI/dt} = \frac{24 \times 10^3 \text{ V}}{3.0 \times 10^4 \text{As}^{-1}} = 0.8\text{H}$$

8. The energy per unit volume is

$$\frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 r^2}$$

The volume of the cylindrical shell of length l, radius r, and thickness dr is $(2\pi r\, dr)\, l$. Therefore, the energy stored in the cylindrical shell is

$$\frac{\mu_0 I^2 l}{8\pi^2 r^2}(2\pi r\, dr) = \frac{\mu_0 I^2 l}{4\pi}\frac{dr}{r}$$

Integrating from a to b we have:

$$U = \frac{\mu_0 I^2 l}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I^2 l}{4\pi} \ln\frac{b}{a}$$

Since $U = \frac{1}{2} L I^2$, we get an expression for L:

$$L = \frac{\mu_0 l}{2\pi} \ln\frac{b}{a}$$

The self-inductance per unit length is $\frac{\mu_0}{2\pi} \ln\frac{b}{a}$.

Terminal Questions

1. To find the induced current we must know the induced emf which is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Now the flux is

$$\Phi_B = \int_S \mathbf{B} \cdot d\mathbf{S}$$

Since the field is uniform in space and the loop is at right angles to it we get

$$\Phi_B = \int B\, dS = B\int dS = B\pi r^2$$

where r is the radius of the loop. Thus, with the loop area being constant

$$\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt}$$
$$= \pi (0.2\text{m})^2 (0.1 \text{ Ts}^{-1})$$
$$= 1.2 \times 10^{-2}\text{V}$$

We can obtain the magnitude of the induced current through the loop using Ohm's law:

$$I = \frac{\varepsilon}{R} = \frac{1.2 \times 10^{-2}\text{V}}{5.0\Omega} = 2.4 \text{ mA}$$

Since the magnetic field, pointing into the page, is increasing, the direction of the induced current is such as to oppose this increase. Thus, the current's magnetic field should be in the opposite direction, i.e., the current should flow counterclockwise in the loop as viewed from top of the page.

2.a) Before the current is switched on, the flux through the ring is zero. When the current is switched, a flux appears upward in the diagram. Due to the change in flux, an emf and a current is induced in the metal ring. The direction of the current is such that its magnetic field is directed opposite to that of the solenoid. Thus the current in the loop is opposite to the current in the solenoid. You must have studied that the force between two conductors carrying currents in the opposite directions is repulsive. This causes the ring to jump.

b) The current in the coil 1 on the left flows counterclockwise in it, so that its magnetic field points towards the second coil. As the resistance increases, the current in coil 1 decreases, causing a decrease in the magnetic flux linked by coil 2. The induced current opposes this decrease, so its magnetic field should point to the right of the coil 2. Thus, from the right- hand rule the induced current should flow counterclockwise in coil 2, i.e., it should flow in the resistor R from right to left.

3.a) You know that the magnetic field within the toroid is given by

$$B = \frac{\mu_0 i_1 N_1}{2\pi r}$$

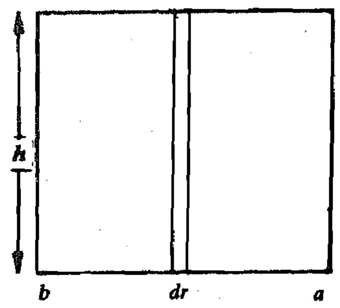


Fig. 13.21

where i_1 is the current in the toroid windings. We must now find the flux through each turn of the toroid, which is the flux through the toroid (Fig. 13.21).

Let us consider an elementary strip of area h dr. The flux through the strip is

$$d\Phi = \mathbf{B} \cdot d\mathbf{S} = (B)(h\,dr) = \frac{\mu_0 i_1 N_1 h}{2\pi r} dr$$

since B is normal to the toroid cross-section. The total flux through the toroid cross-section is found by integrating $d\Phi$ from r = a to r = b :

$$\Phi = \frac{\mu_0 i_1 N_1 h}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 i_1 N_1 h}{2\pi} \ln \frac{b}{a}$$

The self-inductance of the toroid having N_1 turns is

$$L = \frac{N_1 \Phi}{i_1} = \frac{\mu_0 N_1^2 h}{2\pi} \ln \frac{b}{a}$$

The total magnetic energy stored in the coil is

$$U = \frac{1}{2} L i_1^2 = \frac{\mu_0 N_1^2 h\, i_1^2}{4\pi} \ln \frac{b}{a}$$

b) The flux linkage through the coil C of N_2 turns is due to the flux within the toroidal coil. Therefore, it is given by

$$N_2 \Phi_2 = \frac{\mu_0 i_1 N_1 N_2 h}{2\pi} \ln \frac{b}{a}$$

Thus, the mutual inductance for this arrangement is

$$M = \frac{N_2 \Phi_2}{i_1} = \frac{\mu_0}{2\pi} N_1 N_2 h \ln \frac{b}{a}$$

4. The changing magnetic field induces an electric field. Since the magnetic field is decreasing, the E field will be in the counterclockwise direction; to oppose the decrease, The induced electric field exerts a force on the charges on the rim driving them around. This causes the wheel to rotate in the counterclockwise direction, as seen from above. Quantitatively, from Faraday's law we have

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

$$= -\pi R^2 \frac{d\mathbf{B}}{dt}$$

The torque on a small element $d\mathbf{l}$ of the rim referred to the centre of the wheel is ($\mathbf{R} \times \mathbf{F}$) or (Rq E dl). The magnitude of total torque on the wheel is

$$\tau = Rq\oint E\, dl = -\pi R^3 q \frac{dB}{dt}$$

The total angular momentum imparted to the wheel is

$$L = \int \tau\, dt = -\pi R^3 q \int_{B_0}^{0} dB = \pi R^3 q B_0$$

5.a) As we try to pull the sheet of copper out of the magnetic field, induced current appear in it. Since the flux through tbe sheet is decreasing, the direction of the current in it is clockwise to oppose the decrease. The magnetic force (= I d $\mathbf{l}$ × B) due to the induced current will be towards the left, i.e., it will oppose the motion of the loop.

Similarly, when we push the sheet in, induced current in the counterclockwise direction appears. The magnetic force due to this current points towards the right opposing the direction of motion. The currents induced in solid conductors due to changing magnetic fields are termed **eddy** currents. **As** you have seen here, eddy currents can make it difficult to move a conductor through a magnetic field.

b) The **energy** U stored in the solenoid is ½ L i^2. The magnitude of the magnetic field of the solenoid of length 1 is

$$B = \frac{\mu_0 N i}{l}$$

Thus the current through the solenoid is

$$i = \frac{lB}{\mu_0 N}$$

You know that the inductance of a long solenoid is

$$L = \frac{\mu_0 N^2 A}{l}$$

Thus

$$U = \tfrac{1}{2} L i^2 = \tfrac{1}{2}\left(\frac{\mu_0 N^2 A}{l}\right)\left(\frac{l^2 B^2}{\mu_0^2 N^2}\right)$$

$$= \frac{1}{2\mu_0} l B^2 A$$

$$= \frac{1\,(2.2\text{m})\,(0.4\text{ T})^2\,\pi\,(0.45\text{ m})^2}{2 \times 1.26 \times 10^{-6}\text{ Hm}^{-1}}$$

or $$U = 8.9 \times 10^4\text{ J}$$