UNIT 10 MOTION OF CHARGES IN ELECTRIC AND MAGNETIC FIELD

Structure

10.1 INTRODUCTION

By now you are familiar with three kinds of force, namely, gravitational, electric and magnetic forces. These forces are best described in terms of fields, All forces have a property by virtue of which they act on a suitable kind of particle located in the region occupied by the field. If once you know exactly how the fields affect the particles on which they act, you are in a position to understand the nature of the field and, hence, the nature of the force.

You are all aware of the way in which objects move in a gravitational field. In such fields, when an object is thrown upward through the air, it follows a parabolic path. What kind of path is followed by a particle in an electric field or magnetic field? What is the behaviour of a particle in electric and magnetic fields? This Unit attempts to answer these questions. Later, in this Unit, you will find how the behaviour of a charged particle in electric and magnetic field is put to several applications.

With this Unit we complete our study of magnetostatics — the magnetic field associated with steady currents, and its effect on other currents and on isolated moving charges. So far, we considered the magnetic field in vacuum. In the next two Units, we turn our attention to the study of the magnetic fields when matter is present.

Objectives

After studying this unit you should be able to:

- carry out simple calculations involving the motion of charged particles in a uniform electric field,
- describe the main features of the motion of charge in a magnetic field, and define the term cyclotron frequency,
- explain the helical trajectory of a charged particle moving in a uniform magnetic field,
- explain the working principle of Cathode Ray Oscilloscope,
- appreciate the applications of the combined electric and magnetic fields acting perpendicular to each other.

10.2 MOTION IN AN ELECTRIC FIELD

In Block 1, we defined the electric field at a point as the force per unit charge at that point. This definition reminds us that electric fields are important because of the effect they have on charged particles. In this section, we consider the problem: how do charges respond, when placed in a known electric field? We will investigate the motion of a particle moving through a uniform electric field.

You are already acquainted with one important experiment that involved the motion of a charged particle in a uniform electric field. This experiment is Millikan's oil drop experiment, about which you have learnt in your school physics course, and which has been also mentioned in Unit 1. In that experiment, the electric force, due to the uniform field between two charged metal plates, is used to prevent a charged oil drop from falling under the influence of gravity. Let us now consider how such a drop or any other charged particle would behave if there were no gravitational force, and if initially the charged particle was moving in the direction of field.

10.2.1 Initial Velocity in the Direction of the Field

Suppose a uniform electric field E is set up between the two charged plates as shown in Fig. 10.1 Let us consider the particle with charge q moving in the direction of the field with velocity v. From Eq. (1.7) of Unit 1, the force acting on this particle will be given by

$$\mathbf{F} = q\mathbf{E} \qquad (10.1)$$

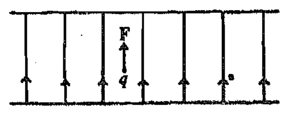

Fig. 10.1: The force on a (positively) charged particle in an electric field. The velocity of the particle has no influence on the force.

Eq. (10.1) shows that the force is independent of both the velocity and position of the particle. This constant force gives the particle a constant acceleration. From Newton's second law (F = ma), this constant acceleration is given by

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q\mathbf{E}}{m} \qquad (10.2)$$

where m is the mass of the particle.

Eq. (10.2) says that the acceleration is in the same direction as the electric field. This equation also shows that it is the ratio of charge to mass that determines a particle's acceleration in a given electric field. This explains why electrons much less massive (about 2000 times) than protons but carrying the same charge, are readily accelerated in electric fields. Many practical devices, like electron microscope and TV tubes make use of the high accelerations possible with electrons even in electric field of modest strength.

You know how to solve problems involving constant acceleration from your mechanics course studied in school days. Such problems arose while discussing the motion of an object in the uniform gravitational field near the earth's surface. In such problems the constant acceleration is the acceleration due to gravity (g); the velocity acquired and the distance travelled in a given time interval is calculated by using one or more of three simple equations known as constant (or uniform) acceleration equations. These equations are

$$v = u + at \qquad (10.3)$$

$$s = ut + \frac{1}{2}at^2 \qquad (10.4)$$

$$v^2 = u^2 + 2as \qquad (10.5)$$

In a given electric field, Eq. (10.2) shows that the particle undergoes constant acceleration. Hence Eqs. (10.3) to (10.5) can be used to study the motion of particles in the electric field. After solving the following SAQs you will realise that there is a subtle difference between the constant acceleration caused by gravity and that of a uniform electric field.

SAQ 1

Suppose the electric field shown in Fig. 10.1 is of the strength 2.0 NC^{-1}. An electron is released from rest in this field. How far and in what direction does it move in 1.0 μs?

SAQ 2

Tick (√) the correct answer and give reasons. In the problem given above, if a proton (instead of an electron) is released, the distance travelled by it would have been

a) more than distance travelled by an electron

b) less than distance travelled by an electron

c) equal to the distance travelled by an electron

d) proton would not be affected.

In what direction would the proton move?

In an electric field the acceleration varies from object to object depending on the ratio of charge to mass whereas for gravity the acceleration is the same for all objects, no matter what their mass, This is a useful phenomena, for it allows us to separate charged objects (ions) according to their charge- to-mass ratios.

10.2.2 Initial Velocity in any Direction

Till now, we have studied the effect of an electric field on those charged particles which are at rest or whose direction of motion is along the electric field. Let us now consider the case in which the charged particle is moving in the electric field with a velocity in the direction shown in Fig. 10.2.

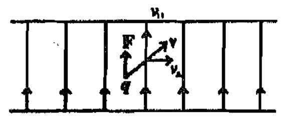

Fig. 10.2 :The direction of the initial velocity of the positively charged particle is *not* in *the* direction of the field.

You already know that when a particle (or a projectile) moves with constant acceleration under the earth's gravitational field, it follows a parabolic path. A similar thing happens here (Fig. 10.3). In such cases, the velocity v is regarded as a sum of two other velocities; one parallel to the field denoted by $\mathbf{v}_{\parallel}$, and the other perpendicular to it denoted by $\mathbf{v}_{\perp}$. So the total velocily v can be written as follows:

$$\mathbf{v} = \mathbf{v}_{\parallel} + \mathbf{v}_{\perp}$$

The horizontal position of the charged particle at any time t is given by

$$x = v_{\perp} t \qquad (10.6)$$

The vertical position of the charged particle is given by

$$y = v_{\parallel} t + \frac{1}{2}\frac{qE}{m}t^2 \qquad (10.7)$$

Substituting the valucof t from Eq. (10.6) into (10.7) we get

$$y = \frac{v_{\parallel}}{v_{\perp}}x + \frac{1}{2}\frac{qE}{m\,v_{\perp}^2}x^2 \qquad (10.8)$$

This is the equation of the particle in electric field. Because v, q, E and m are constants, Eq. (10.8) is of the form $y = ax + bx^2$, in which a and b are constants. This is the equation of a parabola. Suppose a particle that sets out from point A with velocity v_A describes a parabolic path $ABCD$ as shown in Fig. 10.3. To understand the motion of such particle we should be able to calculate the velocity of the particle at any point on the parabolic path. Let us find out the velocity v_B at the point B. Here again, v_B is the sum of the two other vectors: $v_{\perp B}$ and $v_{\parallel B}$. Since $v_{\perp}$ has been defined in such a way that it is perpendicular to the acceleration, it remains unaffected by the electric field. Therefore,

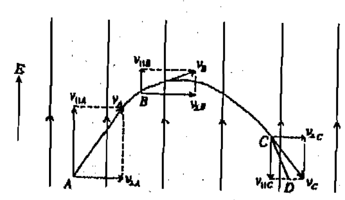

Fig. 10.3: Parabolic trajectory of a (negatively) charged particle in a uniform electric field.

$$v_{\perp B} = v_{\perp A} \qquad (10.9)$$

This is because, as the particle moves $v_{\perp}$-component does not change, as there is no acceleration in that direction. This is true for any other point on the path.

In other words, perpendicular component of the velocity remains constant in both magnitude and direction throughout the motion of the particle. As $v_{\parallel B}$ is along the direction of electric field i.e. along the constant acceleration, so it will be affected by the electric field. The magnitude of $v_{\parallel B}$ can be found out by applying any of the Eqs. (10.3) to (10.5). If the time taken to travel from A to B in Fig. 10.3 is t, and if $s_{\parallel}$ is the distance from A to B measured parallel lo E then

$$v_{\parallel B} = v_{\parallel A} + \left(\frac{qE}{m}\right)t \qquad (10.10)$$

$$v_{\parallel B}^{2} = v_{\parallel A}^{2} + 2\left(\frac{qE}{m}\right)s_{\parallel} \qquad (10.11)$$

$$s_{\parallel} = v_{\parallel A}\,t + \frac{1}{2}\left(\frac{qE}{m}\right)t^{2} \qquad (10.12)$$

Using Eqs. (20.9) to (10.12) you can solve wide range of problems concerning the motion of a charged particle in a uniform electric field. You will come across such problems while studying the Cathode ray oscilloscope in Sec. 10.4, where the electrons are allowed lo pass through the electric field region and then the electron beam strikes the fluorescent screen. However, to make certain that whatever said in this section is clear to you try all the parts of the following SAQ.

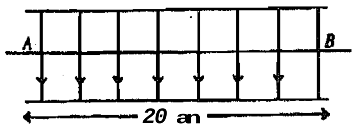

Fig. 10.4: The electron is deflected from its straight-line path by a uniform electric field.

SAQ 3

An electron is moving horizontally lo the right at a speed of 4.0 x 10^6 ms^{-1}. It enters a region of length 3.0 cm in which there is an electric field of 1.0 x 10" NC^{-1} pointing downward as shown in Fig. 10.4. Answer the following questions:

i) How long does it spend travelling from *A* to *B*?

ii) By how much and in what direction is the electron deflected, when it leaves the electric field'? Describe the motion of the particle in the clectric field and draw rough sketch of the path.

iii) What is the magnitude of the vertical component of the velocity of the electron, when it leaves the electric field ?

iv) What is the speed o l the electron when it leaves the electric field ?

v) Through what angle has the electron been deflected when it leaves the electric field?

vi) Describe and draw the rough sketch of its subsequent motion.

vii) In the similar situation, what will happen lo a positively charged particle of same charge and mass as that of an electron.

viii) For what purpose can such an arrangement be used?

10.3 MOTION IN A MAGNETIC FIELD

We have dealt with motion in a uniform electric field, let us now turn lo the problem of motion in a uniform magnetic field. If a charged particle is moving along the magnetic field, it will move as it is, because it will experience no force. Let us now consider the case in which the particle is initially moving in a plane normal to the magnetic field.

10.3.1 Initial Velocity Perpendicular to the Field

In the last unit, you have learnt that a particle having charge *q* and moving with a velocity v in a magnetic field **B** experiences a magnetic force given by

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B} \qquad (10.13)$$

From Eq. (10.13) it follows that magnetic force always acts perpendicular lo the direction of motion. This means that the magnetic field can do no work on a charged particle. Because no work is done, the kinetic energy of the particle cannot change – both the speed *v* and kinetic energy $\left(\frac{1}{2}mv^{2}\right)$ remain constant. Therefore, the magnetic force changes only the direction of particle's motion but not its speed.

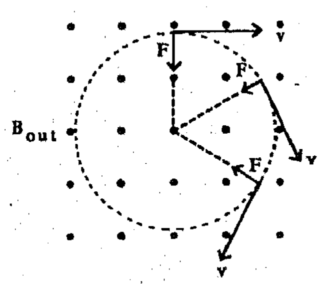

Fig. 10.5: A charged particle moving at right angles to a uniform magnetic field describes a circular path.

To understand how the direction of particle's motion is changed let us consider the case of a particle of charge *q* moving at right angles to a uniform magnetic field as shown in Fig. 10.5. Suppose at some instant at the point *A*, the velocity v points lo the right, so with the field being out of the page, the cross product v x B points downward according to right hand rule (see Unit 9). If the particle is positive it will experience a downward force. This force changes the direction of the particle's motion, but not its speed. A little while later, the particle is moving downward and to the right. Now the force points downwards and to the left. Since the speed of the particle is still v and the velocity is still at right angles to the field, so the magnitude of the force remains the same. Thus, the particle describes a path in which the force always has the same magnitude and is always at right angles to its motion. Each time, under the influence of the force the particle is deflected from the rectilinear path resulting in the simplest possible curved path — a circle. Now in any circular path, the particle experience a centripetal force F_c directed towards the centre of the circle. It is given by

$$F_c = \frac{mv^2}{r} \qquad (10.14)$$

where r is the radius of the circular orbit and v the tangential speed of the particle. Therefore, in the present case, the centripetal force being the magnetic force we can write

$$F_c = qvB = \frac{mv^2}{r}$$

so that

$$r = \frac{mv}{qB} \qquad (10.15)$$

The larger the particle's momentum mv, the larger the radius of the orbit. On the other hand, if the field or charge is made larger, the orbit becomes smaller. Therefore, observation of a charged particle's trajectory in a magnetic field is the standard technique for measuring momentum of the particle. A charged particle can traverse a circular path either in clockwise direction or anticlockwise direction. In Fig. 10.5 the particle is describing a circular path in clockwise direction. Solve the following SAQ. You will understand that a circular path traversed in the anticlockwise direction is also possible.

SAQ 4

In Fig. 10.5, if the particle is negatively charged, what will the circular orbit look like'? Draw with pencil, the orbit of the negatively charged particle on Fig. 10.5.

Thus, it is possible to identify the sign of the charge on the particle as well as its momentum by observing the particle's trajectory.

Since the circumference of the orbit is $2\pi r$, the time taken by the particlc'to complete one full orbit is

$$T = \frac{2\pi r}{v} \qquad (10.16)$$

Using Eq. (10.12) for the radius r gives

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{v}\frac{mv}{qB} = \frac{2\pi m}{qB} \qquad (10.17)$$

The frequency of rotation of a moving charge is given by

$$f = \frac{qB}{2\pi m} \qquad (10.18)$$

This quantity is called cyclotron frequency. It is so called because it is the frequency at which the charged particles circulate in a cyclotron particle accelerator. Using the known value of charge-to-mass ratio (e/m) of an electron, the magnetic field strength can be determined by measuring the cyclotron frequency of the electron. You will learn more about cyclotron later in this unit.

Now let us find out what path, a charged particle will have, if initially its velocity is neither perpendicular nor parallel to the field,

10.3.2 Initial Velocity in any Direction

In this case the velocity can be resolved into two vectors: $\mathbf{v}_\perp$ perpendicular to tlie field and $\mathbf{v}_\parallel$ along the field. Then Eq.(10.13) becomes :

$$\mathbf{F} = q(\mathbf{v}_\perp + \mathbf{v}_\parallel) \times \mathbf{B} = q\mathbf{v}_\perp \times \mathbf{B} + q\mathbf{v}_\parallel \times \mathbf{B}$$

Since the second tenn on the right hand side of this equation is the cross product of two parallel vectors, it is zero. Therefore,

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = q\mathbf{v}_\perp \times \mathbf{B}$$

or $$m\frac{d\mathbf{v}_\perp}{dt} + m\frac{d\mathbf{v}_\parallel}{dt} = q\mathbf{v}_\perp \times \mathbf{B}$$

The force **F** is clearly perpendicular to **B**, i.e., there is no acceleration in the direction parallel to B. This means

$$m\frac{d\mathbf{v}_\parallel}{dt} = 0$$

$$\therefore \quad \frac{d\mathbf{v}_\perp}{dt} = q\mathbf{v}_\perp \times \mathbf{B} \qquad (10.19)$$

Eq. (10.19) shows that the force is perpendicular to the field i.e., it influences the particle's motion in a plane perpendicular to the field.

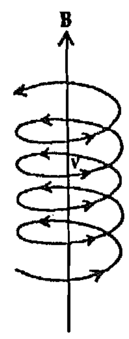

Fig. 10.6: Motion of a particle in a uniform magnetic field.

But we know that the particle's motion perpendicular to the magnetic field is circular. Eq. (10.19) further shows that no force acts along the magnetic field. Therefore, the component of the velocity which is along the field remains unaffected by the field. Thus the particle moves with a uniform velocity $\mathbf{v}_\parallel$ along the magnetic field, even as it executes a circular motion with velocity $\mathbf{v}_\perp$ perpendicular to the field. The resulting path is a helix shown in Fig. 10.6. The radius of the helix is given by Eq. (10.15) if we replace v by $\mathbf{v}_\perp$. The motion of the charged particle can be visualized like beads strung on a wire, with the wire being the magnetic field as shown in Fig. 10.7.

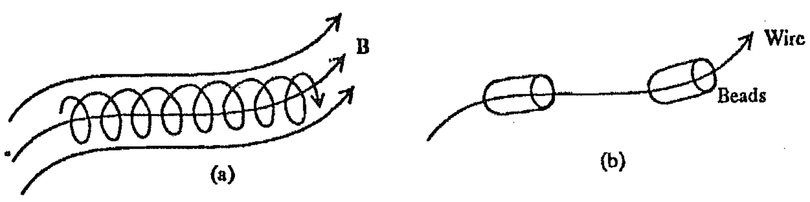

Fig. 10.7: (a) Charged particles undergoing helical motion about magnetic field lines are like (b) beads that are free to move along a wire but not at right angles to it.

SAQ **5**

An electron with a velocity of 10^7 ms^{-1} enters a magnetic field of strength 1.5×10^{-3} Wb m^{-2} at an angle of 30° with it. Calculate the radius of the helical path and the time taken by the electron for one revolution? Take $e/m = 17.6 \times 10^{11}$ C kg^{-1}.

In the next section we illustrate the working principle of Cathode Ray Oscilloscope (CRO) so that you understand the role played by the electric and magnetic fields.

10.4 CATHODE RAY OSCILLOSCOPE (CRO)

The Cathode Ray Oscilloscope (CRO) is a very useful and versatile laboratory instrument used for display, measurement and analysis of waveforms and other phenomena in electrical and electronic circuits. It is used for a number of purposes, such as, measurement of current, voltage, observation of waveforms of alternating voltages; recreation of television images; as indicator in radar for visual presentation of target data such as distance, height, etc. It is based on the following two principles:

1) When fast moving electrons strike the glass screen coated with zinc sulphide, they cause fluorescence.

2) Since the mass of electrons is very small, they are easily deflected by the electric and magnetic fields and follows their variation with practically no time lag.

The CROs are infact very fast X – Y plotters, displaying an input signal versus another signal (or versus time). The plotter is a luminous spot which moves over the display area in response to an input voltage. The luminous spot is produced by a beam of electrons striking a fluorescent screen. The extremely low mass of the electron enables the beam of electrons to follow the changes of the rapidly varying voltages.

Normally CRO uses a horizontal input voltage, which moves the luminous spot periodically in a horizontal direction left to right on the screen. The vertical input voltage moves the luminous spot up and down. The luminous spot thus traces the waveform. When the input voltage repeats itself at a fast rate, the trace or display on the screen appears stationary on the screen. The CRO, thus, provides a means of visualizing voltage waveforms.

A Cathode Ray Oscilloscope consists of a cathode ray tube, which is the heart of the device. In that tube, as shown in Fig. 10.8, the electrons produced by the cathode are accelerated towards the final anode of the electron gun under the influence of the anode potential V_A. By proper focussing, the electrons are compressed into narrow high velocity beam.

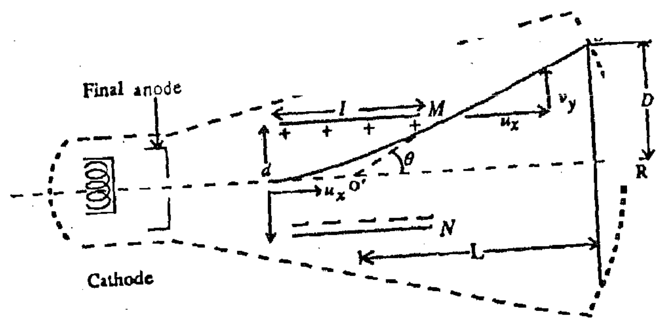

Fig. 10.8 : Electrostatic deflection.

The electrons leave the final anode with a velocity along the x - axis. The velocity is given by

$$u_x = \sqrt{\frac{2qV_A}{m}} \qquad (10.20)$$

If the beam next passes through a field free space, the velocity of the electrons remains constant. In that case, the electrons go straight and strike the screen at R as shown in Fig. 10.8. But if the beam of electrons is made to pass through an electric field or magnetic field then the electrons will get deflected' from their rectilinear path. The amount of deflection will depend upon the strength of the field. Let us see how the beam of electrons is deflected by an electric field.

10.4.1 Electrostatic Deflection

If the electron beam passes between two plane parallel charged plates M and N, which have a uniform electric field at right angles to the direction of motion of the electron beam, the electrons will experience an acceleration along the y-axis. The beam of electrons will get deflected vertically and, hence, strike the screen at point S. Since there is no force and, hence, no acceleration along the x-axis, the x-component of the velocity of electrons remains constant.

If V is the potential difference between two deflecting plates, d their separation, then the acceleration along the vertical direction in this region is given by

$$a_y = \frac{qE}{m} = \frac{q}{m}\frac{V}{d} \qquad (10.21)$$

If l is the length of the plates, then the time for which each electron remains in'the region between the two plates is

$$t = \frac{l}{u_x} \qquad (10.22)$$

while leaving the field, the final velocity along y-axis is given by

$$v_y = 0 + a_y t = \frac{q}{m}\frac{V}{d} t \qquad \text{Using Eq. (10.10)}$$

$$= \frac{q}{m}\frac{V}{d}\frac{l}{u_x} \qquad (10.23)$$

While leaving the field, the electrons get deflected by the distance given by

$$y = 0 + \frac{1}{2} a_y t^2 \qquad \text{Using Eq. (10.12)}$$

$$y = \frac{1}{2} a_y \left(\frac{l}{u_x}\right)^2$$

Substituting the value of a_y from Eq. (10.21), we get

$$y = \frac{1}{2}\frac{q}{m}\frac{V}{d}\frac{l^2}{u_x^2} \qquad (10.24)$$

It shows that the electron moves along a parabolic path in the region between the two plates. This vertical component of velocity remains constant after the electron move out of the electric field. From then onwards, the electron travel in a straight line, because the space is field free.

Outside the field, the electron velocity has two components, i.e., u_x and v_y. Hence, the resultant electron velocity is

$$v = \sqrt{u_x + v_y}.$$

When the electron leaves the electric field region it will travel in straight line towards the screen. This straight line when produced backwards meets the x-axis at point O'. If the line makes an angle of θ with the x-axis, then

$$\tan\theta = \frac{v_y}{u_x} = \frac{q}{m}\frac{V}{d}\frac{l}{u_x^2} \qquad \text{using Eq. (10.23)}$$

Since $\tan\theta = \frac{y}{OC'}$ or $OC' = \frac{y}{\tan\theta} = \dfrac{\frac{1}{2}\frac{q}{m}\frac{V}{d}\frac{l^2}{u_x^2}}{\frac{q}{m}\frac{V}{d}\frac{l}{u_x^2}} = \frac{l}{2}$

Point O' is at the centre of the deflecting plates. The vertical deflection D of the beam on a screen which is at a distance L from the point O' is given by

$$D = L\tan\theta$$

or

$$D = \frac{qVlL}{mdu_x^2}$$

Substituting the value of u_x from Eq. (10.20), we get

$$D = \frac{LlV}{2dV_a} \qquad (10.25)$$

From Eq. (10.25) we conclude that for a given accelerating voltage V_a and for particular dimensions of the cathode ray tube, the deflection of the electron beam is directly proportional to the deflecting voltage. The deflecting voltage may be a time varying quantity and thus the image on the screen follows the variations of the deflecting voltage in a linear manner,

10.4.2 Magnetic Deflection

Magnetic deflection is used where a wide angle of deflection is required, as in television tubes. The **magnetic** field is produced in such a way that it is perpendicular to the electron beam as shown in Fig. 10.9.

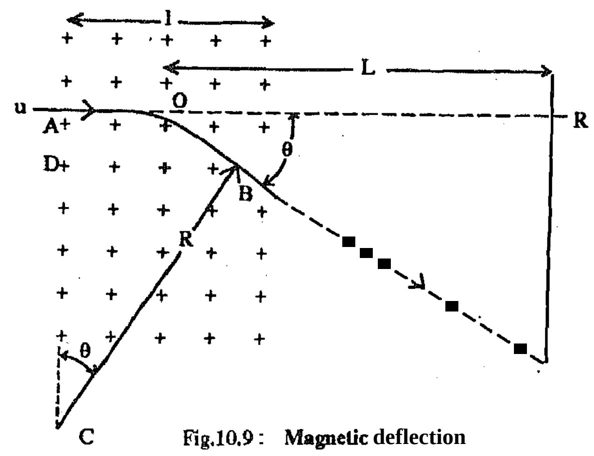

Fig.10.9 : **Magnetic deflection**

In the absence of the **magnetic** field, the electron **beam** goes straight and strikes the fluorescent screen at a **point** R. When the magnetic field is set up in the region of **length** l the beam in this region moves **along** a circular path of radius R. After leaving the region of the magnetic field at **the** point B it continues along a straight path and strikes the screen at point S. The vertical deflection traced out on the screen is D.

Let the circular **path** AB **subtends** an angle 8 at the **point** C. **When** SB is produced backwards it cuts thex-axis at point O which can be taken as **the** mid-point of the magnetic field for small deflection. Since triangles SOR and ACB are similar, we have

$$D = L \tan\theta = L\frac{l}{R}$$

Now

$$R = \frac{mv}{qB}$$

If the **electron has** been accelerated by the final anode to cathode **potential** V_a, then

$$v = \sqrt{\frac{2q\,V_a}{m}} \quad \therefore\ R = \frac{m}{q}\frac{1}{B}\sqrt{\frac{2q\,V_a}{m}} = \frac{1}{B}\sqrt{\frac{2m\,V_a}{q}}$$

$$\therefore\ D = LlB\left(\frac{q}{2m\,V_a}\right)^{\frac{1}{2}} \qquad (10.26)$$

Thus, the deflection on the screen is proportional to B and inversely proportional to square root of the accelerating potential V_a.

In the next section we will study the effect of **combined** electric and magnetic fields on the motion of charged particles. The general equation for force containing both **electric** and magnetic fields is called **Lorentz** equation, We will also discuss a few important applications of the **Lorentz** force law. Before moving to the **next** section, try the following SAQ.

SAQ 6

A beam of protons is deflected by an electric field and also by a magnetic field. If either could be responsible, how would you be able to **tell** which was present?

10.5 LORENTZ FORCE AND ITS APPLICATIONS

Suppose a particle having charge q is moving with velocity v through a space, in which both magnetic and electric fields exist simultaneously, then the force exerted on such a particle is given by

$$\mathbf{F} = q\,\mathbf{E} + q\,\mathbf{v} \times \mathbf{B} \qquad (10.27)$$

Eq. (10.27) brings together Eqs. (10.1) and (10.13). Eq. (10.27) is called the **Lorentz force** equation and F as **Lorentz** force. It is the vector sum of the electric force $q\mathbf{E}$ and the magnetic force $q\,\mathbf{v} \times \mathbf{B}$.

Let us now discuss important applications of the combined electric and magnetic fields, acting perpendicular to each other, in devices, such as, the velocity selector and cyclotron.

10.5.1 Velocity Selector

In a large class of experiments, in which the motion of charged particles or ions or electrons is to be studied, it is important to have a source of particles, all having the same velocity. Since most sources of electrons or ions emit particles with a wide range of velocities, a velocity selector is often essential. Let us understand the action of one such selector which uses both electric and magnetic force. Here, a capacitor like arrangement provides a uniform electric field downward in the plane of paper and magnetic field is provided perpendicular to the paper pointing into it, as shown in Fig. 10.10.

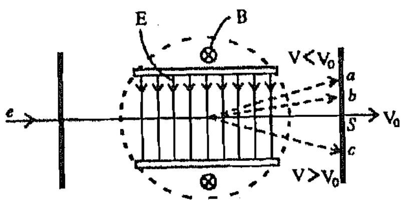

Fig. 10.10 : Crossed electric and magnetic fields act as a velocity selector. Only electrons with $v_o = E/B$ pass through the field region undeflected.

Suppose a narrow beam of identical charged particles (for example, a beam of electrons), travelling in a vacuum enters a region which contains a uniform electric field E and a uniform magnetic field **B**, with E and B perpendicular to each other. The particles in the beam have a spectrum of velocities, but they all enter the field region perpendicular to both Geld vectors, as indicated in Fig. 10.10. The electric field produces an upward force qE on the electrons in the beam, whereas the magnetic field produces a downward force qvB (check this by using the right hand method). There is a unique velocity v_o for which the electric and magnetic forces exactly cancel. The value of v_o is obtained by making these two forccs equal. We find

$$qE = qv_oB$$

or

$$v_o = E/B \qquad (10.28)$$

Electrons that have velocities less than v_o are deflected upward and strike the exit wall at points such as a and b. Electrons that have velocities greater than v_o are deflected downward and strike the wall at points such as c, Electrons that have the velocity v_o (and only these electrons) are undeviated and pass through the exit slit S. The arrangement of crossed electric and magnetic fields is called a velocity selector.

10.5.2 Cyclotron

Cyclotron is the most familiar of all the machines for accelerating charged particles and ions to a high velocity, It is based on the fact that electric and magnetic fields exert force on the ions. A sketch of cyclotron is shown in Fig. 10.11.

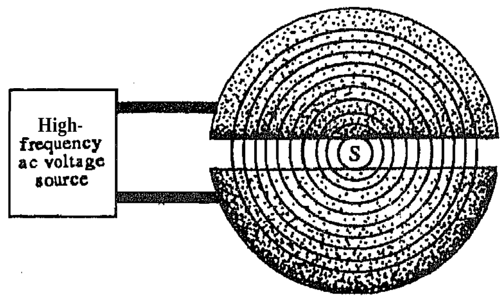

Fig. 10.11: Cyclotron. Top view of cyclotron electrodes placed in an evacuated chamber between the poles of an electromagnet Positive ions emitted by the source S travel in circular orbits inside the hollow electrodes perpendicular to the magnetic field. Each time the ions traverse the gap, between the electrodes, they are accelerated by a potential difference due to an applied alternating voltage synchronized with the ion motion. As the ions gain energy, the radius of their path increases. Finally, they are brought out of the magnetic field region by a negatively charged deflector plate.

The charged particles starting from a central source are caused to move in circular paths by a magnetic field perpendicular to their motion as shown in Fig. 10.11. They travel inside the two hollow electrodes, between which an alternating voltage is applied. This alternating voltage is synchronized with the ion motion so that the ions that start out at the right time feel an accelerating electric field each time they pass from one electrode to the other. Such ions make larger and larger orbits as they gain kinetic energy. However, they continue to stay in step with the alternating voltage, since their angular frequency is constant given by Eq. (10.18). Finally, they are brought out of the magnetic field region by a negatively charged deflector plate..

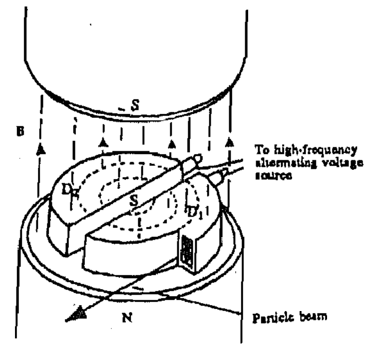

Fig.10.12 : Essential elements of cyclotron.

The essential elements of a cyclotron are shown in Fig. 10.12. It consists of two electrodes D_1 and D_2 which are hollow metallic semi cylindrical chambers shaped like a pill-box cut in half along a diameter. These are called the "Dees" because of their resemblence to the letter D in shape. The two Dees are connected to a high frequency oscillator capable of generating voltages of 10,000-100,000 V. Thus potential difference appears across the narrow gap between the two Dees and, thus, in this region a strong electric field is established which reverses its direction at regular intervals. The Dees are enclosed within a vacuum chamber (not shown in Figure), which is then placed between the poles of a powerful electromagnet. At the center of the machine there is an' ion source S, which releases the charged particles or ions which are to be accelerated.

Suppose the strength of the magnetic field has been so set that the cyclotron frequency, given by Eq. (10.18), of the ion emitted by the source S just matches the frequency of the high frequency oscillator connected to the Dees. At any instant, if D_1 is at a peak negative potential, then a positive ion released from the source will get accelerated towards D_1 by the electric field in the gap. While inside the Dee, the ion describes a circular orbit under the influence of the magnetic field and returns to the gap at the end of a half circle. (Within the Dees there is no electric field because the Dees are metallic). By then, the potential produced by the oscillator has also completed half a cycle so that now D_1 is at the positive potential while D_2 at the negative potential. Hence, ions get further accelerated by the electric field in the gap and enter D_2. There they follow a circular path of increased radius and again arrive at the gap when D_1 is once more negative.

Each time the ion crosses the gap, it gains energy qV_0, where q is the charge on the ion and V_o is the maximum potential difference generated by the oscillator between the two Dees. As it gains energy, its speed increases and the radius of circular orbit also increases. After ions have reached their outermost orbit, they are deflected out of Dee's by means of a deflecting electrode.

We are providing a solved example, which will help you in designing the cyclotron.

Example 1

The pole faces of a cyclotron magnet are 120 cm in diameter; the field between the pole faces is 0.80 T. The cyclotron is used to accelerate protons. Calculate the kinetic energy, in eV, and the speed of a proton as it emerges from the cyclotron. Determine the frequency of the alternating voltage that must be applied to the Dees of this accelerator.

$$(1\,eV = 1.6 \times 10^{-19}\,J);\ \text{mass of the proton} = (1.67 \times 10^{-27}\,kg)$$

Solution

The kinetic energy of the particle is given by

$$KE = \frac{1}{2} mv^2$$

and from Eq. **(10.15)**

$$v = \frac{Bqr}{m}$$

$$\therefore\ KE = \frac{(Bqr)^2}{2m} J = \frac{(Bqr)^2}{2me} eV$$

For protons, $q = e = 1.6 \times 10^{-19}$ C and $m = m_p = 1.67 \times 10^{-27}$ kg.

Thus

$$KE_p = \frac{B^2 r^2 e}{2m_p} = \frac{(0.80\,T)^2 (0.60\,m)^2 (1.6 \times 10^{-19}\,C)}{2(1.67 \times 10^{-27}\,kg)}$$

$$= 11 \times 10^6\,eV = 11\,MeV$$

and $$v = \frac{(0.80\,T)(1.6 \times 10^{-19}\,C)(0.60\,m)}{1.67 \times 10^{-27}\,kg} = 4.6 \times 10^7\ ms^{-1}$$

The frequency of the voltage applied to the Dees is the cyclotron frequency given by **Eq. (10.18).** For protons

$$f_{cp} = \frac{Be}{2\pi m_p} = \frac{(0.80\,T)(1.6 \times 10^{-19}\,C)}{2\pi(1.67 \times 10^{-27}\,kg)}$$

$$= 1.22 \times 10^7\ Hz = 12.2\ MHz.$$

Before summing up what you have learnt, answer the following SAQ.

SAQ 7

What is the primary function of electric Geld and magnetic field in the cyclotron?

Let us now sum up what we have learnt in this unit.

10.6 SUMMARY

- The force on a charged particle in an electric field is simply the product of the charge and the electric field

  $$\mathbf{F} = q\,\mathbf{E}$$

  When no other forces act on the particle, the resulting acceleration, given by Newton's law, is

  $$\mathbf{a} = \frac{q}{m}\mathbf{E}$$

- A charged particle moving in a uniform electric field follows a parabolic path because it is subjected to a constant acceleration. Given the initial velocity of the particle, the velocity at any other point on the parabolic path can be computed using Eqs. **(10.9)** to **(10.12).**
- A charged particle, velocity of which is **v** in a plane perpendicular to a magnetic field B, describes a circular trajectory. The radius r of this circular path is given by

  $$r = \frac{mv}{Bq}$$

  where m is the mass of the charged particle.
- The number of revolutions made by this particle per second is known as the cyclotron frequency and is given by

  $$f = \frac{Bq}{2\pi m}$$

- When the direction of motion of the charged particle is neither parallel nor perpendicular to the direction of magnetic field it describes a helical trajectory.
- In CRO, the electron beam can be deflected either by an electric field or by a magnetic field. In both cases, the deflection of the electron beam is proportional to the applied electric (or magnetic) field.
- The motion of a charged particle, moving through a combination of the electric and magnetic fields, is described by the Lorentz force

$$\mathbf{F} = q\,\mathbf{E} + q\,\mathbf{v} \times \mathbf{B}.$$

10.7 TERMINAL QUESTIONS

1) A particle with charge q and mass m is shot with kinetic energy K into the region between two plates as shown in Fig. 10.13. If the magnetic field between the plates is B and directed as shown, how large must B be, if the particle is to miss collision with the opposite plate?

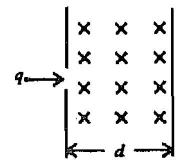

Fig. 10.13

2) In Fig. 10.14, a proton ($q = +e$, $m = 1.67 \times 10^{-27}$ kg) is shot with speed 8×10^{6} ms^{-1} at an angle of 30" to anx-directed field B = 0.15 T. Describe the path followed by the proton (including the radius, pitch etc.)

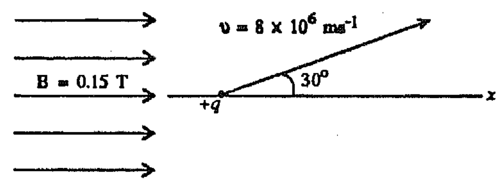

Fig. 10.14

3) As shown in Fig. 10.15, a beam of particles of charge q enters a region where an electric field is uniform and directed downward. Its value is 80 kVm^{-1}. Perpendicular to E and directed into the page is a magnetic field B = 0.4 T. If the speed of the particles is properly chosen, the particles will not be deflected by these crossed electric and magnetic fields. What speed is selected in this case? (This device is called a velocity selector.)

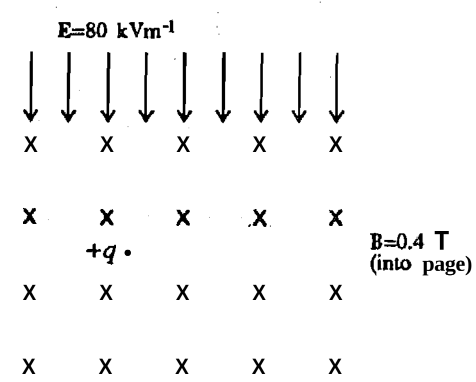

Fig. 10.15

4) A beam of electrons passes undeflected through two mutually perpendicular electric and magnetic fields, the electric field is cut off and the same magnetic field is maintained, the electrons move in the magnetic field in a circular path of radius 1.14 cm. Determine the ratio of the electronic charge to mass if E = 81 Vm^{-1} the magnetic field has flux density 2 x 10^{-3} T.

5) A cyclotron is being used to accelerate protons to a kinetic energy of 5.0 MeV. If the magnetic field in the cyclotron is 2.0 T, what must be the radius of the cyclotron and the frequency at which the Dee voltage is alternated?

10.8 SOLUTIONS AND ANSWERS

SAQs

1) Using Eq. (10.4) we have

$$y = 0 + \frac{1}{2}at^2 \qquad \text{(distance is measured along vertical direction)}$$

with the acceleration given by Eq. (10.2), we have

$$y = \frac{1}{2}\frac{qE}{m}t^2$$

$$= \frac{(-1.6 \times 10^{-19}\,C)(2.0\,N\,C^{-1})}{2 \times (9.1 \times 10^{-31}\,kg)}(1.0 \times 10^{-6}\,s)^2$$

$$= -0.18\ m.$$

The minus sign indicates that motion is downward, opposite to the field direction. It is expected because electron carries a negative charge.

2) b) A proton in the same situation would not move as far because its acceleration is much less due to its much higher mass. The proton will move in the upward direction.

3) i) The electric field acts along the y-axis. The horizontal component of velocity v_x of electron remains unaffected by the electric field. Thus, the time spent travelling from *A* to B is

$$t = \frac{x}{v_x} = \frac{2.0 \times 10^{-2}\,m}{4.0 \times 10^{6}\,ms^{-1}} = 0.50 \times 10^{-8}\,s$$

ii) During this time the electron experiences acceleration $\frac{qE}{m}$ in vertical direction and undergoes a vertical deflection (y) given by

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}t^2 = \frac{1}{2}\frac{qE}{m}\frac{x^2}{v_x}$$

Thus the path of the charged particle within the electric field is a parabola in the *xy* plane. Now

$$y = \frac{1}{2}\frac{qE}{m}t^2$$

or $$y = \frac{(1.6 \times 10^{-19}\,C)(-1.0 \times 10^{2}\,NC^{-1})}{(2)(9.1 \times 10^{-31}\,kg)}(0.50 \times 10^{-8}\,s)^2$$

$$= 2.2\ mm.$$

The positive value of y means that electron is deflected upward. In the field region, it follows an upward-curving parabola, as shown in Fig. 10.16.

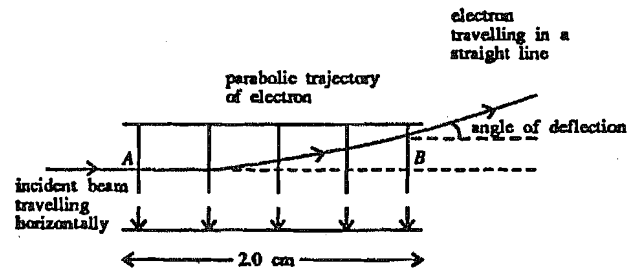

Fig. 10.16

iii) At the time of **entrance** there was no component of v along the y-axis, hence, using Eq. (10.10) we get

$$v_y = 0 + \frac{(-1.6 \times 10^{-19}\,C)}{(9.1 \times 10^{-31}\,kg)} \times (-1.0 \times 10^{3}\,NC^{-1})\ (0.50 \times 10^{-8}\,s)$$

$$= 8.8 \times 10^{5}\,ms^{-1}.$$

iv) The electron leaves the region with a speed v given **by**

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(4 \times 10^{2}\,ms^{-1})^2 + (8.8 \times 10^{5}\,ms^{-1})^2}$$

$$= 4.1 \times 10^{6}\,ms^{-1}$$

v) The angle of deflection at B is $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

$$= \tan^{-1} \frac{8.8 \times 10^{5}\,ms^{-1}}{4.4 \times 10^{6}\,ms^{-1}}$$

$= 12°$ above the horizontal.

vi) Once it leaves the field region, the electron will **again move** in a straight line along the tangent to the parabola at the point **where** the electron leaves the field region as **shown** in Fig. 10.16.

vii) Positively **charged** particle is also deflected **by 12°**, but because of their **sign** they are deflecled downwards.

viii) Such an arrangement can be used for separating positive **and** negative particles in a beam.

4) When the sign of the charge is negative, the right hand rule shows that the force experienced **by** it will be in upward direction. The **Fig. 10.17** shows what your answer should look like.

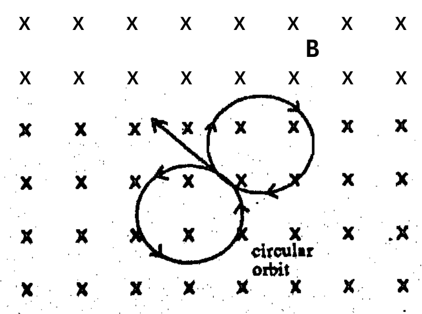

Fig. 10.17

5) $v = 10^7\,ms^{-1}$, $v_{\parallel} = v\cos 30°$, $v_{\perp} = v\sin 30°$

$$R = \frac{m}{e}\frac{v_{\perp}}{B} = \frac{10^7 \sin 30°}{1.5 \times 10^{-3} \times 1.76 \times 10^{11}} = \frac{1}{52.8}\ m.$$

$$T = \frac{2\pi}{1.5 \times 10^{-3} \times 1.76 \times 10^{11}} = 2.38 \times 10^{-8}\ s.$$

6) On reversing the direction of the flow of protons, if the protons are deflected in the same direction, the deflection is due to electric field, if the protons are deflected in the opposite direction, the deflection is due to magnetic field.

7) In a cyclotron, the purpose of the electric field is to energize the beam and that of magnetic field to give it the circular motion.

Terminal Questions

1) To just miss the opposite plate, the particle must move in a circular path with radius r so from $Bqr = mv$; and using $K = (mv^2)/2$, we have $B = (2mK)^{1/2}/(qr)$.

2) We resolve the particle velocity into components parallel to and perpendicular to the magnetic field. The magnetic force due to $v_{\parallel}$ is zero ($\sin\theta = 0$); the force due to $v_{\perp}$ has no x component. Therefore, the x motion is uniform, at speed $v_{\parallel} = (0.86)(8 \times 10^6\ ms^{-1}) = 6.88 \times 10^6\ ms^{-1}$, while the transverse motion is circular with radius

$$r = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27}\ kg)(0.5 \times 8 \times 10^6\ ms^{-1})}{(1.6 \times 10^{-19}\ C)(0.15\ T)} = 0.28\ m$$

The proton will spiral along the x-axis; the radius of the spiral (or helix) will be 28 cm. To find the pitch of the helix (the x distance travelled during one revolution), we note that the time taken to complete one circle is

$$\text{period} = \frac{2\pi r}{v} = \frac{2\pi(0.28\ m)}{(0.5)(8 \times 10^6\ ms^{-1})} = 4.4 \times 10^{-7}\ s$$

During that time, the proton will travel x distance of pitch given by $(v_{\parallel})(\text{period}) = (6.88 \times 10^6\ ms^{-1})(4.4 \times 10^{-7} s) = 3.0\ m$

3) The electric field causes a downward force Eq on the charge if it is positive. The right-hand rule tells that the magnetic force, $qvB \sin 90°$, is upward if q is positive. If these two forces are to balance so that the particle does not deflect, then

$$Eq = qvB \sin 90° \text{ or } v = \frac{E}{B} = \frac{80 \times 10^3\ Vm^{-1}}{0.4\ T} = 2 \times 10^5\ ms^{-1}$$

When q is negative, both forces are reversed, so the result $v = E/B$ still holds.

4) If the beam is undeflected, when the crossed fields are on, we have $evB = eE$ and $v = E/B$. When the electric field is cut off, the electrons move in a circle with: $e/m = v/RB = E/(RB^2) = (8 \times 10^3)/[(0.0114)(2 \times 10^{-3})^2] = 1.75 \times 10^{11}\ C\ kg^{-1}$

5) The cyclotron frequency is given by

$$f = \frac{qB}{2\pi m} = \frac{(1.6 \times 10^{-19}\ C)(2.0\ T)}{(2\pi)(1.7 \times 10^{-27}\ kg)} = 3.0 \times 10^7\ Hz.$$

This is the frequency required to accelerate protons at each crossing of the Dee gap. An energy of 5.0 MeV is equal to

$$(5.0 \times 10^6\ eV)\ 1.6 \times 10^{-19}\ J\ (eV)^{-1} = 8.0 \times 10^{-13}\ J.$$

so the proton kinetic energy is

$$KE = \frac{1}{2}mv^2 = 8.0 \times 10^{-13}\,J.$$

Solving for the speed v, gives

$$v = \sqrt{\frac{2K}{m}} = \frac{(2)(8.0 \times 10^{-13}\,J)}{1.7 \times 10^{-27}\,kg} = 3.1 \times 10^{7}\,ms^{-1}$$

The radius needed to accommodate 5-MeV protons is given by

$$r = \frac{mv}{qB} = \frac{(1.7 \times 10^{-27}\,kg)(3.1 \times 10^{7}\,ms^{-1})}{(1.6 \times 10^{-19}\,C)(2.0\,T)} = 0.16\,m.$$

UNIT 11 MAGNETISM OF MATERIALS-I

Structure

11.1 INTRODUCTION

In the last two Units, we have discussed the magnetic fields produced by moving charges or currents in conductors. There, the moving charges and conductors were considered to be placed in vacuum (i.e., in air). In Units 11 and 12, we learn how the magnetic field affects materials and how some materials produce magnetic field. You must have learnt in your school Physics Course that in equipment such as generator and motor, iron or iron alloy is used in their structure for the purpose of enhancing the magnetic flux and for confining it to a desired region. Therefore, we will study the magnetic properties of iron and a few other materials called ferromagnets, which have similar properties as iron. We shall also learn that all the materials are affected by the magnetic field to some extent, though the effect in some cases is weak.

When we speak of magnetism in everyday conversation, we almost certainly have in mind an image of a bar magnet. You may have observed that a magnet can be used to lift nails, tacks, safety pins, and needles (Fig. 11.1a) while, on the other hand, you cannot use a magnet to pick up a piece of wood or paper (Fig. 11.1b).

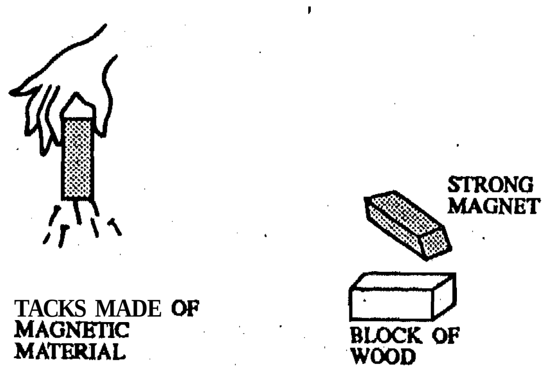

Fig.11.1: a) Materials that are attracted to a magnet are called magnetic materials. b) Materials that do not react to r magnet are called nonmagnetic materials.

Materials such as nails, needles etc., which are influenced by a magnet are called **magnetic materials** whereas other materials, like wood or paper, are called **non-magnetic** materials. However, this does not mean that there is no effect of magnetic field on non-magnetic materials. The difference between the behaviour of such materials and iron like magnetic materials is that the effect of magnetic field on non-magnetic material is very weak.

There are two types of non-magnetic materials: diamagnetic and paramagnetic. Unit 11 deals with diamagnetic and paramagnetic effects. The ideas, concepts and various tenns