Cyclotron frequency ω_L is

$$\omega_L = \frac{qB}{2\pi m}$$

$$\therefore \frac{\omega_p}{\omega_L} = \frac{qB}{2m} \times \frac{2\pi m}{qB} = \pi .$$

UNIT 12 MAGNETISM OF MATERIALS-II

Structure

12.1 INTRODUCTION

In Block 2 of this course you have studied the behaviour of dielectric materials in response to the external electric fields. This was done by investigating their properties in terms of electric dipoles, both natural and induced, present in these materials and their lining up in the electric field. The macroscopic properties of these materials were studied using the so-called polarization vector P, the electric dipole moment per unit volume.

The magnetic properties of materials has a similar kind of explanation, albeit in a more complicated form, due to the absence of free magnetic monopoles. The magnetic dipoles in these materials are understood in terms of the so-called Amperian current loops, first introduced by Ampere.

All materials are, in some sense, magnetic and exhibit magnetic properties of different kinds and of varying intensities. As you know, all materials, can be divided into three main categories: (i) Diamagnetic; (ii) Paramagnetic and (3) Ferromagnetic materials. In this unit, we shall study the macroscopic behaviour of these materials.

We understood the macroscopic properties of the dielectric materials using the fact that the atoms and molecules of these substances contain electrons, which are mobile and are responsible for the electric dipoles, natural and induced, in these substances. The polarisation of these substances is the gross effect of the alignment of these dipoles. Similarly we describe the magnetic properties of various materials in terms of the magnetic dipoles in these materials.

In Unit 11, we have already explained diamagnetism and paramagnetism in terms of magnetic dipoles. In this unit, first, we will mention the origin of ferromagnetism. Later, we will develop a description of the macroscopic properties of magnetic material.

With Unit 12, we end our study of magnetism, In the next Block we will deal with the situation where both electric and magnetic fields will vary with time. This will lead, ultimately, to the 'four differential equations known as Maxwell's equations.

Objectives

After studying this unit you should be able to :

- understand and explain the terms: ferromagnetism, amperian current, magnetisation, magnetic intensity **H**, magnetic susceptibility, magnetic permeability, relative permeability,
- relate magnetisation M (which is experimentally measureable) and the atomic currents (which is not measureable) within the material,
- derive and understand the differential and integral equations for M and H and apply these to calculate fields for simple situations,

- interrelate B, H, M, μ_0, μ and χ,
- relate B & H for various magnetic and non-magnetic materials,
- derive an equation in analogy with Ohm's law for a magnetic circuit.

12.2 FERROMAGNETISM

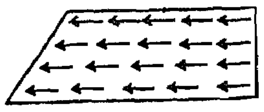

Fig. 12.1: Domain

Ferromagnetic materials are those materials, which respond very strongly to the presence of magnetic fields. In such materials, the magnetic dipole moment of the atoms arises due to the spins of unpaired electrons. These tend lo line up parallel lo each other. Such a line-up does not occur over the whole material, but it occurs over a small volume, known as 'domain', as shown in Fig. 19.1. However, these volumes are large compared to the atomic or molecular dimensions. Such line-ups take place even in Ihe absence of an external magnetic field. You must be wondering about the nature of forces that cause the spin magnetic moments of different atoms to line up parallel to each other. This can be explained only by using quantum mechanical idea of "exchange forces". We will not go into the details of exchange forces. About this, you will study in other courses of physics, but we are giving you some idea of exchange forces in the margin remark.

Cansider two electrons on atoms that are close to each other. If the electron spins are parallel, they stay away from each other due to Pauli principle, therehy reducing their coulomb energy of repulsion. On the other hand, if these spins are anti-parallel. the electrons can come close lo eachother and their coulomb energy is higher. 'Thus, by making their spins parallel, the electrons can reduce their energy.

In an unmagnetized ferromagnetic material, the magnetic moments of different domains are randomly oriented, and the resulting magnetic moment of the material, as a whole is zero, as shown in Fig. 12.9. However, in the presence of an external magnetic field, the magnetic moments of the domains line-up in such a manner as to give a net magnetic moment to the material in the direction of the field. The mechanism by which this happens is that the domains with the magnetic moments in the favoured directions increase in size at the expense of the other domains, as shown in Fig. 12.3a.

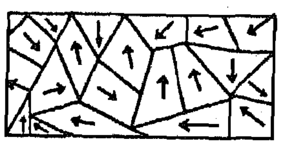

Fig. 122: **The domains in an unmagnetised bar of iron. The arrows show the alignment direction of the magnetic moment in each domain.**

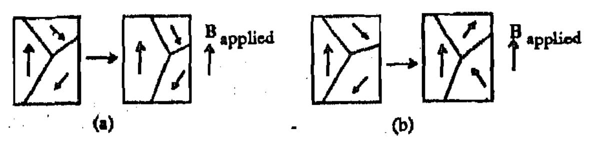

Fig.12.3 : In a ferromagnetic material domain changes, resulting in a net magnetic moment, occur through (a) domain growth and (b) domain realignment,

In addition, the magnetic moments of the entire domains can rotate, as shown in Fig. 12.3b. The material is thus magnetised. If, after this, the external magnetic field is reduced to zero, there still remains a considerable amount of magnetization in the material. The material gets permanently magnetized. The behaviour of ferromagnetic materials, under the action of changing magnetic fields, is quite complicated and exhibits the phenomenon, of hysteresis which literally means 'lagging behind'. You will study more about this in Sec. 12.5.

Above a certain temperature, called 'Curie Temperature', because the forces of thermal agitation dominate 'exchange' forces, the domains lose their dipole moments. The ferromagnetic material begins to behave like a paramagnetic material. When cooled, it recovers its ferromagnetic properties.

Finally, we briefly mention two other types of magnetism which are closely related to ferromagnetism. These are anti-ferromagnetism and **ferrimagnetism** (also called ferrites). In this course, we will not study the physics of antiferro- and ferrimagnetism. The main reason for mentioning these materials is that they are of technological importance, being used in magnetic recording tapes, antenna and in computer memory.

In antiferromagnetic substances, the 'exchange' forces, as we mentioned earlier, play the role of setting the adjacent atoms into antiparallel alignment of their equal magnetic moments, that is, adjacent magnetic moments are set in opposite directions, as shown in Fig. 12.4 a.

Such substances exhibit little or no evidence of magnetism present in the body. However, if these substances are heated above the temperature known as **Neel temperature,** the exchange force ceases to act and the substance behaves like any other paramagnetic material.

In ferrimagnetic substances, known generally as femtes, the exchange coupling locks the magnetic moments of the atoms in the material into a pattern, as shown in Fig. 12.4b. The external effects of such an alignment is intermediate between ferromagnetism and antiferromagnetism. Again, here the exchange coupling disappers above a certain temperature.

(a) (b)

Fig. 12.4 :Relative orientation of electron spins in (a) antiferromagetic material and (b) ferrite.

Thus, we find that the magnetization of the materials is due to permanent (and induced) magnetic dipoles in these materials. The magnetic dipole moments in these materials are due to the circulating electric currents, known as amperian currents at the atomic and molecular levels. You arc expected lo understand the correct relationship between magnetization in a material and the amperian currents, togetherwith the basic difference (and somelimes similarities) between the behaviour of the magnetic materials in magnetic fields, and dielectrics (and conductors) in electric fields.

Though physics of paramagnetic and ferromagnetic materials have analogues in the electric case, diamagnetism is peculiar to magnetism. The student is advised to read the matter in this unit and find the analogies and appreciate the differences, if any, by referring back to the units on dielectrics. In the next section, we will find out the relationship between the macroscropic quantity M, which is experimentally measurable and the atomic currents (a microscopic quantity) within the material which is not measurable. With the help of this relationship, we can find out the magnetic field that magnetised matter itself produces.

12.3 MAGNETIC FIELD DUE TO A MAGNETISED MATERIAL

In Unit 5, we have described the macroscopic properties of dielectric materials in terms of the polarization vector **P**, the origin of which is in the dipole moments of its natural or induced electric dipoles. We shall adopt a similar procedure in the study of magnetic materials. You would be tempted to say that we should carry over all the equations in the study of dielectrics to magnetic materials. One way of doing this would be to replace the electric field vector E by B, then replace P by an analogous quantity which we shall call magnetization vector M which is the magnetic dipole moment per unit volume. Further, we replace the polarization charge density ρ_p by magnetic 'charge' density ρ_m whatever that means, by writing $V \cdot M = -\rho_m$ just as we had $V \cdot P = \rho_p$. In fact, people did something Iike this, and they believed that magnetic charges or monopoles exist. They have built a whole theory of electromagnetism on this assumption. However, we know that magnetic 'charges' or monopoles have not yet been detected in any experiment so far, despite a long search for them. Now, we know that the magnetization of matter is due lo circulating currents within the atoms of the materials, This was originally suggested by Ampere, and we call these circulating currents as 'amperian' current loops. These currents arise due to either the orbilal motion of electrons in the atoms or their spins. These currents, obviously, do not involve large scale charge transport in the magnetic materials as in the case of conduction currents. These currents are also known as magnetization currents, and we shall relate these currents to the magnetization vector M.

Let us consider a slab of uniformly magnetised material, as shown in Fig. 12.5a. It contains a large number of atomic magnetic dipoles (evenly distributed throughout its volume) all pointing in the same direction. If μ is the magnetic moment of each dipole then the magnetisation M will be the product of μ and the number of oriented dipoles per unit volume. You know that the dipoles can be indicated by tiny current loops. Suppose the slab consists of many tiny loops, as shown in Fig. 12.5b. Let us consider any tiny loop of area a, as shown in Fig. 12.5c. In terms of magnetisation M, the magnitude of dipole moment μ is written as follows:

$$\mu = Madz \qquad (12.1)$$

where dz is the thickness of the slab.

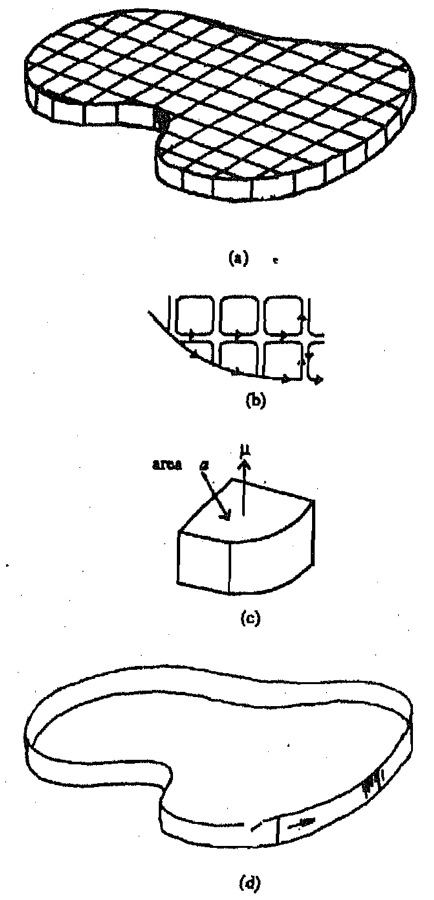

Fig.12.5: (a) A thin slab of uniformly magnetized material, with the dipoles indicated by (b) and (c) tiny current loops is equivalent to (d) a ribbon of current I flowing around the boundary.

If the tiny loop has a circulating current I, then dipole moment of the tiny loop is given by

$$\mu = Ia \qquad (12.2)$$

Equating (12.1) and (12.2) we get

$$M = \frac{I}{dz} \text{ or } I = Mdz. \qquad (12.3)$$

Here we have assumed that the current loops corresponding to magnetic dipoles are large enough so that magnetisation does not vary appreciably from one loop to the next, so Eq. 12.3 shows that the current is the same in all current loops of Fig. 12.5b. Notice that within the slab, currents flowing in the various loops cancel, because everytime if there is one going in one particular direction, then a continuous one is going in the exactly opposite direction. At the boundary of the slab, there is no adjacent loop to do the cancelling. Hence the whole thing is equivalent to the single loop of current I flowing around the boundary, as shown in Fig. 12.5d. Therefore, the thin slab of magnetised material is equivalent to a single loop carrying the current Mdz. Hence, the magnetic field at any point external to the slab, is the same as that of the current Mdz.

In case there is non-uniform magnetization in the material, the atomic currents in the (amperian) circulating current loops do not have the same magnitude at all points inside the material and, obviously, they do not cancel each other out inside such a material. Still we will find that magnetised matter is equivalent to a current distribution J = curl M. Let us see how we have arrived at this relation.

In the non-uniformly magnetised material consider two little blocks of the volume $\Delta x \, \Delta y \, \Delta z$, cubical in shape adjacent to each other along y-axis (see Fig. 12.6a). Let us call these blocks '1' and '2' respectively. Let the z-component of M in these blocks be $M_z(y)$ and $M_z(y + \Delta y)$ respectively.

Let the amperian currents circulating round the block '1' be $I(1)$ and round the block '2' be $I(2)$. Using Eq. (12.3) and refemng to Fig. 12.6a we write,

$$I_x(1) = M_z(y)\,\Delta z$$

and

$$I_x(2) = M_z(y + \Delta y)\,\Delta z$$

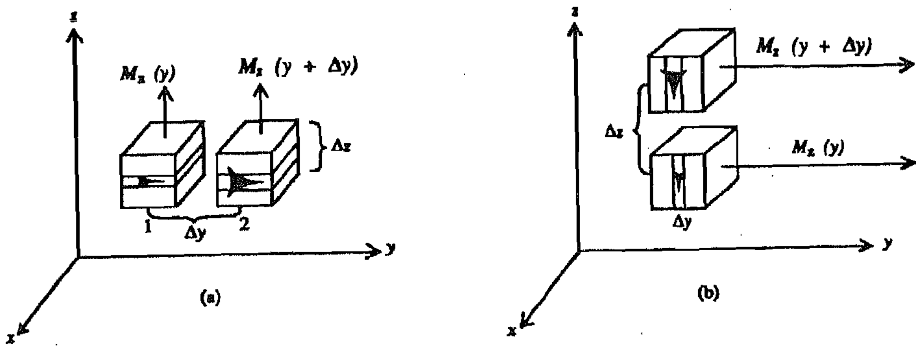

Fig. 12.6: Two adjacent chuncks of magnetised material, with a larger arrow on the one to the right in (a) and above in (b), suggesting greater magnetisation at that point. On the surface where they join them is a net current in the x-direction

At the interface of the two blocks, there will be two contributions to the total current: $I(1)$ flowing in the negative x-direction, produced due to block 1, and $I(2)$ flowing in the positive x-direction produced due to Block 2. The total current in the positive x-direction is the sum :

$$I_x(2) - I_x(1) = \left[M_z(y + \Delta y) - M_z(y) \right] \Delta z$$

or

$$\Delta I_x = + \frac{\partial M_z}{\partial y} \Delta y\, \Delta z \qquad (12.4)$$

Eq. (12.4) gives the net magnetization current in the material at a point in the x-direction in terms of the z-component of **M**. The current per unit area, i.e., current density J_m flowing in the x-direction is given as follows :

$$(J_m)_x = \frac{\Delta I_x}{\Delta y\, \Delta z}$$

where $\Delta y\, \Delta z$ is the area of cross-section of one such block for the current ΔI_x. Hence

$$(J_m)_x = + \frac{\partial M_z}{\partial y} \qquad (12.5)$$

In these equations, we have put suffixes x to the currents to indicate that, at the interface of the blocks, the current is along the x-axis.

There is another way of obtaining the current flowing in x-direction by considering these two tiny blocks, one above the other, along the z-axis, as shown in Fig. 12.6b. We obtain the relation as

$$(J_m)_x = - \frac{\partial M_y}{\partial z} \qquad (12.6)$$

By superimposition of these two situations, we get

$$(J_m)_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z} = (\nabla \times \mathbf{M})_x \qquad (12.7)$$

Eq. (12.7) is obviously the x-component of a vector equation relating $\mathbf{J}_m$ and the curl of **M**. Combining this with y and z components, we obtain

$$\mathbf{J}_m = \nabla \times \mathbf{M} \qquad (12.8)$$

Eq. (12.8) is a more general expression representing the relationship between the magnetisation and the equivalent current. We see from Eq. (12.8) that inside a uniformly magnetized material in which case **M** = constant; we have $\mathbf{J}_m = 0$. This is true. See Eq. (12.8), the current is only at the surface of the material where the

If the magnetisation in the first block is $M(x, y, z)$, the magnetisation in the second block is

$$M(x, y, z) + \frac{\partial M}{\partial y} \Delta y + \text{higher order terms.}$$

The z-component of magnetisation of the first block in terms of $I_x(1)$ is written as

$$M_z \Delta z = I_x(1)$$

Similarly, the z-component of magnetisation of the second block neglecting high-order terms which vanish in the limit where each block becomes very small, is given by

$$\left(M_z + \frac{\partial M_z}{\partial y} \Delta y \right) \Delta z = I_x(2)$$

magnetization has a discontinuity (droping from a finite M to zero). Inside a non-uniformly magnetized material, J_m is nonzero.

We shall see in the next section that J_m, which is introduced to explain the origin of magnetisation in a material, is made to make its exit from the equation, and only the conductioncurrent density indicating the actual charge transport and which is experimentally measurable remains.

12.4 THE AUXILIARY FIELD H (MAGNETIC INTENSITY)

So far we have been considering that magnetisation is due to current associated with atomic magnetic moments and spin of the electron. Such currents are known as bound currents or magnetisation amperian current. The current density J_m in Eq. (12.8) is the bound current set up within the material. Suppose you have a piece of magnetised material. What field does this object produce? The answer is that the field produced by this object is just the field produced by the bound currents established in it. Suppose we wind a coil around this magnetic material and send through this coil a certain current I. Then the field produced will be the sum of the field due to bound currents and the field due to current I. The current *I* is known as the free current because It is flowing through the coil and we can measure it by connecting an ammeter in series with the coil. (In case the magnetic material happens to be conductor, the free current will be the current flowing through the material itself.) Remember that free currents are those caused by external voltage sources, while the internal currents arise due to the motion of the electrons in the atoms. The current is free, because someone has plugged a wire into a battery and it can be started and stopped with a switch. Therefore, the total current density J can be written as

$$\mathbf{J} = \mathbf{J}_f + \mathbf{J}_m \tag{12.9}$$

where $\mathbf{J}_f$ represents the free current density.

Let us use Ampere's law to find the field. In differential form, it is written as (See Unit 9)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{9.46}$$

Using Eq. (12.9), Ampere's law would then take the form as follows:

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{J}_f + \mathbf{J}_m)$$

As mentioned earlier, we have no way to measure $\mathbf{J}_m$ experimentally, but we have a way to express it in terms of a measurable quantity, the magnetization vector M through the Eq. (12.8). We then have

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_f + \mu_0 (\nabla \times \mathbf{M})$$

or

$$\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{J}_f \tag{12.10}$$

Eq. (12.10) is the differential equation for the field $\left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right)$ in terms of its source $\mathbf{J}_f$, the free current density. This vector is given a new symbol H, i.e.,

$$\frac{\mathbf{B}}{\mu_0} - \mathbf{M} = \mathbf{H} \tag{12.11}$$

The vector **H** is called the magnetic 'intensity' vector, a name that rightly belongs to **B**, but, for historical reasons, has been given to H. Using Eq. (12.11). Eq. (12.10) becomes

$$\nabla \times \mathbf{H} = \mathbf{J}_f \tag{12.12}$$

In other words, H is related to the free current in the way B is related to the total current, bound plus free. This surely has made you think over the purpose of introducing the new vector field H. For practical reasons the vector H is very useful as it can be calculated from the knowledge of external current only; whereas B is related to the total current which is not known. Eq. (1312) can also be written in the integral form as

$$\oint \mathbf{H}.d\mathbf{l} = I_f \qquad (11.13)$$

where I_f is the conduction current through the surface bounded by the path of the line integral on the left. Here the line integral of **H** is around the closed path which may or may not pass through the material. This equation can be used to calculate W, even in the presence of the magnetic material.

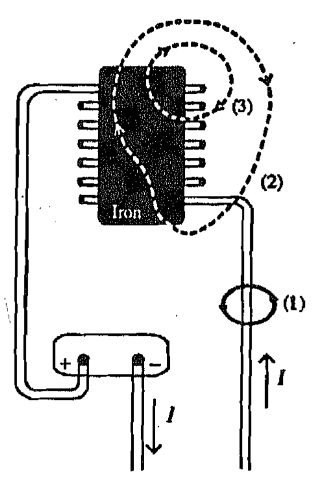

Fig. 12.7 : SAQ1

SAQ I

Fig. 12.7 shows a piece of imn wound by a coil carrying a current of 5A. Find the value of $\oint \mathbf{H} \cdot d\mathbf{l}$ around the path (1), (2) and (3). Also state for which path(s) $\mathbf{B} = \mathbf{H}$ and $\mathbf{B} \neq \mathbf{H}$.

From Eq. (12.3), we see that the units in which **M** is measured is amperes per meter. Eq. (12.11) shows that the vector **H** has the units as **M**, hence **H** is also measured in amperes per metre. The electrical engineers working with electromagnets, transformers, etc., call the unit of **H** as ampere turns per metre. Since 'turns', which is supposed to imply the number of turns in the coil carrying a current, is dimensionless, it need not confuse you.

Magnetic properties of substance

In paramagnetic and diamagnetic materials, the magnetisation is maintained by the field. When the field is removed, M disappears. In fact, it is found that **M** is proportional to **B**, provided that the field is not too strong. Thus

$$\mathbf{M} \propto \mathbf{B} \qquad (12.14)$$

It is conventional to express Eq. (12.14) in terms of H instead of B. Thus we have

$$\mathrm{M} = \chi_m \mathbf{H} \qquad (12.15)$$

The constant of proportionality χ_m is called the magnetic susceptibility of the material. It is a dimensionless quantity, which varies fmrn one substance to another. We can characterise the magnetic properties of a substance by χ_m. It is negative for diamagnetic substances and positive for paramagnetic materials. Its magnitude is very small compared to unity, that is $|\chi_m| << 1$. For vacuum χ_m is zero, since M can only exist in magnetised matter. We give below a short table giving the values of χ_m for diamagnetic and paramagnetic substances at room temperature.

	Material	χ_m
Paramagnetic	Aluminium	2.1×10^{-5}
Paramagnetic	Sodium	0.84×10^{-5}
Paramagnetic	Tungsten	7.6×10^{-5}
Paramagnetic	Oxygen	190×10^{-5}
Diamagnetic	Bismuth	-1.64×10^{-5}
Diamagnetic	Copper	-0.98×10^{-5}
Diamagnetic	Silver	-2.4×10^{-5}
Diamagnetic	Gold	-3.5×10^{-5}

We have not given a table for the susceptibilities of ferromagnetic substances as χ_m, depends not only on H but also on the previous mangetic history of the material. Using Eq. (12.11) in the form

$$\mathrm{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

we have

$$\mathbf{B} = \mu_0 (1 + \chi_m) \mathbf{H} \qquad (12.16)$$
$$= \mu_0 K_m \mathbf{H}$$

$$\therefore \boxed{B = \mu H} \tag{12.17}$$

where $\mu = \mu_0 K_m = \mu_0 (1 + \chi_m)$

$$\therefore \boxed{K_m = \frac{\mu}{\mu_0}} \tag{12.18}$$

μ is called the **permeability** of the medium and K_m is called the 'relative' **permeability**. We see that μ has the same dimensions as μ_0 and K_m is dimensionless. *In* vaccum $\chi_m = 0$ and $\mu = \mu_0$. Relative permeability K_m differs from unity by a very small amount as $K_m = (1 + \chi_m)$. K_m for para- and ferromagnetic materials are greater than unity and for diamagnetic material it is less than unity.

The magnetic properties of a material are completely specified if any one of the three quantities, magnetic susceptibility, χ_m, relative permeability K_m or permeability μ is known.

Example 1

A toroid of aluminium of, length 1m, is closely wound by 100 turns of wire carrying a steady current of 1 A. The magnetic field B in the toroid is found to be 1.2567×10^{-4} wbm^{-2}. Find (i) H, (ii) χ_m, and K_m (iii) M in the toroid and (iv) equivalent surface magnetization current I_s.

Solution

i) According to Eq. (12.13)

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_f$$

To evaluate H produced by the current, we consider a circular integration path along the toroid. H is constant everywhere along this path of length 1m. The number of current turns threading this integration path is 100 × 1A. Since H is everywhere parallel to the circular integration path, we get

$$H \times 1\,m = 100 \times 1A$$

$$\text{or} \quad H = \frac{100 \times 1A}{1m} = 100\ A\ m^{-1}$$

(ii) From Eq. (12.16)

$$B = \mu_0 K_m H$$

$$\text{or} \quad K_m = \frac{B}{\mu_0 H} = \frac{1256.7 \times 10^{7}}{4\pi \times 10^{-7}} \times \frac{1}{100} = 1.00005$$

And $(1 + \chi_m) = K_m$

$$\therefore \chi_m = K_m - 1 = 1.00005 - 1 = 5 \times 10^{-5}$$

(iii) From Eq. (12.15)

$$M = \chi_m H$$

$$= 5 \times 10^{-5} \times 100\ A\ m^{-1} = 5 \times 10^{-3}\ Am^{-1}$$

(iv) $I_m = ML$

$$= 5 \times 10^{-3}\ Am^{-1} \times 1m = 5mA.$$

In this solution, we have assumed B, H and M to be **uniform** over the cross-section of the toroid and along the **axis** of the toroid.

Try to do the following SAQ.

SAQ 2

An air-core solenoid wound with 20 turns per centimetre carries a current of 0.18 A. Find H and B at the center of the solenoid. If an iron core of absolute permeability 6×10^{-3} H m^{-1} is inserted in the solenoid, find the value of H and B?
($\mu_0 = 4\pi \times 10^{-7}$ H m^{-1})

12.5 RELATIONSHIP BETWEEN B AND H FOR MAGNETIC MATERIAL

The specific dependence of M on B will be taken up in this section. The relationship between M and B or equivalently a relationship between B and H depend on the nature of the magnetic material, and are usually obtained from experiment.

A convenient experimental arrangement is a toroid with any magnetic material in its interior. Around the toroid, two coils (primary and secondary) are wound, as shown in Fig.12.8.

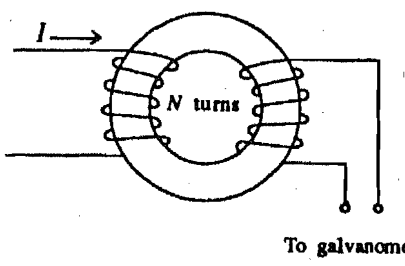

Fig.12.8 : Arrangement for investigating the relation between B and M, or B and H, in a magnetic material.

If we consider the radius of the cross-section of the toroidal windings to be small in comparison with the radius of the toroid itself, the magnetic field within the toroid can be considered to be approximately uniform. A current passing through the primary coil establishes H. The establishment of the current in the primary coil induces an electromotive force (emf). By measuring the induced voltage, we can determine changes in flux Φ and hence, in B inside the magnetic material. If we take H as the independent variable, and if we keep the track of the changes in B starting from B = 0, we can always know what B is for a particular value of H. In this way, we can obtain a B-H curve for different types of magnetic material.

The experiment described above can be carried out for diamagnetic and paramagnetic materials by commencing with I = 0 and slowly increasing the value of I to obtain a series of values of B and H . A plot of B against H for these substances is shown in the Fig. 12.9(a). We see that the graph is a straight line as expected from the relation

$$B = \mu_0 (1 + \chi_m) H \qquad (12.16)$$

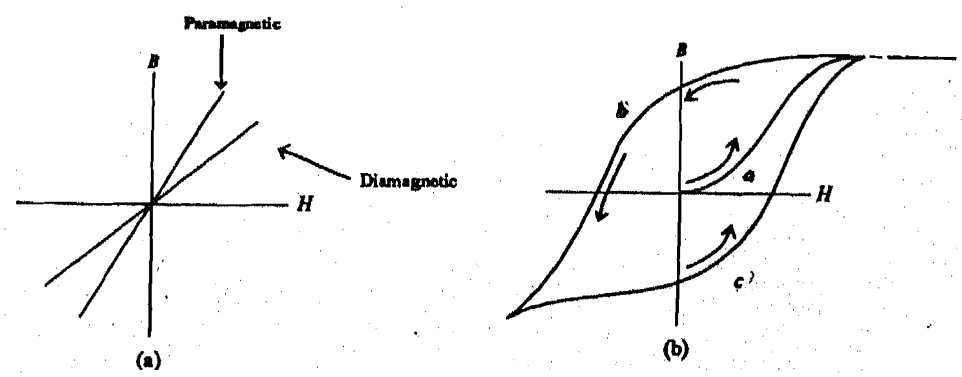

Fig.12.9 : Internal magnetic field (B) versus applied magnetic field (H) for different types of magnetic materials. (a) In diamagnetic and paramagnetic materials, the relationship is linear. (b) In ferromagnetic materials, the relationship depends on the strength of the applied field and on the past history of the material. In (b), the field strengths along the vertical axis are much greater than along the horizontal axis. Arrows indicate the direction in which the fields are changed.

where μ_0 and χ_m are constants. The slope of the graph is given by $(1+\chi_m)$ from which χ_m can be determined using the following relation:

$$\chi_m = \frac{\text{slope}}{\mu_0} - 1.$$

For diamagnetic substances, slope $< \mu_0$ making $\chi_m < 0$. For paramagnetic materials slope $> \mu_0$ so that $\chi_m > 0$.

If in the experiment givenabove we use ferromagnetic materials like iron, we obtain a typical B – Hcurve as shown in Fig. 12.9(b).

i) At $I = 0$, i.e., when $H = 0$, B is zero. When I is increased, B and H are detennined for increasing values of I. At first, B increases with H along the curve 'a'. At some high value of H, the curve (shown by the dashed line in the figure) becomes linear, indicating that M ceases to increase, as the material has reached saturation with all the domain dipole moments in the same direction.

ii) If, afterreaching saturation, we decrease the current in the coil to bring H back to zero, the $B-H$ curve falls along the curve '*b*'. When H reaches zero, there is still some B left implying that even when $I = 0$, there is still some magnetization or M left in the specimen. The material is permanently magnetized. The value of B for $H = 0$ is called **remanence**.

iii) If the current is reversed in the primary coil and made to increase its value, the B–H curve runs along the curve '*b*' until B becomes zero at a certain value of H. This value of H is called the coercive force. If we continue to increase the value of the current in the negative direction, the curve continues along '6' until the saturation is reached again.

iv) The current is now decreased until it becomes zero once again. This corresponds to $H = 0$, but B is not zero and has magnetization in the opposite direction. Here we reverse the current again, so that the current in the coil is once more along the positive direction. With the increasing current in this direction, the curve continues along the curve '*c*' to meet the curve '*b*' at saturation.

If we alternate the current between large positive and negative values, the B–H curve goes back and forth along '*b*' and '*c*' in a cycle. This cycle curve is called hysteresis curve. It shows that B is not a single valued function of H, but depends on the previous treatment of the material.

The shape of the hysteresis loop varies very widely from one substance to another. Those substances, like steel, alnico, etc., from which permanent magnets are made, have a very wide hysteresis loop with a large value of the coercive force (see Fig. 12.10). However, those substances, like soft iron, permalloy, etc., from which electromagnets (temporary magnet) are made, should have large remanance but very small coercive force. Those ferromagnetic materials, which are used in the cores of transformers, like iron-silicon (0.8-4.8%) alloys, have very narrow hysteresis loop.

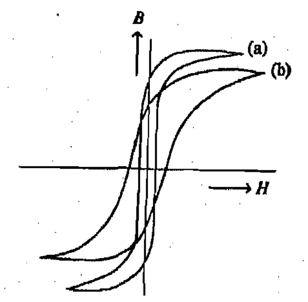

Fig. 12.10: The hysteresis curves for a few materials. Curves (a) aod (b) are respectively for specimen of soft iron and steel materials.

12.6 MAGNETIC CIRCUITS

A magnetic circuit is the closed path taken by the magnetic flux set up in an electric machine or apparatus by a magnetising force. (The magnetising force may be due to a current coil or a permanent magnet.)

In order to study the resemblance between a magnetic circuit and an electric circuit, we will develop a relation corresponding to Ohm's law for a magnetic circuit. Let us consider the case of an iron ring (Fig. 12.11) magnetised by a current flowing through a coil wound closely over it. Suppose,

I = current flowing in the coil

N = number of turns in the coil

l = length of the magnetic circuit (mean circumference of the ring)

A = area of cross-section of the ring

μ = permeability of iron.

In this case, all the magnetic flux produced is confined to the iron ring with very little leakage (we shall see the reason for this later). We have seen earlier that H inside the ring is given by

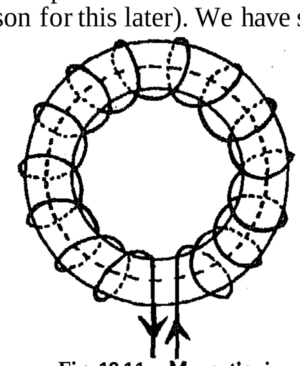

Fig. 12.11: Magnetic circuit

$$\oint \mathbf{H} \cdot d\mathbf{l} = NI \qquad \text{(from Ampere's law)}$$

where the path of integration is along the axis of the ring. As the line integral of electric field **E** over a circuital path is the electromotive force (e.m.f), by analogy, the line integral of H is termed as magnetomotive force (M.M.F.)

$$\therefore \text{M.M.F.} = \oint \mathbf{H} \cdot d\mathbf{l} = NI.$$

At every point along this path in the ring, we write

$$H = \frac{B}{\mu}$$

Further if Φ is the magnetic flux given by Φ = BA, then H = $\Phi/\mu A$, hence

$$\text{M.M.F.} = \oint \mathbf{H} \cdot d\mathbf{l} = \Phi \int \frac{dl}{\mu A} = NI \tag{12.19}$$

where we have taken Φ outside the integral as it is constant at all cross-sections of the ring. Eq. (12.19) reminds us of a similar equation for an electric circuit containing a source of E.M.F., namely,

$$\text{e.m.f.} = \text{current} \times \text{resistance} = I \int \frac{\rho dl}{A} \tag{12.20}$$

The Eqs. (12.19) and (12.20) suggest that:

i) The magnetomotive force ($\oint \mathbf{H} \cdot d\mathbf{l}$) is analogous with e.m.f. ($\oint \mathbf{E} \cdot d\mathbf{l}$).

ii) The magnetic flux Φ is analogous with current I in Ohm's law.

iii) The magnetic resistance known as reluctance $\left(\int \frac{dl}{\mu A}\right)$ is analogous with electric resistance $\left(\int \frac{\rho dl}{A}\right)$

$$\therefore \text{M.M.F} = \text{flux} \times \text{reluctance}$$

or

$$\text{Total flux } \Phi = \frac{M.M.F.}{\text{reluctance}} = \frac{NI}{\int \frac{dl}{\mu A}} \tag{12.21}$$

If we take μ to be constant throughout the ring then

$$\text{reluctance} \quad \Re = \int \frac{dl}{\mu A} = \frac{L}{\mu A} \tag{12.22}$$

where L is the length of the ring. However, we must recognise the significant difference between an electric circuit and a magnetic circuit:

i) Energy is continuously being dissipated in the resistance of the electric circuit, whereas no energy is lost in the reluctance of the magnetic circuit.

ii) The electric current is a true flow of the electrons but there is no flow of such particle in a magnetic flux.

iii) At a given temperature, the resistivity ρ is independent of current, while the corresponding quantity $\frac{1}{\mu}$ in reluctance varies with magnetic flux Φ.

Reluctances in Series : Let us assume that the toroid is made of more than one ferromagnetic material, each of which is of the same cross-sectional area A, but with different permeabilities $\mu_1, \mu_2, \ldots$

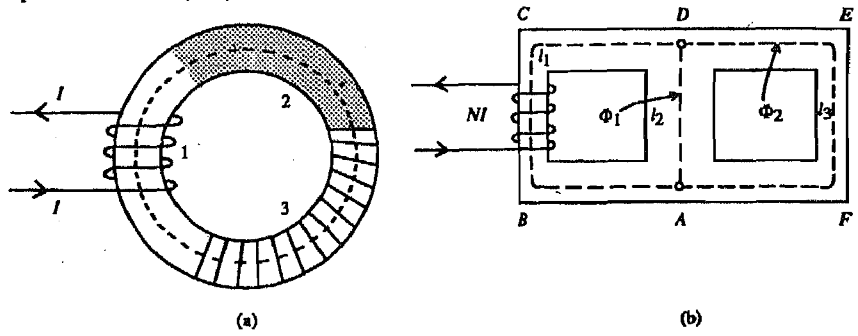

Fig.12.12 : (a) A magnetic circuit composed of several materials: Reluctances in series. (b) Magnetic circuit consisting of two loops: Reluctances in parallel.

Then, (see Fig. 12.12a) as before, we have

$$NI = \int \mathbf{H} \cdot d\mathbf{l}$$

$$= \int_1 \mathbf{H} \cdot d\mathbf{l} + \int_2 \mathbf{H} \cdot d\mathbf{l} + \ldots,$$

where the integrals on the right are taken over axial paths in the materials 1,2, ...
Therefore,

$$\text{M.M.F} = \int_1 \frac{\Phi}{\mu A} dl + \int_2 \frac{\Phi}{\mu A} dl + \ldots$$

$$= \Phi \int_1 \frac{dl}{\mu_1 A} + \Phi \int_2 \frac{dl}{\mu_2 A} + \ldots$$

$$= \Phi \left[\frac{L_1}{\mu_1 A} + \frac{L_2}{\mu_2 A} + \ldots \right]$$

$$= \Phi (\Re_1 + \Re_2 + \ldots) = \Phi \Re$$

so that the total reluctance of the given magnetic circuit is given by

$$\Re = \Re_1 + \Re_2 + \ldots \tag{11.23}$$

Reluctances in **Parallel** : We shall next illustrate the case of a magnetic circuit in which the reluctances are in parallel. Fig. 12.12b shows such a magnetic circuit. The current carrying coils have N turns each, carrying a current I amperes. The magnetic flux Φ threading the coil splits into two paths with fluxes Φ_1 and Φ_2 as shown in the figure. Obviously, $\Phi = \Phi_1 + \Phi_2$. We assume that the area of cross-section A is constant everywhere in the circuit.

Let the lengths of the paths ABCD, DA and DEFA shown in the figure be L, L_1, L_2 respectively. For the path $ABCDA$, we have

$$NI = \int_{ABCD} \frac{\Phi}{\mu A} dl + \int_{DA} \frac{\Phi}{\mu A} dl$$

$$= \frac{\Phi}{\mu A} L + \frac{\Phi_1}{\mu_1 A} L_1 \tag{12.24}$$

Similarly for the closed path $ADEFA$, we have

$$0 = -\int_{AD} \frac{\Phi}{\mu A}\, dl + \int_{DEFA} \frac{\Phi}{\mu A}\, dl$$

$$= -\frac{\Phi_1}{\mu_1 A} L_1 + \frac{\Phi_2}{\mu_2 A} L_2 \qquad (12.25)$$

Notice that we have used μ_1 and μ_2 for the paths ***AD*** and ***DEFA***, As Φ's being different for these paths, Hs would be different. This inakes μs different in these paths. Using $\Phi = \Phi_1 + \Phi_2$ and Eq. (12.25), we write

$$\Phi = \Phi_1 + \Phi_1 \frac{\mu_2}{\mu_1} \cdot \frac{L_1}{L_2}$$

$$= \Phi_1 \left(1 + \frac{\mu_2}{\mu_1} \cdot \frac{L_1}{L_2} \right)$$

Substituting the value of Φ_1 from the above equation in the Eq. (12.24), we have

$$NI = \frac{\Phi}{\mu A} L + \Phi \left(\frac{\frac{L_1}{\mu_1 A} \cdot \frac{L_2}{\mu_2 A}}{\frac{L_1}{\mu_1 A} \cdot \frac{L_2}{\mu_3 A}} \right)$$

or
$$NI = \Phi \left(\Re + \frac{\Re_1 \Re_2}{\Re_1 \Re_2} \right) \qquad (12.26)$$

This shows that the reluctances of the paths *DA* and *DEFA* are in parallel as the magnetic flux Φ splits into Φ_1 and Φ_2 along these paths respectively. The combined reluctance $\Re$ of these paths is given, in terms of the reluctances $\Re_1$ and $\Re_2$ of these paths, as follows

$$\frac{1}{\Re} = \frac{1}{\Re_1} + \frac{1}{\Re_2} \qquad (12.27)$$

Notice that the Eq. (12.24), (12.25) and $\Phi = \Phi_1 + \Phi_2$ are the statements of Kirchoff's laws for the inagnetic circuits.

Now we see why the magnetic flux does not leak through the air. Air forms a parallel path for the flux, for air, $\mu = \mu_0$ and for a ferromagnetic material $\mu \approx 10^4\, \mu_0$, hence the air path is a very high reluctance path compared to that through the ferromagnetic material. The magnetic flux will follow the path of least reluctance, a situation similar to that in the electric circuit.

The magnetic circuit fonnulae are used by the electrical engineers in calculations relating electromagnets, motors and dynamos. The problem is usually to find the number of turns and the current in the winding of a coil, which is required to produce a certain flux density in the air gap of all electromagnet. Knowing the reluctance of the circuit, M.M.F. is calculated from the relation:

M.M.F. = flux × reluctance

Since M.M.F. is also NI (see Eq. (12.19)), the magnitude of ampere turns can be calculated. Let us illustrate it by studying the magnetic circuit of an electromagnet.

Magnetic Circuit of an Electromagnet

The magnetic circuit of an electromagnet consists of the yoke which forms the base of the magnet, the limbs on which the coil is wound, the pole pieces and the air gap. See **Fig. 12.13.** Let l_1 be the effective length and a the area of cross-section of **the** yoke. If μ_1 is the **permeability** of its iron, then $\frac{l_1}{\mu_1 a_1}$ is the **reluctance** of the yoke. Similarly the reluctance of each limb is $\frac{l_2}{\mu_2 a_2}$ and the reluctance of each pole piece is $\frac{l_3}{\mu_3 a_3}$, while the reluctance of the **air gap** is $\frac{l_4}{\mu_0 a_4}$ (because $\mu_{air} \approx \mu_0$). Hence the total reluctance of

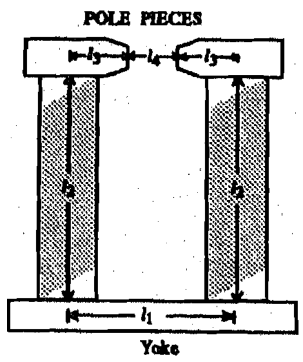

Fig. 12.13: Magnetic circuit of an electromagnet.

the magnetic circuit is

$$\frac{l_1}{\mu_1 a_1}+\frac{2l_2}{\mu_2 a_2}+\frac{2l_3}{\mu_3 a_3}+\frac{l_4}{\mu_0 a_4}$$

If the magnetic circuit carries one and the same flux Φ across all its parts, then according to **Eq. (12.19),** the number of ampere turns is:

$$\Phi\left(\frac{l_1}{\mu_1 a_1}+\frac{2l_2}{\mu_2 a_2}+\frac{2l_3}{\mu_3 a_3}+\frac{l_4}{\mu_0 a_4}\right) \qquad (12.28)$$

Let us take another example of calculating the magnetic field B in the air gap of a toroid **of** Fig. *12.14.* Here the toroid is of a **ferromagnetic material** (soft iron) with a small air gap **of** width '*d*' which is **small** compared to the length L of the toroid. For this case, **we** have

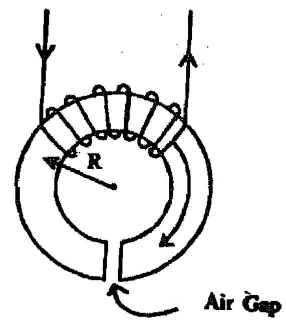

Fig. 12.14 : Magnetic field in the air gap.

$$NI = \Phi\left[\frac{(L-d)}{\mu A}+\frac{d}{\mu_0 A}\right]$$ Φ being the flux through **this** magnetic **circuit.**

$$= \frac{B}{\mu\mu_0}\left[\mu_0(L-d)+\mu d\right]$$

or

$$B = \frac{NI\,\mu\mu_0}{\mu_0 L+(\mu-\mu_0)\,d} \qquad (12.29)$$

This is the value of the magnetic field in the air gap. Read the following example which shows bow the air gap effectively increases the length of the toroid.

Example 2

Compare the examples of a complete toroid of length L wound with a coil of N **turns** each carrying a current I amperes and of a toroid of **lenth** (L − d) with an air gap of length d (d << L) **.** Show **that** the air gap effectively increases the length of the toroid **by** $(K_m - 1)\,d$, where K_m is relative permeability.

Solution

In the case of a complete toroid without the air gap, we have $B = NI/\left(\frac{L}{\mu}\right)$. In the event of an air gap of length d, we have fmm **Eq.** *(12.29)*:

$$B = \frac{NI\,\mu\mu_0}{\mu_0 L+(\mu-\mu_0)\,d}$$

Dividing both the numerator and the denominator by $\mu\mu_0$, we get

$$B = \frac{NI}{\frac{L}{\mu}+\left(\frac{1}{\mu_0}-\frac{1}{\mu}\right)d} = \frac{NI}{\frac{1}{\mu}\left[(L-d)+\frac{d}{\mu_0}\right]}$$

$$= \frac{NI}{\frac{1}{\mu}\left[(L-d)+\frac{\mu}{\mu_0}d\right]} = \frac{NI}{\frac{1}{\mu}\left[(L-d)+K_m d\right]}$$ so that

$$B = \frac{NI}{\frac{1}{\mu}\left[L+(K_m-1)\,d\right]}$$

If we **compare** this formula with that for the complete **toroid,** we see that L is effectively increased by $(K_m - 1)\,d$.

Before ending this unit solve the following SAQ.

SAQ 3

A soft iron ring with a *1.0* cm air gap is wound with a coil of 500 turns and carrying a **current** of *2* A. The mean length of iron ring is 50 cm, its cross-section is 6 cm^2, its permeability is 2500 μ_0. Calculate the magnetic induction in the air gap. **Find** also B and H in the iron ring.

Let us now sum up what we have learnt in this unit.

12.7 SUMMARY

- The behaviour of the ferromagnetic materials is complicated on account of the permanent magnetization and the phenomenon of hysteresis. This behaviour is explained by the presence of the domains in these materials. In each domain the dipole moments are locked to remain parallel due to 'exchange' force. However, in the unmagnetised state, the magnetisation directions of different domains are random, resulting in a zero net magnetisation. There also exist two other kinds of magnetic materials : antiferromagnetic and femmagnetic.
- For non-uniform magnetisation, magnetised matter is equivalent to a current distribution $\mathbf{J}$ = curl $\mathbf{M}$, where $\mathbf{M}$ is magnetisation or magnetic moment per unit volume.
- The magnetic field produced by the magnetised material is obtained by Ampere's law as follows:

  $$\nabla \times \mathbf{B} = \mathbf{J}_f + \mathbf{J}_m$$

  where $\mathbf{J}_f$ is the free current density which flows through the material and $\mathbf{J}_m$ is the bound current density which is associated with magnetisation. This gives

  $$\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{J}_f$$

  where $\mathbf{H} \left(= \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right)$ is a new field vector.
- For paramagnetisin and diamagnetism B, M and H are linearly related to each other except for ferromagnetic materials which exhibit hysteresis, a non-linear behaviour.
- The study of the electromagnets, motors and dynamos involves the problem of current carrying coils containing ferromagnetic materials, i.e., it involves the study of magnetic circuits. We speak of the magnetic circuits when all the magnetic flux present is confined to a rather well-defined path or paths.
- Magnetic circuit formula is:

  magnetomotive force (M.M.F.) = flux x reluctance
- M.M.F. is also equal to NI where N is the number of turns of the coil wound over the magnetic material and I the current flowing through each coil.
- Reluctance $\mathfrak{R} = \frac{l}{\mu a}$

  where l, a and μ are the length, area of cross-section and permeability of the material. Additions of reluctances obey the same rules as additions of resistances.

12.8 TERMINAL QUESTIONS

1) Find the magnetizing field H and the magnetic flux density B at (a) a point of 105 mm from a long straight wire carrying a current of 15 A and (b) the center of a 2000-turn solenoid which is 0.24m long and bears a current of 1.6A. ($\mu_0 = 4\pi \times 10^{-7}$ H m^{-1})

2) A toroid of mean circumference 0.5 m has 500 turns: each carrying a current of 0.15 A. (a) Find H and B if the toroid has an air core. (b) Find B and the magnetization M if the core is filled with iron of relative permeability 5000.

3) A toroid with 1500 turns is wound on an iron ring 360 mm^2 in cross-sectional area, of 0.75-m mean circumference and of 1500 relative permeability. If the windings carry 0.24A, find (a) the magnetizing field H (b) the m.m.f., (c) the magnetic induction B, (d) the magnetic flux, and (e) the reluctance of the circuit.

12.9 SOLUTIONS AND ANSWERS

SAQs

1) Path (1) encloses $I = 5A$

$$\therefore \oint \mathbf{H} \cdot d\mathbf{l} = I = 5A$$

For Path (2), $\oint \mathbf{H} \cdot d\mathbf{l} = 7I = 35A$

For Path (3), $\oint \mathbf{H} \cdot d\mathbf{l} = 2I = 10A$

$B = H$ for Path (1)

$B \neq H$ for Paths (2) and (3) because these paths pass through iron.

2) $H = nI = (2000\ m^{-1})(0.18A) = 360\ Am^{-1}$

$B = \mu_0 H = (4\pi \times 10^{-7}\ Hm^{-1})(360A\ m^{-1}) = 0.45\ mT$

If an iron core of absolute permeability $6 \times 10^{-3}\ H\ m^{-1}$ is inserted in the solenoid then H remains unchanged **i.e.,**

$H = 360\ A\ m^{-1}$ (unchanged)

and $B = \mu H = (6 \times 10^{-3}\ H\ m^{-1})(360\ A\ m^{-1}) = 2.16\ T$

3) The expression for the magnetic induction in the air gap is.

$$B = \frac{NI\mu}{[L + (K_m - 1)d]}$$

Substituting the values, given in question, we get

$$B = \frac{500 \times 2 \times 2500 \times 4\pi \times 10^{-7}}{0.50 + (2500 - 1)\, 0.01}$$

$$= \frac{10^3 \times 10^4 \times \pi \times 10^{-7}}{0.50 + 25} = \frac{\pi}{25.5} = 0.123\ Wb\ m^{-2}$$

B in the iron ring has the same value as in air, but H in iron is given by

$$H = \frac{B}{K_m \mu_0}$$

or $H = 0.123/2500 \times 4\pi \times 10^{-7}$

$= 39.1\ A\ m^{-1}$

Terminal Questions

1) (a) $H = \frac{B}{\mu_0} = \frac{\mu_0 I}{2\pi r} \times \frac{1}{\mu_0} = \frac{I}{2\pi r} = \frac{15A}{(2\pi)(0.105m)} = 22.7\ A\ m^{-1}$

$\left(\because B = \frac{\mu_0 I}{2\pi r}\right.$, see Unit 9)

$$B = \frac{(2)(15)}{10^7 \times 0.105} = 28.57\ \mu T$$

(b) $H = nI = \frac{2000}{0.24m} \times 1.6A = 13.33\ A\ m^{-1}$

$$B = \frac{4\pi}{10^7} \times \left(\frac{2000}{0.24m}\right) 1.6A = 0.0168T$$

2) For a toroid $H = nI$, and we use $B = (4\pi/10^7)(K_m H) = \mu H$. Thus

a) $H = \frac{500\ \text{turns}}{0.5m}\, 0.15\ A = 150\ A\ m^{-1}$ and

$B = (4\pi \times 10^{-7}\ H\ m^{-1})(150A\ m^{-1}) = 0.188\ mT$

(b) $B = 5000\,(0.188\,\text{mT}) = 0.94\text{T}$

Using $B/\mu_0 = H + M$

$$\frac{0.94}{4\pi \times 10^{-7}} = 150 + M \qquad M = 7.5 \times 10^{5}\ \text{A m}^{-1}$$

3) a) $H = nI = \dfrac{1500}{0.75\ \text{m}}(0.24\text{A}) = 480\ \text{Am}^{-1}$

b) m.m.f. $= Hl = (480\ \text{A m}^{-1})(0.75\ \text{m}) = 360\ \text{A}$

c) $B = \dfrac{4\pi K_m H}{10^7} = \dfrac{4\pi}{10^7}(1500)(480) = 0.905\text{T}$

d) $\Phi = BA = (0.905\ \text{Wb m}^{-2})\dfrac{360\ \text{m}^2}{10^6} = 3.26 \times 10^{-4}$ Wb.

(e) Reluctance $= \dfrac{\text{m.m.f.}}{\Phi} = \dfrac{360}{3.26 \times 10^{-4}} = 1.1 \times 10^{6}\ \text{H}^{-1}$

$= 1.1\ \mu\text{H}^{-1}$