so the proton kinetic energy is

$$KE = \frac{1}{2}mv^2 = 8.0 \times 10^{-13}\ \text{J}.$$

Solving for the speed v, gives

$$v = \sqrt{\frac{2K}{m}} = \frac{(2)(8.0 \times 10^{-13}\ \text{J})}{1.7 \times 10^{-27}\ \text{kg}} = 3.1 \times 10^{7}\ \text{ms}^{-1}$$

The radius needed to accommodate 5-MeV protons is given by

$$r = \frac{mv}{qB} = \frac{(1.7 \times 10^{-27}\ \text{kg})(3.1 \times 10^{7}\ \text{ms}^{-1})}{(1.6 \times 10^{-19}\ \text{C})(2.0\ \text{T})} = 0.16\ \text{m}.$$

UNIT 11 MAGNETISM OF MATERIALS-I

Structure

11.1 INTRODUCTION

In the last two Units, we have discussed the magnetic fields produced by moving charges or currents in conductors. There, the moving charges and conductors were considered to be placed in vacuum (i.e., in air). In Units 11 and 12, we learn how the magnetic field affects materials and how some materials produce magnetic field. You must have learnt in your school Physics Course that in equipment such as generator and motor, iron or iron alloy is used in their structure for the purpose of enhancing the magnetic flux and for confining it to a desired region. Therefore, we will study the magnetic properties of iron and a few other materials called ferromagnets, which have similar properties as iron. We shall also learn that all the materials are affected by the magnetic field to some extent, though the effect in some cases is weak.

When we speak of magnetism in everyday conversation, we almost certainly have in mind an image of a bar magnet. You may have observed that a magnet can be used to lift nails, tacks, safety pins, and needles (Fig. 11.1a) while, on the other hand, you cannot use a magnet to pick up a piece of wood or paper (Fig. 11.1b).

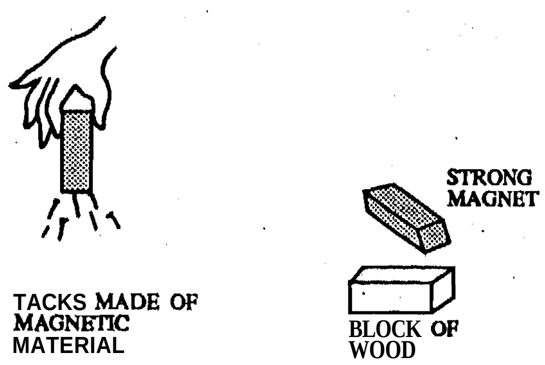

Fig. 11.1: a) Materials that are attracted to a magnet are called magnetic materials. b) Materials that do not react to a magnet are called nonmagnetic materials.

Materials such as mils, needles etc., which are influenced by a magnet are called **magnetic materials** whereas other materials, like wood or paper, are called **non-magnetic** materials, However, this does not mean that there is no effect of magnetic field on non-magnetic materials. The difference between the behaviour of such materials and iron like magnetic materials is that the effect of magnetic field on non-magnetic material is very weak.

There are two types of non-magnetic materials: diamagnetic and paramagnetic. Unit 11 deals with diamagnetic and paramagnetic effects. The ideas, concepts and various terms that you become familiar with in this Unit would help you in the study of ferromagnetism in the next Unit. In this unit, we present a simple classical account of the magnetism, based on notion of classical physics. But you must keep in mind that it is not possible to understand the magnetic effects of materials from the point of view of classical physics. The magnetic effects are a completely quantum mechanical phenomena. Only modem quantum physics is capable of giving a detailed explanation of the magnetic properties of matter because the study requires the introduction and utilization of quantum mechanical properties of atom. For a complete explanation, one must take recourse to quantum mechanics; however, a lot, though incomplete, of information about matter can be extracted by combining classical and quantum concepts.

Basically, in this unit, we will try to understand, in a general way, the atomic origin of the various magnetic effects. The next unit is an extension of this unit. There, we will try to develop a treatment of magnetised matter based on some observed relations between the magnetic field and the parameters which characterise the material. Finally, we consider the analysis of the magnetic circuit, which is of particular importance in the design of the electromagnets.

Objectives

After studying this unit you should be able to:

- understand and explain: gyromagnetic ratio, paramagnetism, diamagnetism, Larmor frequency,
- relate the magnetic dipole moment of an atomic magnet with its angular momentum,
- explain the phenomena of diamagnetism in terms of Faraday induction and Lenz's principle,
- explain paramagnetism in terms of the torque on magnetic dipoles,
- find the precessional frequency of an atomic dipole in a magnetic field,
- appreciate that a lot of information about magnetism of matter can be obtained from the classical ideas of atomic magnetism.

11.2 RESPONSE OF VARIOUS SUBSTANCE TO A MAGNETIC FIELD

To show how the magnetic materials respond to a magnetic field, consider a strong electromagnet, which has one sharply pointed pole piece and one flat pole piece as shown in Fig. 11.2.

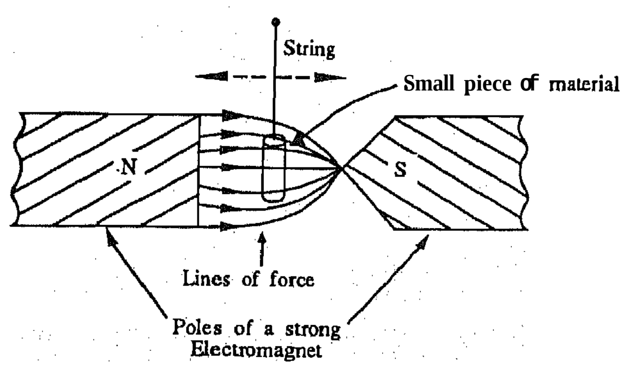

Fig.11.2: A small cylinder of bismuth is weakly repelled by the sharp pde; a piece of aluminium is attracted.

The magnetic field is much stronger in the region near the pointed pole whereas near the flat pole the field is weaker. This is because the lines must concentrate on the pointed pole. When the current is passed through the electromagnet (i.e., when the magnet is turned on), the hanging material is slightly displaced due to the small force acting on it. Some materials get displaced in the direction of increasing field, i.e., towards the pointed pole. Such materials are paramagnetic materials. Examples of such material are aluminium and liquid oxygen. On the other hand, there are materials like bismuth, which are attracted in the direction of the decreasing field, i.e., it gets repelled from the pointed pole. Such materials are called diamagnetic. Finally, there is a small class of materials which feel a considerable stronger force ($10^3 - 10^5$ times) towards the pointed pole. Such substances are called ferromagnetic materials. Examples are iron and magnetite.

How does a substance experience a force in a magnetic field? And why does the force act in a particular direction for some substance while in opposite direction for other substance? If we can answer these questions, we will understand the mechanisms of paramagnetism, diamagnetism and ferromagnetism. In Unit 9, you have already learnt that the magnetic fields are due to electric charges in motion. In fact, if you could examine a piece of material on an atomic scale, you would visualize tiny current loops due to (i) electrons orbiting around nuclei and (ii) electrons spinning on their axes. For, macroscopic purposes, these current loops are so small that they are regarded as the magnetic dipoles (see Section 9.2 of Unit 9) having magnetic moment. It is this magnetic moment, via which the atoms at a substance interact with the external field, and give rise to diamagnetic and paramagnetic effects. In this unit, you will understand the origin of paramagnetism and diamagnetism. Ferromagnetism has been left to be explained in the next unit. Let us first find out the value of the magnetic moment and see how it is related to the angular momentum of the atom.

11.3 MAGNETIC MOMENT AND ANGULAR MOMENTUM OF AN ATOM

Study comment: You may find it useful to look back at Unit 9, Section 9.2.2, in which the idea of magnetic dipoles has been introduced.

Electrons in an atom are in constant motion around the nucleus. To describe their motion, one needs quantum mechanics, however, in this unit we shall use only classical arguments to obtain our results, though we repeat here that our description of the physical world is incomplete as we shall be leaving out quantum mechanics.

We consider an electron in the atom to be moving, for simplicity, in a circular orbit around the nucleus under the influence of a central force, known as the electrostatic force, as shown in Fig. 11.3(a). As a result of this motion, the electron will have an angular momentum L about the nucleus.

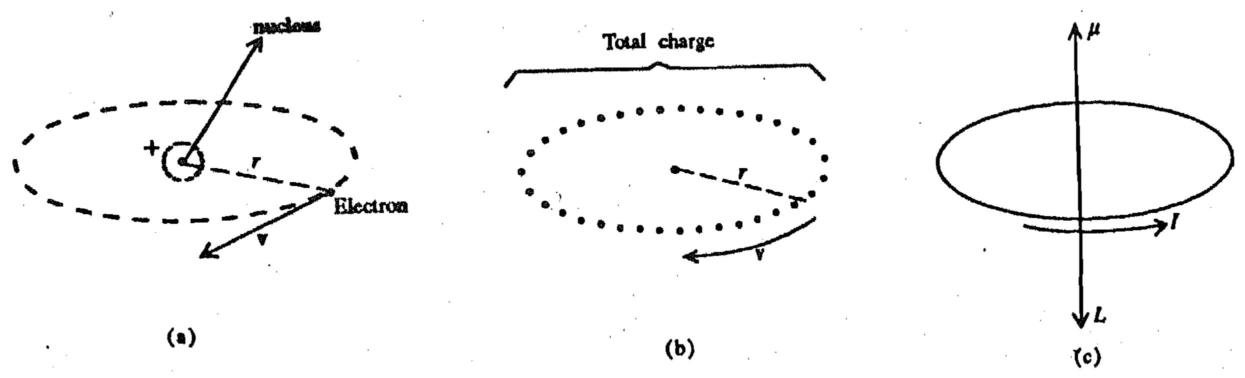

Fig.11.3: a) Classical model of an atom in which an electron moves at speed v in a circular orbit. b) The average electric current is the same as if the charge - e were divided into small bits, forming a rotating ring of charge. c) The orbital angular momentum vector and the magnetic moment vector both point in opposite directions.

The magnitude of this angular momentum is given by the product of the mass m of the electron, its speed v and the radius r of the circular path (see Fig. 11.3), i.e.,

$$L = mvr \qquad (11.1)$$

Its direction is perpendicular to the plane of the orbit. As you have already read Unit 9, and worked out the terminal questions given at the end of that unit, the fact that orbital motion of the electron constitutes an electric current will immediately strike your mind. The average electric current is the same as if charge on electron were distributed in small bits, forming a rotating ring of charge, as shown in Fig. 11.3(b). The magnitude of this current is the charge times the frequency as this would equal to the charge per unit time passing through any point on its orbit. The frequency of rotation is the reciprocal of the period of rotation $2\pi r/v$, hence the frequency of rotation has the value $v/2\pi r$. The current is then

$$I = -\frac{ev}{2\pi r} \tag{11.2}$$

The magnetic moment due to this current is the product of the current and the area of which the electron path is the boundary, that is, $\mu = I\pi r^2$. Hence we have

$$\mu = -\frac{evr}{2} \tag{11.3}$$

It is also directed perpendicular to the plane of the orbit. Using Eq. (11.1) in Eq. (11.3) we get as follows:

$$\boxed{\mu = -\frac{e}{2m}\mathbf{L}} \tag{11.4}$$

The negative sign above indicates that μ and L are in opposite directions, as shown in Fig. 11.3(c). Note that L is the **orbital** angular momentum of the electron. The ratio of the magnetic moment and the angular momentum is called the **gyro-magnetic ratio.** It is independent of the velocity and the radius of the orbit.

According to quantum mechanics, $L = \hbar\sqrt{l(l+1)}$, where l is a positive integer and $\hbar = \frac{h}{2\pi}$, h being Planck's constant. However, in some physical cases the applicability of classical models is close to reality, therefore, we will go ahead with the classical ideas. Further, the early work on the nature of magnetic materials was based on classical ideas which gave intelligent guesses at the behaviour of these materials.

SAQ 1

a) Show that the magnetic dipole moment can be expressed in units of JT^{-1} (Joule per Tesla).

b) In the Bohr hydrogen atom, the orbital angular momentum of the electron is quantized in units of $\hbar$, where $h = 6.626 \times 10^{-34}$ Js is Planck's constant. Calculate the smallest allowed magnitude of the atomic dipole moment in JT^{-1}. (This quantity is known as Bohr magneton.) Mass of the electron is 9.109×10^{-31}kg.

In addition to its orbital motion, you know that, the electron in an atom behaves as if it were rotating around an axis of its own as shown in Fig. 11.4.

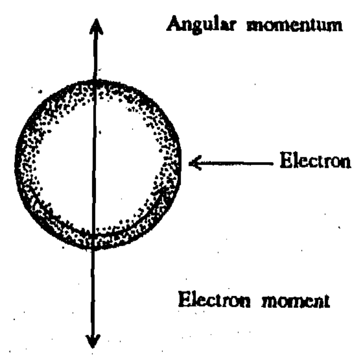

Fig.11.4: The spin and the associated magnetic moment of the electron.

This property is called spin. Though strictly it is not possible to visualise the spin of a point particle like electron, for many purposes it helps to regard the electron as a ball of negative charge spinning around its axis. Then you can say that it is a current loop. Spin is entirely a quantum mechanical idea. Nevertheless, the spin of the electron has associated with it an angular momentum and a magnetic moment. For purely quantum mechanical reasons with no classical explanation, we have

$$\mu = -\frac{e}{m}\mathbf{S} \qquad (11.5)$$

where S is the spin angular momentum and μ is the spin magnetic moment. The gyromagnetic ratio in this case is twice that in the orbital case.

In general, an atom has several electrons. The orbital and spin angular momenta of these electrons can be combined in a certain way, the rules of which are given by quantum mechanics, to give the total angular momentum J and a resulting total magnetic moment. It so happens that the direction of the magnetic moment is opposite to that of the angular momentum in this case as well, so that we have

$$\boxed{\mu = -g\frac{e}{2m}\mathbf{J}} \qquad (11.6)$$

where g is a numerical factor known as Lande g-factor which is a characteristic of the state of the atom. The rules of quantum mechanics enable us to calculate the g-factor for any particular atomic state. $g = 1$ for the pure orbital case and $g = 2$ for the pure spin case.

The atoms and molecules interact with the external magnetic field due to its magnetic moment. But there is another way in which atomic currents and hence moments are affected by the field. In this case the magnetic moment is induced by the field. This effect leads to diamagnetism which we study in the next section. But before moving to the next section, try the following SAQ.

SAQ 2

a) Compare Eq. (11.6) with (11.4) and (11.5), to find the value of g for (i) pure orbital case and for (ii) pure spin case.

b) The experimentally measured electron spin magnetic moment is 9.27×10^{-24} $\mathrm{Am^2}$. Show that this value is consistent with the formula given by the Eq. (11.5).

(Hint : According to Bohr's theory $S = \frac{\hbar}{2}$, Here $\hbar = \frac{h}{2\pi}$, h being Planck's constant.)

11.4 DIAMAGNETISM AND PARAMAGNETISM

In many substances, atoms have no permanent magnetic dipole moments because the magnetic moments of various electrons in the atoms of these substances tend to cancel out, leaving no net magnetic moment in the atom. The orbital and spin magnetic moments exactly balance out. These materials exhibit diamagnetism. If a material of this type is placed in a magnetic field, little extra currents are induced in their atoms, according to the laws of electromagnetic induction (to be discussed in detail in Unit 13), in such a direction as to oppose the magnetic-field already present, Hence, in such a substance, the magnetic moments (on account of induced currents) are induced in a direction opposite to that of the external magnetic field. This effect is diamagnetism. It is a weaker effect. However, this effect is universal.

There are other substances of which the atoms have permanent magnetic dipole moments. This is due to the fact that the magnetic moments due to orbital motion and spins of their electrons do not cancel out, but have a net value. When such a substance is placed in a magnetic field, besides possessing diamagnetism, which is always present, the dipoles of such a material tend to line up along the direction of the magnetic field. This is paramagnetism and the material is called paramagnetic In a paramagnetic substance, the paramagnetism usually masks the ever present property of diamagnetism in every substance.

Diamagnetism involves a change in the magnitude of the magnetic moment of an atom whereas paramagnetism involves change in the orientation of the magnetic moment of an atom. Let us see how.

11.4.1 Diamagnetism — Effect of Magnetic Field on Atomic Orbits

We consider an atom, which has no intrinsic magnetic dipole moment, and imagine that a magnetic field is slowly turned on in the space occupied by the atom. The act of switching the magnetic field introduces change in the magnetic field which, in turn, generates an electric field given by Faraday's law of induction (to be discussed in detail in Unit 13). It states that the line integral of E around any closed path equals the rate of change of the magnetic flux Φ through the surface enclosed by the path.

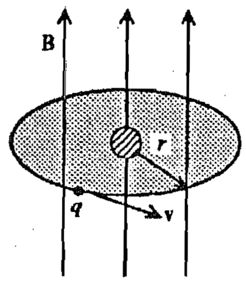

Fig.11.5: An electron moving in circular orbit in a uniform magnetic field that is normal to the orbit.

For simplicity, we choose a circular path along which the electron in the atom is moving (see Fig. 11.5). The electric field around this path is given by Faraday's law as

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

or $$E \cdot 2\pi r_{\perp} = -\frac{d}{dt}(B \cdot \pi r_{\perp}^2) \qquad (11.7)$$

where $r_{\perp}$ is the radius of the circular path perpendicular to B. The above equation gives the circulating electric field whose strength is

$$E = -\frac{r_{\perp}}{2}\frac{dB}{dt} \qquad (11.8)$$

This electric field exerts a torque $\tau = -e\,Er_{\perp}$ on the orbiting eleclron which must be equal to the rate of change of its angular momentum $\frac{dL}{dt}$, that is,

$$\frac{dL}{dt} = -e\,Er_{\perp}$$

or $$\frac{dL}{dt} = -e\left(-\frac{r_{\perp}}{2}\frac{dB}{dt}\right)r_{\perp}$$

or $$\frac{dL}{dt} = \frac{er_{\perp}^2}{2}\frac{dB}{dt} \qquad (11.9)$$

The change in angular momentum, ΔL due to turning on the field is obtained by integrating Eq. (11.9) with respect to time from zero field as follows:

$$\Delta L = \frac{er_{\perp}^2}{2}\Delta B \qquad (11.10)$$

Thus Eq. (11.10) shows that a build up of a magnetic field B causes a change in the angular momentum of the electron, ΔL and hence a change in the magnetic moment governed by Eq. (11.4) as follows:

$$\Delta \mu = -\frac{e}{2m}\Delta L$$

$$\Delta \mu = -\frac{er_{\perp}^2}{4m}\mathbf{B} \qquad (11.11)$$

The direction of the induced magnetic moment is opposite to that of B, which produces it as can be seen from the negative sign in the Eq. (11.11). In this equation, we have the term $r_{\perp}^2$ which is the square of the radius of the particular electron orbit whose axis is along **B**. If **B** is along the z-axis, we put $r_{\perp}^2 = x^2 + y^2$. Thus, the average $\langle r_{\perp}^2 \rangle$ would be $2\langle x^2 \rangle$, since $\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle$ due to spherical symmetry. Further $\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle = \frac{1}{3}\langle x^2 + y^2 + z^2 \rangle = \frac{1}{3}\langle r^2 \rangle$ gives $\langle r_{\perp}^2 \rangle = \frac{2}{3}\langle r^2 \rangle$. Hence the Eq. (11.11), which we shall write as

$$\Delta \mu = -\frac{e^2 \langle r_{\perp}^2 \rangle}{4m} \mathbf{B}$$

becomes

$$\Delta \mu = -\frac{e^2}{6m} \langle r^2 \rangle \mathbf{B}. \qquad (11.12)$$

We find that the induced magnetic moment in a diamagnetic atom is proportional to **B** and opposing it. This is diamagnetism of matter. If each molecule has n electrons each with an orbit of radius r, then the change in the magnetic moment of the atom is

$$\Delta \mu = -\frac{e^2}{6m} \sum_{\text{all electrons}} \langle r^2 \rangle \mathbf{B}.$$

There is an alternate way of understanding the origin of diamagnetism which is based on the fact that electron either speeds up or slows down depending on the orientation of the magnetic field. Let us see how. As shown in Fig. 11.6, in the absence of the magnetic field, centripetal force $\frac{mv^2}{r}$ is balanced by the electrical force as follows:

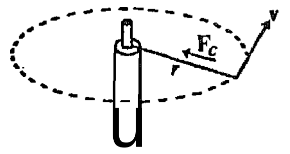

Fig. 11.6: There is no external magnetic Field. Centripetal force is balanced by the electrical force.

$$\frac{1}{4\pi \varepsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r} \qquad (11.13)$$

Let us find out what happens to one of the orbits when an external magnetic field is applied as shown in Fig. 11.7.

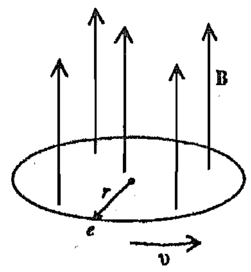

Fig. 11.7: Magnetic Field is perpendicular to the plane of the orbit.

In the presence of the magnetic field there is an additional term $e(\mathbf{v} \times \mathbf{B})$ and under these conditions speed of the electron changes. Suppose the new speed is v_1, then

$$e v_1 B + \frac{1}{4\pi \varepsilon_0} \frac{e^2}{r^2} = \frac{m v_1^2}{r}$$

or $$e v_1 B = \frac{m}{r}(v_1^2 - v^2) = \frac{m}{r}(v_1 + v)(v_1 - v)$$

If we assume that the change $\Delta v = v_1 - v$ is small, we get

$$e v_1 B = \frac{m}{r}(2 v_1)\Delta v$$

or $$\Delta v = \frac{erB}{2m} \qquad (11.14)$$

A change in orbital speed means a change in the dipole moment given by Eq. (11.3) as follows:

$$\Delta \mu = -\frac{1}{2} e (\Delta v) r = -\frac{e^2 r^2}{4m} \mathbf{B} \qquad (11.15)$$

This shows that change in μ is opposite to the direction of B. In the absence of an external magnetic field, the electron orbits are randomly oriented and the orbital dipole moments cancel out. But in the presence of a magnetic field, the dipole moment of each atom changes and all get aligned antiparallel to the external field. This is the mechanism responsible for diamagnetism. This property of magnetic material is observed in all atoms. But as it is much weaker than paramagnetism it is observed only in those material where paramagentism is absent.

11.4.2 Paramagnetism—Torque on Magnetic Dipoles

Paramagnetism is exhibited by those atoms which do not have the magnetic dipole moment. The magnetic moment of an atom is due to moment produced by the orbital currents of electrons and their "unpaired spins". In Unit 9 you have learnt that a current loop having μ as its magnetic dipole moment when placed in a uniform field experiences a torque τ which is given by Eq.(9.16), i.e.,

$$\tau = \mu \times B$$

The torque tends to align the dipoles so that the magnetic moment is lined up parallel to the field (in the way the permanent dipoles of dielectric are lined up with electric field). It is this torque which accounts for paramagnetism. You might expect every material to be paramagnetic since every spinning electron constitutes a magnetic dipole. But it is not so, as various electron of the atom are found in pairs with opposing spins. The magnetic moment of such a pair of electrons is cancelled out. Thus paramagnetism is exhibited by those atoms or molecules in which the spin magnetic moment is not cancelled. That is why the word "unpaired spins" is written above. Paramagnetism is generally weak because the lining up forces are relatively small compared with the forces from the thermal motion which try to destroy the order. At low temperatures, there is more lining up and hence stronger the effect of paramagnetism.

SAQ 3(a)

Of the following materials, which would you expect to be paramagnetic and which diamagnetic?

Copper, Bismuth, Aluminium, Sodium, Silver

SAQ 3(b)

Would it be possible to prepare an alloy of, say, a diamagnetic material like copper and a paramagnetic material like aluminium so that the alloy will neither be paramagnetic nor diamagnetic?

11.5 THE INTERACTION OF AN ATOM WITH MAGNETIC FIELD–LARMOR PRECESSION

In the last subsection, while explaining paramagnetism we, considered an atom as a magnet with the magnetic moment μ. When placed in a uniform magnetic field B, it is acted upon by a toque $\tau = \mu \times B$, which tends to line it up along the direction of the magnetic field. But it is not so for the atomic magnet, because it has an angular momentum J like a spinning top. We already know that a rapidly spinning top or a gyroscope in the gravitational field is acted upon by a torque, the result of which is that it precesses about the direction of the field. (To know more about precession you can read Unit 9 of the course 'Elementary Mechanics', PHE-01). Similarly, instead of lining up with the direction of the magnetic field, the atomic magnet precesses about the field direction. The angular momentum and with it the magnetic moment precess about the magnetic field, as shown in Fig. 11.8a.

Due to the presence of the magnetic field, the atom will feel a toque τ whose magnitude is given by

$$\tau = \mu B \sin\theta \qquad (11.16)$$

where θ is the angle which μ makes with B. The direction of the torque is perpendicular to the direction of magnetic field and also of μ as shown in Fig. 11.8b,

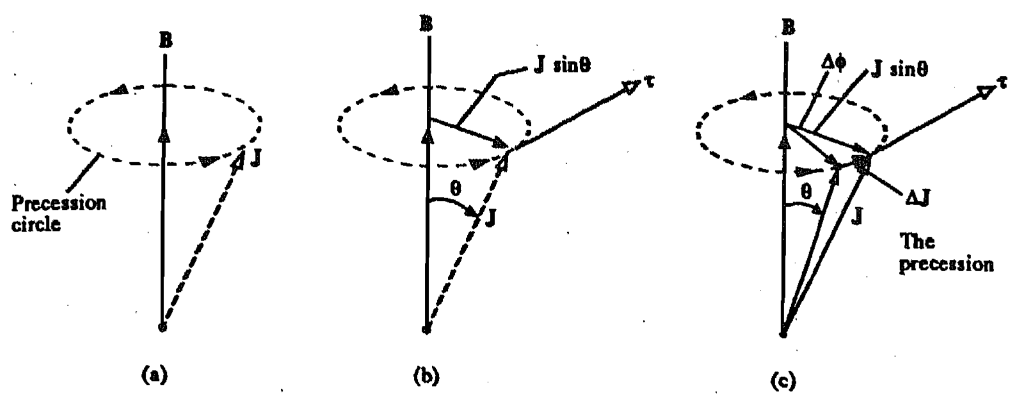

Fig.11.8: a) The angular momentum associated with atomic magnet precesses about magnetic field (b) The presence of magnetic field results in the torquer. It is at right angles to the angular momentum. c) The torque changes the direction of the angular momentum vector, causing precession.

Notice that the torque is perpendicular to the vector **J**. Now according to Newton's second law

$$\tau = \frac{d\mathbf{J}}{dt} \tag{11.17}$$

For small changes, we can write it as

$$\Delta \mathbf{J} = \tau \Delta t \tag{11.18}$$

In other words, the torque will produce a change in the angular momentum with time. Suppose that A **J** is the change in the angular momentum in an interval of time A t. This A J will be in the direction of τ. This will result in the tip of J moving in a circle about B as the axis. This is, in fact, a precession of **J** (so also of μ) about the direction of B. The magnitude of A **J** can be written by using Eq. (11.16) in Eq. **(11.18) as** follows:

$$\Delta J = \tau \Delta t = (\mu B \sin\theta)\Delta t \tag{11.19}$$

Although the toque τ, being at right **angles** to **J**, **cannot** change the magnitude of **J**, it can change its direction. Fig. **11.8c** shows how the vector AJ adds **vectorially** onto the vector J to bring this about. If ω_p is the angular velocity of the precession and A ϕ is angle of precession in time A t, then

$$\omega_p = \frac{\Delta\phi}{\Delta t} \tag{11.20}$$

From Fig. **11.8c** we see that

$$\Delta\phi = \frac{\Delta J}{J\sin\theta} = \frac{(\mu B\sin\theta)\Delta t}{J\sin\theta}$$

Dividing above by A t, approaching the differential limit and putting $\omega_p = \frac{d\phi}{dt}$, we get

$$\omega_p = \frac{\mu B}{J} \tag{11.21}$$

Substituting for μ/J from the **Eq. (11.6),** we get

$$\boxed{\omega_p = g\frac{e}{2m}B} \tag{11.22}$$

as the angular speed of precession of an atomic magnet about the direction of B. If in Eq. (11.22) $g = 1$, then ω_p **is** called the Larmor frequency, and is proportional to B. **It** should be borne in mind that this is the classical picture.

Now you may wonder if the atomic magnets (dipoles) precess about magnetic field, **how** many of these dipoles get aligned along the direction of magnetic field. **We** know **that** the potential energy of a dipole in the applied field is given by $-\mu . \mathbf{B} = -\mu B \cos\theta$. Therefore, an unaligned dipole **has** a greater potential energy than an aligned one. If **the** energy of the dipole is conserved then it cannot change its direction with respect to the field, i.e. the value of angle θ remains constant. So it keeps precessing about the field. However, by losing energy the atomic dipole gets aligned with the field. In a solid, the dipole can lose energy in various ways as its energy is transferred to other degrees of freedom and so it gets aligned with the field depending upon the temperature of the solid. To change the orientation of the dipole, the maximum energy required is $2\mu B$. If μ is about 10^{-23} Am^{-2} and a large field, say, 5T is applied then the potential energy will be of the order of 10^{-22} joules. This is comparable to the thermal energy kT at room temperature. Thus only a small fraction of the dipoles will be aligned parallel to B. In the next section it will be shown, using statistical mechanics, what fraction of dipoles is aligned along B.

In the presence of the magnetic field, when the tiny magnetic dipoles present in the material get aligned along a particular direction we say that material becomes magnetized or magnetically polarized. The state of magnetic polarization of a material is described by the vector quantity called magnetisation, denoted by **M**. It is defined as the magnetic dipole moment per unit volume. It plays a role analogous to the polarization **P** in electrostatics. In the next section we will also find the expression of magnetisation for paramagnets. But before proceeding do the following SAQ.

SAQ 4

Water has all the electron spins exactly balanced so that their net magnetic moment is zero, but the water molecules still have a tiny magnetic moment of the hydrogen nuclei. In the magnetic field of 1.0 wb m^{-2} protons (in the form of H- nuclei of water) have the precession frequency of 42 MHz. Calculate the g - factor of the proton.

11.6 MAGNETISATION OF PARAMAGNETS

In the presence of an external magnetic field, the magnetic moment tends to align along the direction of the magnetic field. But the thermal energy of the molecules in a macroscopic piece of magnetic material tends to randomize the direction of molecular dipole moments. Therefore, the degree of alignment depends both on the strength of the field and on the temperature. Let us derive the degree of alignment of the molecular dipoles, quantitatively, using statistical methods.

According to Boltzmann's law the probability of finding molecules in a given state varies exponentially with the negative of the potential energy of that state divided by KT. In this case the energy E depends upon the angle θ that the moment makes with the magnetic field. So probability is proportional to $\exp(-U(\theta)/KT)$.

Suppose there are N magnetic molecules per unit volume each of magnetic moment μ, at a temperature T. Classically, the magnetic dipole can make any arbitrary angle with the field direction (Fig. 11.9). In the absence of an external field, the probability that the dipoles will be between angles θ and θ + dθ is proportional to $2\pi \sin\theta \, d\theta$, which is the solid angle $d\Omega$ subtended by this range of angle. This probability leads to a zero average of the dipoles. When a magnetic field **B** is applied in the z - direction, the probability becomes also proportional to the Boltzmann distribution $e^{-U/kT}$. Here $U = -\mu . B = -\mu B \cos\theta$ is the magnetic energy of the dipole when it is making an angle θ with the magnetic field, k is the Boltzmann constant and T is the absolute temperature.

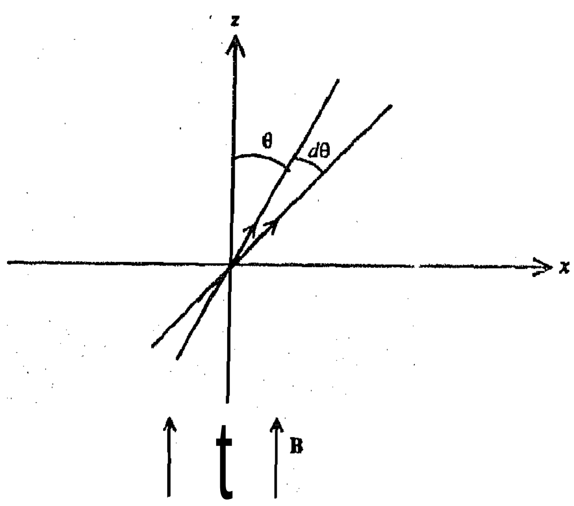

Fig. 11.9: Calculation of the paramagnetic properties of materials in external magnetic field.

Hence, the number of atoms (or molecules) dN per unit volume for which μ makes angles between θ and $\theta + d\theta$ with B, is given by

$$dN = 2\pi K e^{+\mu B \cos\theta / kT} \sin\theta \, d\theta \qquad (11.23)$$

where K is a constant.

Calling $\mu B/kT$ as a, the total number of dipoles per unit volume of the specimen is

$$N = \int dN = \int_0^{\pi} 2\pi K e^{+a\cos\theta} \sin\theta \, d\theta$$

Putting $\cos\theta = x$, we have

$$N = 2\pi K \int_{-1}^{+1} e^{+ax} dx$$

$$= \frac{2\pi K}{a}(e^{a} - e^{-a}) \qquad (11.24)$$

The magnetic dipole, making an angle θ with B, makes a contribution $\mu \cos\theta$ to the intensity of magnetization M of the specimen. Hence, the magnetization of the specimen obtained by summing the contributions of all the dipoles in the unit volume is given by

$$M = \int dN \mu \cos\theta$$

$$= 2\pi K e^{+\mu B \cos\theta / kT} \mu \cos\theta \sin\theta \, d\theta$$

$$= 2\pi K \int_{-1}^{+1} e^{+ax} x \, dx$$

where, again, we have put $\cos\theta = x$ and $\mu B/kT = a$. Evaluating the above integral, we obtain

$$M = 2\pi K \mu \left[\frac{1}{a}(e^{a} + e^{-a}) + \frac{1}{a^2}(e^{a} + e^{-a}) \right]$$

Substituting for $2\pi K$ from the Eq. (11.24), we get

$$M = \mu N \left(\frac{(e^{a} + e^{-a})}{(e^{a} - e^{-a})} - \frac{1}{a} \right)$$

$$\therefore M = M_s \left(\coth a - \frac{1}{a} \right) \qquad (11.25)$$

where $M_s = \mu N$ is the saturation magnetization of the specimen when all the dipoles align with the magnetic field. The expression $\coth a - \frac{1}{a}$ is called the Langevin function which is denoted by L (a).

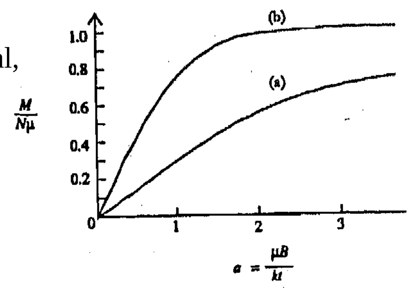

Fig.11.10 : The magnetisation of paramagnetic material placed in a magnetic field B as a function of $a = \frac{\mu B}{kT}$.

(i) is based on classical calculation with no restriction on the direction of dipole

(ii) is based on quantum mechanical calculation with restriction on (he direction of dipole.

We now consider two cases : (i) when $\frac{\mu B}{kT}$ is very large. This would happen if the temperature were very low and/or B very large. For this case,

$$L(a) = \coth a - \frac{1}{a} = \frac{e^{a} + e^{-a}}{e^{a} - e^{-a}} - \frac{1}{a} = \frac{1 + e^{-2a}}{1 - e^{-2a}} - \frac{1}{a} = 1.$$

Hence $M = M_s$. These would be saturation.

(ii) When $\frac{\mu B}{kT}$ is small which means that T is large and / or B is small. In this case

$\coth a - \frac{1}{a} = \frac{a}{3}$ and $M = M_s(\mu B/3kT) = \mu^2 NB/3kT.$

The complete dependence of M on B is shown in Fig. 11.10. For your comparison, the dependence of M on B based on quantum mechanical calculation is also shown.

SAQ 5

Evaluate the integral $\int_{-1}^{+1} e^{ax} x \, dx$.

SAQ 6

Show that when $a = \mu B/kT$ is small, $M = M_s\left(\coth a - \frac{1}{a}\right) = \frac{M_s a}{3}$.

Let us now sum up what we have learnt in this unit.

11.7 SUMMARY

- All materials are, in some sense, magnetic and respond to the **presence** of a magnetic field. Materials can be classified into mainly three groups: diamagnetic, paramagnetic and **ferromagnetic.** Diamagnetism is displayed by those materials in which the atoms have no permanent magnetic dipole moments. Paramagnetism and **ferromagnetism** occurs in those materials in which the atoms have permanent magnetic dipoles.
- The orbital motion of the **electron** is associated with a magnetic moment μ, which is proportional to its orbital angular momentum **J**. We write this as

  $$\mu = -g\left(\frac{e}{2m}\right)J$$

  where e is the **charge** on electron, m the mass of electron and g is **Lande** g - **factor** which has a value ≈ 1 for orbital case and 2 for spin case.
- The ratio of the magnetic dipole moment to the angular **momentum** is called the **gyromagnetic** ratio.
- The magnetic dipoles in the magnetic materials are due to atomic currents of electrons in their orbits and due to their intrinsic spins.
- Change in the magnitude of the magnetic moment of atoms is responsible for diamagnetism whereas change in the **orientation** of the magnetic moment accounts for paramagnetism.
- Because the magnetic moment is associated with angular momentum, in the presence of a magnetic field, the atom does not simply turn along the magnetic field but precesses around it with a frequency $\omega_p = g(e/2m)$ B. This is called **the Larmor** precession.
- **When** a **dimagnetic** atom is placed in an external magnetic field normal to its orbit, the field induces a magnetic moment opposing the field itself (Lenz's law) as

  $$\Delta\mu = \frac{er^2}{4m}B$$

  where r **and** m are tbe radius of tbe orbit and mass of the electron.
- When atoms of magnetic moment μ are **placed** in a magnetic field B, then the Magnetisation M is given by

  $$M = M_s(\coth a - 1/a)$$

  where $a = \frac{\mu B}{kT}$ **and** $M_s = \mu N$ **is** the saturation magnetisation **when all** the dipoles **are** aligned in the direction of field.

11.8 TERMINAL QUESTIONS

1. A uniformly charged **disc** having the charge q and radius r is rotating with constant angular velocity of magnitude **w.** Show that the magnetic dipole moment **has the magnitude** $\frac{1}{4}(q\omega r^2)$

(**Hint :** Divide the sphere into narrow rings of rotating charge; find the current to which each ring is equivalent, its dipole moment and then integrate over all rings.)

2. Compare the precession frequency and the cyclotron frequency of the proton for the same value of the magnetic field B.

11.9 SOLUTIONS AND ANSWERS

SAQs

1) a) Potential energy U of the magnetic dipole is given by the relation : $U = \mu . B$.
where. μ is the dipole moment and B is the magnetic field.

Since U is expressed in Joules and B in Tesla, the above relation gives the unit of magnetic dipole moment as JT^{-1}

b) From Eq. (11.4), $\mu = \frac{e}{2m} L$

$L = \frac{nh}{2\pi}$ (because angular momentum of electron is quantized)

where n is an integer.

Hence minimum allowed magnitude of dipole moment is given by putting $n = 1$, as follows:

$$\mu_{min} = \frac{e}{2m}\frac{h}{2\pi} = \frac{1.602 \times 10^{-19} C}{2 (9.109 \times 10^{-31} kg)} \times \frac{6.626 \times 10^{-34} Js}{2\pi}$$

$$\text{or } \mu_{min} = 9.27 \times 10^{-24} C Js kg^{1}$$

$$= 9.27 \times 10^{-24} J T^{-1}$$

$\therefore$ the Bohr magneton is given by $\frac{eh}{4\pi m} = 9.27 \times 10^{-24} J T^{-1}$

2) a) (i) $g = 1$ (ii) $g = 2$

b) Eq. (11.5) is $\mu = \frac{e}{m} S$

hence

$$9.27 \times 10^{-24} A m^2 = \frac{1.6 \times 10^{-19} C}{9.1 \times 10^{-31} kg} \times S$$

so that

$$S = \frac{9.1 \times 10^{-31} kg \times 9.27 \times 10^{-24} A m^2}{1.6 \times 10^{-19} C}$$

$$= \frac{9.1 \times 9.27}{1.6} \times 10^{-36} \frac{A kg m^2}{C}$$

$$= 52.72 \times 10^{-36} Js$$

$$= 0.5272 \times 10^{-34} Js$$

But the spin angular momentum S is $\frac{\hbar}{2}$, therefore

$$\frac{\hbar}{2} = \frac{h}{4\pi} = 0.5272 \times 10^{-34} Js$$

or $h = 4 \times 3.142 \times 0.5272 \times 10^{-34}$ Js

$= 6.626 \times 10^{-34}$ Js

which is indeed the value of Planck's constant.

3) a) Copper is slightly diamagnetic. Bismuth, Silver - diamagnetic, Aluminium & Sodium - paramagnetic

b) No. Since the diamagnetic material is characterised by the absence of intrinsic magnetic dipoles and paramagnetic substances have magnetic dipoles, the alloy of these materials will be the material with intrinsic magnetic dipoles. Such a material will exhibit the property of paramagnetism which masks the diamagnetism of both components of the alloy.

4) We have the formula

$$\omega_p = g\frac{e}{2m}B,$$

but $2\pi f_p = \omega_p$

hence

$$g = 2\pi f_p \times \frac{2m}{e} \times \frac{1}{B}$$

Now

$$2\pi f_p = 2 \times 3.14 \times 42 \times 10^6 \text{ s}^{-1} = 263.93 \times 10^6 \text{ s}^{-1}$$

For proton, $\frac{2m}{e} = \frac{2 \times 1860 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}$ kg C^{-1}

$= 21.15 \times 10^{-9}$ kg C^{-1}

Given, $\frac{1}{B} = 1 \text{ wb}^{-1} \text{ m}^2$

Using this above we obtain $g = 5.584$, which is the proton g - factor.

5) $\int x e^{ax} dx = \frac{x e^{ax}}{a} - \int \frac{e^{ax}}{a} dx$ integrated by parts

$$= \frac{x e^{ax}}{a} - \frac{1}{a}\frac{e^{ax}}{a} = \frac{x e^{ax}}{a} - \frac{e^{ax}}{a^2}$$

$$\text{Hence} \int_{-1}^{+1} x e^{ax} dx = \left[\frac{x e^{ax}}{a}\right]_{-1}^{+1} - \left[\frac{1}{a^2} e^{ax}\right]_{-1}^{+1} = \frac{1}{a^2}(e^{+a} + e^{-a}) - \frac{1}{a}(e^{a} - e^{-a})$$

6) We have $\coth a = \frac{e^a + e^{-a}}{e^a - e^{-a}}$ and also that

$$e^a = 1 + a + \frac{a^2}{2!} + \frac{a^3}{3!} + \ldots \text{ and } e^{-a} = 1 - a + \frac{a^2}{2!} - \frac{a^3}{3!} + \ldots$$

Hence

$$e^a + e^{-a} = 2\left(1 + \frac{a^2}{2!} + \frac{a^4}{4!} + \ldots\right) \approx 2\left(1 + \frac{a^2}{2!}\right)$$

$$e^a - e^{a} = 2\left(a + \frac{a^3}{3!} + \ldots\right) \approx 2a\left(1 + \frac{a^2}{3!}\right)$$

so that

$$\coth a = \frac{1 + \frac{a^2}{2}}{a\left(1 + \frac{a^2}{6}\right)} \approx \frac{1}{a}\left(1 + \frac{a^2}{2}\right)\left(1 - \frac{a^2}{6}\right)$$

$$= \frac{1}{a}\left(1 + \frac{a^2}{3} - \frac{a^4}{12}\right) \approx \frac{1}{a} + \frac{a}{3}$$

Therefore,

$$\coth a - \frac{1}{a} = \frac{a}{3} \text{ and } M = M_s a/3$$

Terminal Questions

1) The surface charge density is $\frac{q}{\pi r^2}$

The disc can be thought of as made up of number of rings. Let us consider a ring of radius R and width dR.

The charge within this ring is given by

$$dQ = \frac{q}{\pi r^2}(2\pi R dR) = \frac{2q}{r^2}(R dR)$$

The current carried by this ring is its charge divided by the rotation period:

$$dI = \frac{dQ}{(2\pi/\omega)} = \frac{q\omega}{\pi r^2}(R dR)$$

The magnetic moment contributed by this ring has magnitude

$$d\mu = a dI$$

where a is the area of the ring.

Therefore,

$$d\mu = \pi R^2 dI = \frac{q\omega}{r^2}(R^3 dR)$$

Taking into account all the rings (radius varying from 0 to r), we get the magnitude of the magnetic moment as follows:

$$\mu = \int d\mu = \int_{R=0}^{R=r} \frac{q\omega}{r^2}(R^3 dR)$$

$$= \frac{q\omega}{r^2}\left[\frac{R^4}{4}\right]_0^r$$

$$= \frac{q\omega}{r^2} \times \frac{1}{4} \times r^4 = \frac{1}{4} q\omega r^2$$

2) Precession frequency ω_p of a proton in a magnetic field is given by

$$\omega_p = \frac{\mu B}{J} = \frac{q}{2m_p} B$$

(because $\frac{\mu}{J} = \frac{q}{2m_p}$ where q is the charge and m_p is mass of the proton)

Cyclotron frequency ω_L is

$$\omega_L = \frac{qB}{2\pi m}$$

$$\therefore \frac{\omega_p}{\omega_L} = \frac{qB}{2m} \times \frac{2\pi m}{qB} = \pi .$$

UNIT 12 MAGNETISM OF MATERIALS-II

Structure

2.1 INTRODUCTION

In Block 2 of this course you have studied the behaviour of dielectric materials in response to the external electric fields. This was done by investigating their properties in terms of electric dipoles, both natural and induced, present in these materials and their lining up in the electric field. The inacroscopic properties of these materials were studied using the so-called polarization vector P, the electric dipole moment per unit volume.

The magnetic properties of materials lias a similar kind of explanation, albeit in a more complicated form, due to the absence of free magnetic monopoles. The magnetic dipoles in these materials are understood in terms of the so-called Amperian current loops, first introduced by Ampere.

All materials are, in some sense, magnetic and exhibit lnagnetic properties of different kinds and of varying intensities. As you know, all materials, can be divided into three main categories: (i) Diamagnetic; (ii) Paramagnetic and (3) Ferromagnetic materials. In this unit, we shall study the inacroscopic behaviour of these materials.

We understood the macroscopic properties of the dielectric materials using the fact that the atoms and molecules of these substances contain electrons, which are mobile and are responsible for the electric dipoles, natural and induced, in these substances. The polarisation of these substances is the gross effect of the alignment of these dipoles. Similarly we describe the magnetic properties of various materials in terms of the magnetic dipoles in these materials.

In Unit 11, we have already explained diamagnetism and paramagnetism in terms of magnetic dipoles. In this unit, first, we will mention the origin of ferromagnetism. Later, we will develop a description of the macroscopic properties of magnetic material.

With Unit 12, we end our study of magnetism. In the next Block we will deal with the situation where both electric and magnetic fields will vary with time. This will lead, ultimately, to the 'four differential equations known as Maxwell's equations.

Objectives

After studying this unit you should be able to :

- understand and explain the tenns: ferromagnetism, amperian current, magnetisation, magnetic intensity H, magnetic susceptibility, magnetic permeability, relative permeability,
- relate magnetisation M (which is experimentally measureable) and the atomic currents (which is not measureable) within the material,
- derive and understand the differential and integral equations for M and H and apply these to calculate fields for simple situations,