UNIT 6 CAPACITOR

Structure

6.1 INTRODUCTION

You have studied in your earlier classes that the potential of a conductor increases as the charge placed on it is increased. Mathematically we write

$$Q \propto \phi \text{ or } Q = C\,\phi \qquad (6.1)$$

Where C is the proportionality constant.

We call this constant C as the capacity or capacitance. We also call any device that has capacitance as the capacitor (condensoi). You are already familiar with this device.

We change the capacitance in our radio-transistor while operating the 'tuning' knob and get the radio station of our choice. Capacitors are used in many electrical or electronic circuits, they provide coupling between amplifier stages, smoothen the output of power supplies. They are used in motors, fans, in combination with inductances to produce oscillations which when transmitted 'become radio signals/TV signals etc. Besides, these capacitors have a variety of applications in electric power transmission.

In the present unit, we shall learn about capacitance, capacitors of different forms, energy stored in a capacitor, working principle of a capacitor. We have studied the macroscopic properties of dielectrics in Unit 5. Here we will study the effect on the capacitance of a capacitor, when a dielectric is placed between the two plates of a capacitor. Then we will introduce some practical 'capacitors.

In next unit we will study the microscopic properties of the dielectrics.

Objectives:

After going through this unit you will be able to:

- define capacitance of a capacitor,
- describe capacitors of different geometries and obtain mathematical expression for their capacitance,
- able to calculate the energy stored in a capacitor,
- describe the effect of introducing a dielectric material in a capacitor,
- obtain expressions for the effective capacitance of grouping a number of capacitors in series and in parallel,
- describe **practical** capacitors such as a guard condenser and an electrolylic capacitor.

6.2 CAPACITANCE

A capacitor or a condenser is an electronic device for storing electrical energy by allowing charges to accumulate on metal plates. This electrical energy is recovered when these charges are allowed to move away from these plates into the circuit of which the capacitor forms a part. Any device which can store 'charges is a capacitor. For example, an insulated conducting spherical shell of radius R can store charges; hence it can be used as a condenser. Let us see how it works as a capacitor. If a charge Q is placed on it the outer surface of the shell becomes an equipotential surface. The potential of the outer surface of the shell (see Unit 4) is given by

$$\phi = \frac{Q}{4\,\pi\,\varepsilon_0 R} \qquad (6.2)$$

with ,infinity as zero potential. Instead of infinity we can regard the ground (earth) as zero potential. Then the capacitance of this shell (w.r.t. ground) is

$$C = Q/\phi = 4\,\pi\,\varepsilon_0\,R \;\; \frac{\text{Coulomb}}{\text{volt}} \qquad (6.3)$$

The unit of capacitance C in SI system is farad.

$$\text{farad} = \frac{\text{Coulomb}}{\text{volt}} \qquad (6.4)$$

If R = 100 cm in the above spherical shell its capacity in farads is

$$(4\pi\varepsilon_0)\ 100 = 1.1 \times 10^{-10} \text{ farad.}$$

Thus it is clear from this that if a capacitor is to be made with one unit (farad) capacity it has to have huge dimensions (10^{10} m in the above case). Practical form of condensers have small dimensions and smaller units such as picofarad (10^{-12} farad) and microfarad 10^{-6} farad) are more commonly used. The symbolic representation of a capacitor is o–| |–o.

The above example of a spherical conductor as a capacitor is given only to illustrate the concept. However, the most commonly used practical form of condensers always has a system of two metal sheets (circular, cylindrical or rectangular) kept close to each other with an insulator separating the two sheets. This system has the ability to have larger capacity without having the corresponding larger dimensions. You will learn more about this in detail in the next section,

6.3 PARALLEL PLATE CAPACITOR OR CONDENSER

This is the simplest and most commonly used form of a condenser. A parallel plate condenser consists of two rectangular or circular sheets (plates) of a metal arranged parallel to one another separated by a distance d. The value of d is usually very small and an insulating material is normally inserted between the two sheets. See Fig. 6.1. A charge Q (positive) placed on the upper plate distributes equally on this plate to make it an equipotential surface. The lower plate is shown grounded (earthed. the symbol ⏚ used for showing the grounding). The lower plate is therefore at ground potential (zero potential). Because of electrostatic induction an equal amount of negative charge appears on the upper side of the lower plate. This induced negative charge pulls up almost all the positive charge placed on the upper plate to the lower side of the upper plate. Thus the electric field now gets confined to the space between the two plates: the positive charge acting as sources and the negative charge as sink (the lines of force originate on the positive charges and end on negative charges). The induced negative charge is equal to the amount of positive charge because of the zero field requirement inside the material of the conducting sheets. Besides, both the metal sheets are equipotential surfaces. The lines of force field lines are normal to these sheets except at edges. See Fig. 6.1. Since all the field lines originate front the upper plate and end on the lower plate, the value of the electric field, E **is uniform** in the **space** between the plates except at the edge. The edge effects are negligible if the area of the plates, ***A***, is large compared to d. Since E is uniform the potential difference between the upper and the lower.plates is given by

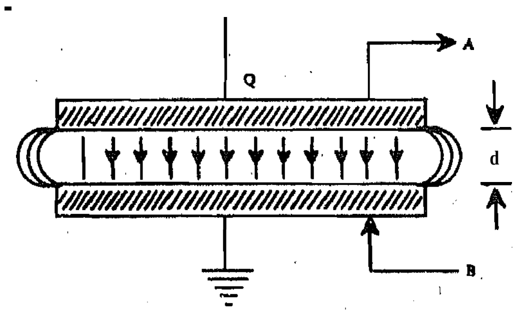


Fig. 6.1: Parallel plate condense. A and B arc the metal plates separated nt a distance '*d*'.

$$\phi_2 - \phi_1 = -E \cdot dl = Ed$$

where ϕ_2, ϕ_1 refer to the potentials of upper and lower plates respectively. As the lower plate is earthed,

$$\phi_1 = 0; \phi_2 = Ed \qquad (6.5)$$

To evaluate E let us use Gauss's theorm. Suppose we evaluate the electric flux for a closed eylindrical surface EFGH of base area S with its axis normal to the plate. See Fig. 6.2

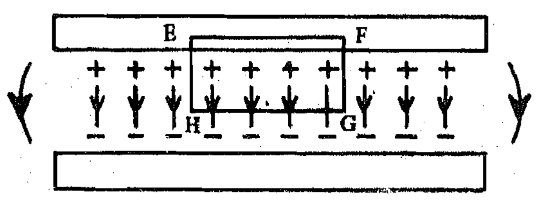


Fig. 6.2: Gusslan closed cylinder EFCH.

One of the horizontal surfaces is inside the metal and the other in the space between the plates, the curved faces are parallet to the field lincs. There is no flux through EF as the field inside the conducting surface is zero. Similarly, there is zero flux through EH and FG as the curved surfaces of the Gaussian cylinder arc parallel to the field lines.

The flux through the surface HG of area S is equal to ES. Since E is along the normal to the area, hence, we can apply Gauss' theorem. According to Gauss theorem

$$ES = \frac{S}{\varepsilon_0}, \; E = \sigma/\varepsilon_0 \qquad (6.6)$$

where σ is the charge per unit area on the condenser plate. The potential ϕ_2 of the upper plate is Ed from Eq (6.1). The total charge Q is σ. A.

$$C = \frac{Q}{\phi} = \frac{\varepsilon_0 A}{d} \qquad (6.7)$$

By keeping a small value for d, the capacity C can be increased. In the above derivation we have taken the medium between the plates to be vacuum. The above arrangement has the advantage of the electric field being unaffected by the presence of other charges or conductors in the neighbourhood of the capacitor. Moreover, if the area A of the plates is much greater than d the correction for the capacitance due to the nonuniform field at the edges is negligible.

SAQ 1

Suppose we have the distance of separation between the plates, what happens to the capacitance?

SAQ 2

Find the charge on a 1000 μF capacitor when charged to a voltage of 24 V.

In next subsection you will learn about the energy stored in capacitor.

6.3.1 Energy Stored in a Capacitor

In Unit 3, it was shown that the work done, W in assembling a charge Q by adding infinitesimal increments of charge is given by

$$W = 1/2\, Q\, \phi \qquad (6.8)$$

Where ϕ is the final potential of the charged body. In the case of a capacitor of capacitance C, this work done in placing a charge Q on the capacitor must also be given by similar expression, i.e.,

$$W = 1/2\, Q\, \phi \qquad (6.9)$$

This can be written in terms of the capacitance $C = Q/\phi$ as

$$W = 1/2\, C\, \phi^2 = Q^2/2C \text{ joules} \qquad (6.10)$$

This work is stored up in the electric field as potential. energy.

Since the potential is defined as the work done per unit charge, the work done in moving a small charge δq against a **charge potential ϕ will be work done = $\phi\delta q$**

But $\phi = \frac{q}{C}$

The total work done in charging a capacitor to Q coulombs is given by

Total work done $= \int_0^Q \frac{q\sigma q}{C} = \frac{q^2}{2C}$

$= \frac{Q^2}{2C}$

SAQ 3

Show that in a parallel plate capacitor of area A and the separation of plates by a distance d in vacuum the energy stored in the (space) volume of the electric field between the plates is given by 1/2 Q ϕ.

6.4 PARALLEL PLATE CAPACITOR WITH DIELECTRICS

When a dielectric slab is inserted between the parallel plates of a condenser the capacity increases. The polarised dielectric slab ABCD (see Fig. 6.3) reduces the electric field E inside the dielectric by a factor $(1/\varepsilon_r)$ where ε_r is the relative permittivity as discussed in the last unit: This can be proved by computing the electric field by using Gauss' law for electric displacement, D inside the dielectric ABCD. Recall the Gaussian cylinder used in evaluating E in Section 6.2. The flux of D is now given by (only free charges contribute to the flux)

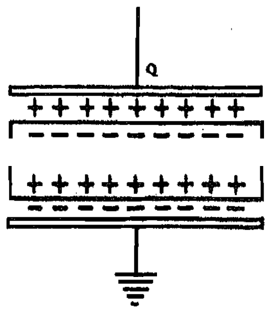


Fig. 6.3: Dielectric slab between capacitor plates.

$$DS = \sigma S \tag{6.11}$$

as the bound surface charges do not contribute to this flux and

$$D = \varepsilon_0 \varepsilon_r E \tag{6.12}$$

for an isotropic uniformly polarised dielectric. Thus the field

$$E = \frac{\sigma}{\varepsilon_0 \varepsilon_r} \tag{6.13}$$

The potential difference between the plates is equal to E.d Where d is now the thickness of the slab filling the entire space between the plates. The capacitance now becomes

$$C = \frac{Q}{\phi} = \frac{\sigma A}{Ed} = \frac{\varepsilon_0 \varepsilon_r A}{Ed} \tag{6.14}$$

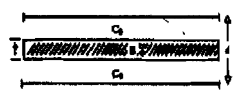
The value of the capacitance C increases by the factor ε_r which is relative permittivity of the dielectric material.

From Eq. (6.14) we note that the capacitance of a parallel plate capacitor increases with the increase in surface area (A) of the plates and also with the decrease of the distance separating the plates.

The effect of introducing a dielectric in between the plates increases the capacitance ($\because \varepsilon_r > 1$). Thus inclusion of a dielectric enables the capacitor to hold more charges at a given potential difference between the plates.

We rewrite Eq. (6.14) as

$$C = \varepsilon_0 A/(d/\varepsilon_r) \tag{6.15}$$



and compare it with Eq. (6.7). We find that a dielectric of thickness d has an equivalent free space thickness (d/ε_r). This observation, will be useful later when we deal with the capacitor in which the space in between the plates is only partially filled with a dielectric.

SAQ 4

Find the capacitance of the parallel plate capacitor consisting of two parallel plates of area 0.04 m^2 each and placed 10^{-3} apart in free space.

A capacitor is shown in Fig. 6.4 in which a dielectric slab of thickness t is inserted between the plates kept apart at a distance d. We write the capacitance of this capacitor, on the basis of the equivalent free space thickness of the dielectric. We find the free space thickness between the plates = $(d-t)$ where t is the thickness of the dielectric material. This t is equivalent to t/ε_r in free space. The,capacitor of Fig. 6.4

is equivalent to a capacitor with free space between the plates, with the separation of $(d - t + t/\varepsilon_r)$. We write the expression for the capacitance as

$$C = \varepsilon_0 A/(d - t + t/\varepsilon_r) \quad (6.16)$$

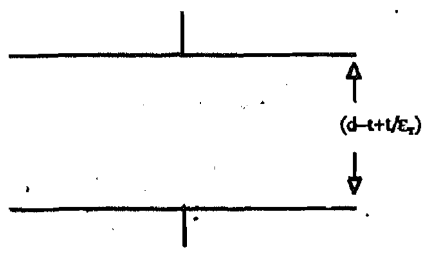


Fig. 6.5: Shows the equivalent capacitor.

Now we will obtained Eq. (6.16) with another simple methods. Let the voltage across the capacitor which is shown in Fig. **6.4** is **V**. When a dielectric of thick 't' is introduced between the two plates of the capacitor, the distance between the positive plate of capacitor and the upper surface of the dielectric is say d_1 and from lower surface of dielectric to negative plate of the capacitor is d_2. Now assume that the voltage between positive plate and upper surface of the dielectric is V_1, the voltage between upper and lower surface of the dielectric is V_2 and lower plate of the dielectric to negative plate of the capacitor is V_2. The total voltage **V** across capacitor is the sum of these three voltages i.e.,

$$V = V_1 + V_2 + V_3$$

Let E be the field inside the dielectric. Then

$$V_1 = d_1 E,\ V_2 = E \,.\, t/\varepsilon_r \text{ and } V_3 = d_2 E$$

$$V = (d_1 + d_2)\, E + E \,.\, t/\varepsilon_r$$

From the figure

$$d = d_1 + d_2 + t$$

$$d_1 + d_2 = (d - t)$$

From the above equation we get

$$V = (d - t)E + E \,.\, t/\varepsilon_r$$

Using Eq. (6.5). we get that in this case

$$d = [(d - t) + t/\varepsilon_r]$$

From Eq. (6.15), we get

$$C = \frac{\varepsilon_0 A}{d}$$

$$= \frac{\varepsilon_0 A}{[(d - t) + t/\varepsilon_0]}$$

We can also find that the ratio of the capacitance with dielectric between the plates to the capacitance with free space between the plates is equal to the relative permittivity, viz.,

$$\varepsilon_r = \frac{\text{Capacitance with dielectric between the plates}}{\text{Capacitance with free space between the plates}}$$

The relative permittivities (ε_r) of some of the most common materials are given in Table 6.1.

Table 6.1: Relative permittivity (ε_r) of some common materials

Air	1.0006
Castor Oil	4.7
Mica	5-9
Glass	4.5-7.00
Bakelite	4.5-7.5
Paper	2 - 2.3
Porcelain	5.5
Quartz	1.5
Water	81

SAQ 5:

A dielectric of relative permittivity 3 is filled in the space between the plates of a capacitor. Find the factor by which the capacitance is increased, if the dielectric is only sufficient to fill up 3/4 of the gap.

6.4.1 Voltage Rating of a Capacitor

Capacitors are designed and manufactured to operate at a certain maximum voltage which depends on the distance between the plates of the capacitor. If the voltage is exceeded, the electrons jump across the space between the plates and this can result in permanent damage to the capacitor. The maximum safe voltage is called the working voltage. The capacity and the working voltage (WV) is marked on the capacitor in the case of bigger capacitors and indicated by the colour code (similar to that of resistance) in the case of capacitors having low values of the capacitance.

6.5 CAPACITANCE OF A CYLINDRICAL CAPACITOR

In Section 6.3, we have calculated the capacitance of a parallel plate capacitor. Another important form of capacitor is a cylindrical capacitor. This is shown in Fig. 6.6a. A section of this capacitor is shown in Fig. 6.6b. It is made up of two hollow coaxial cylindrical conductors of radii a and *b*. The space between the cylinders is filled with a dielectric of relative permittivity ε_r. Practical forms of such capacitors are

In Fig. 6.6b direction of the field lines is radial, viz., normal to the surface of the cylinder. Small lines in between the two cylinders, show the direction of fixed line.

i) a coaxial cable, in which the inner conductor is a wire and the outer conductor is normally a mesh of conducting wire separated from the inner conductor by an insulator (usually plastic)

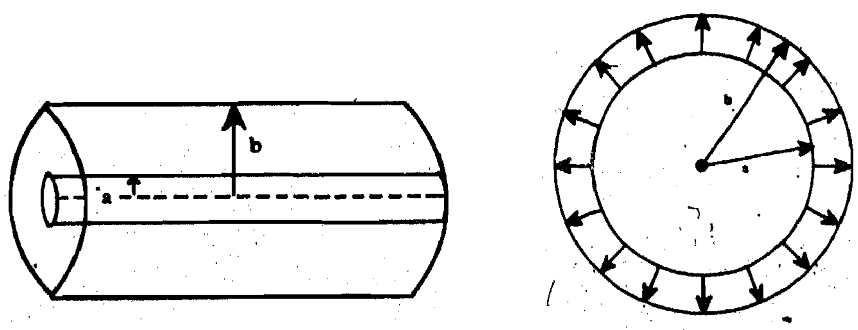


Fig. 6.6: (a) Cylindrical capacitor (b) cross section of the cylindrical capacitor.

ii) the submarine cable, in which a copper conductor is covered by polystyrene (the outer conductor is sea water), Since both the inner and outer cylinders are conductors, they are equipotential surfaces (see Unit 4). The field is radial (normal to the surface of the cylinder): Because of cylindrical symmetry we conclude that the capacitance is proportional to the length of the cylinder (as the length will increase, the area of the plot will increase). We shall now find the capacitance per unit length of the capacitor.

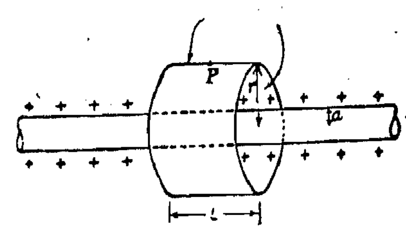


Fig. 6.7: Gaussian surface ABCD.

Let the charge per unit length placed on the inner cylinder of the capacitor be λ. The outer cylinder is grounded. An equal and opposite amount of charge appears on inner side of the outer cylinder. This is because of the zero field in the conductor. To evaluate the field let us consider a coaxial closed cylindrical surface ABCD of unit. length and of radius r. See Fig. 6.7. The electric field is normal to the inner cylindrical surface and is also confined to the space between the cylinders. The flux of electric displacement vector, D, through the bottom and top surfaces of this Gaussian cylinder ABCD is zero as D is parallel to these faces. The flux of D is only through the curved surface of ABCD and as **D** is normal to this at all points, the flux through this closed Gaussian surface is given by

$$\mathbf{D} . d\mathbf{s} = (2\pi r) . \mathbf{D} . \delta l \qquad (6.17)$$

Now $D = \varepsilon_0 \varepsilon_r E$ for isotropic uniformly polarised dielectrics. Using Gauss' law we get

$$2\pi r D = (2\pi r)\, \varepsilon_0 \varepsilon_r E = \lambda \qquad (6.18)$$

where λ is the free charge enclosed by the Gaussian surface. Thus

$$E = \frac{\lambda}{2\pi r \varepsilon_0 \varepsilon_r} \qquad (6.19)$$

To find the capacitance, we require the potential difference between the two cylinders. In Unit 3, you have studied that the expression for potential difference is given by

$$\phi = -\int_b^a E . dr \qquad (6.20)$$

Four our case, Eq. (6.20) becomes

$$\phi_a - \phi_b = -\int_b^a (E dr)$$

$$= \int_a^b \frac{\lambda}{2\pi r \varepsilon_0 \varepsilon_r} dr$$

$$= \frac{\lambda}{2\pi r \varepsilon_0 \varepsilon_r} dr/r$$

$$= \frac{\lambda}{2\pi r \varepsilon_0 \varepsilon_r} \ln (b/a)$$

$$\phi_a - \phi_b = \frac{\lambda}{2\pi r \varepsilon_0 \varepsilon_r} \ln (b/a) \qquad (6.22)$$

As the outer cylinder is grounded $\phi_b = 0$.

Now, capacitance per unit length, C is given by

$$= \lambda/\phi_a$$

$$= 2\pi\varepsilon_0\varepsilon_r/\ln (b/a) \quad (6.23)$$

Note: In the expression for the capacitance per unit length of a cylindrical capacitor, Eq. (6.23), we find that the capacitance depends on the ratio of the radii and on their absolute values.

SAQ 6

Two cylindrical capacitors are of equal length and have the same dielectric. In one of them a radii of the inner and outer cylinders are 8 and 10 cm, respectively and in the other they are 4 and 5 cm. Find the ratio of their capacitances.

6.6 CAPACITORS IN SERIES AND PARALLEL

In Section 6.5, we have seen the method of finding the capacitance per unit length of a cylindrical capacitor. We multiply the capacitance per unit length by the length for cylindrical capacitors and get its capacitance. Now we can consider a cylindrical capacitor of length 2 units as consisting of two cylindrical capacitors of unit length joined end to end so that the inner cylinders are connected together and the outer cylinders also get connected similarly. This is shown in Fig. 6.8,

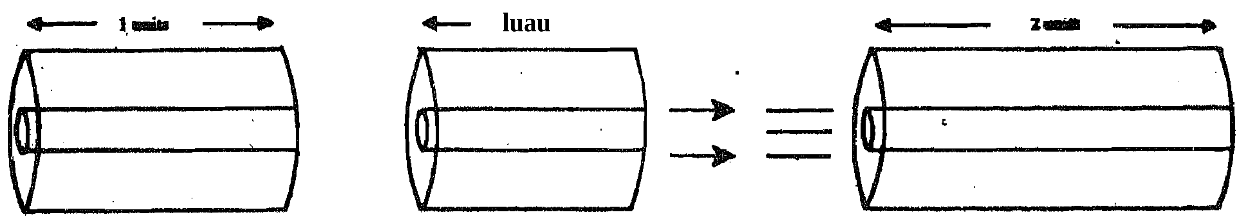


Fig. 6.8: A long cylindrical capacitor seen as a particular combination of unit cylindrical capacitor.

We find immediately that in such a combination the charge on the capacitor is doubled and so the capacitance is also doubled since the potenlial difference remains constant. Two capacitors connected in parallel (symbolic representation) are shown in Fig. 6.8a.

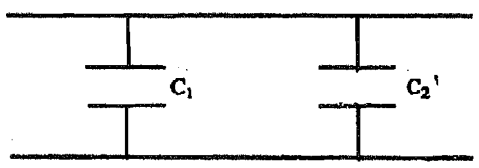


Fig. 6.80: Two cnpncitors connected in parallel.

In this combination, we find that

- the potential difference between the plates remains the same;
- the charge on each capacitor adds up (more area is available for storing charges).

We can find an equivalent capacitor chat holds the same charge when kept at the same potential difference as the combinations of the capacitors. The capacitance of that capacitor is known as the Effective Capacitance of the combination. Before we proceed further, we note that capacitors can be grouped or combined in another way too. Here alternate plates of the capacitors are connected to the succeeding capacitor so that they form a series. Fig. 6.9 shows the combination, it is known as **combination** of capacitors in series.

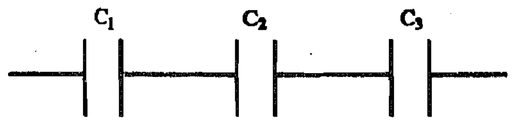


Fig. 6.9: Capacitors In Series.

If a voltage source is connected across the two end plates of the first and last capacitors of the series, **equal** charges will be induced in each capacitor whereas the potential difference across each capacitor will depend upon its capacitance.

We shall find the mathematical formulas for the equivalent capacitance of the combination of capacitances in **parallel** and in series.

6.6.1 Combination of Capacitors in Parallel

Fig. 6.10 shows the combination of three capacitors in parallel.

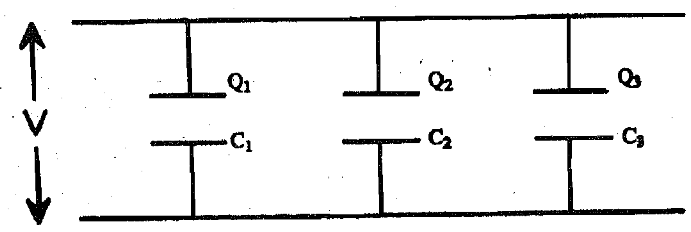


Fig. 6.10: Capacitors in parallel.

Here C_1, C_2 and C_3 are the capacitances of the individual capacitors, Q_1, Q_2 and Q_3 are respective the charges on them **and** ϕ is the potential difference between the **plates** of each capacitor. We take C to be the effective capacitance of the combination. The total charge Q of the **parallel** combination is equal to

$$Q = Q_1 + Q_2 + Q_3 \tag{6.24}$$

Since ϕ is same for this **equivalent** C of the parallel combination

$$\text{Now } C = Q/\phi = \frac{Q_1 + Q_2 + Q_3}{\phi}$$

$$= \frac{Q_1}{4} + \frac{Q_2}{4} + \frac{Q_3}{\phi}$$

$$C = C_1 + C_2 + C_3 \tag{6.24}$$

Thus the effective capacitance of the parallel combination of capacitor is equal to the sum of the individual capacitances.

6.6.2 **Combination** of **Capacitors in Series**

Fig. 6.11, shows the combination of three capacitors in series.

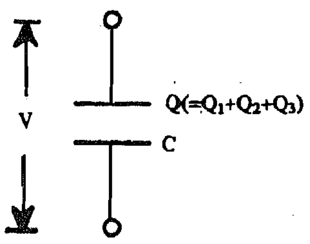


Fig. 6.11: Capacitors in series and the equivalent capacitors.

Here C_1, C_2 and C_3 are the capacitances of the individual capacitors. The application of a voltage will place a charge $+Q$ on one plate which induces a charge $-Q$ to the other plate. The intermediate plates acquire equal and opposite charges, because of electrostatic induction. The potential drop across each will be inversely proportional to its capacitance. (Since $C = Q/\phi$ gives $\phi = Q/C$. Since **Q** is fixed $\phi \propto 1/C$]. Thus ϕ_1, ϕ_2 and ϕ_3 the potential drop across the capacitors are such that $\phi_1 \propto 1/C_1$, $\phi_2 \propto 1/C_2$ and $\phi_3 \propto 1/C_3$. Now we replace the capacitors by a single capacitor of capacitance **C** that holds the charge Q when subjected to a potential difference $\phi = (\phi_1 + \phi_2 + \phi_3)$ This capacitance C is known as the effective capacitance of the combination. We now write $C = Q/\phi$ or $1/C = \phi/Q$. But $\phi = \phi_1 + \phi_2 + \phi_3$. Therefore,

$$\frac{1}{C} = \frac{\phi_1 + \phi_2 + \phi_3}{Q} \text{ i.e.}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad (6.25)$$

Thus for capacitors connected in series the reciprocals of the capacitances add to give the reciprocal of the effective capacitance.

SAQ 7

Determine the equivalent capacitance of the netwrok shows in Fig. 6.12 and the voltage drop across each of the capacitor of the series of capacitors.

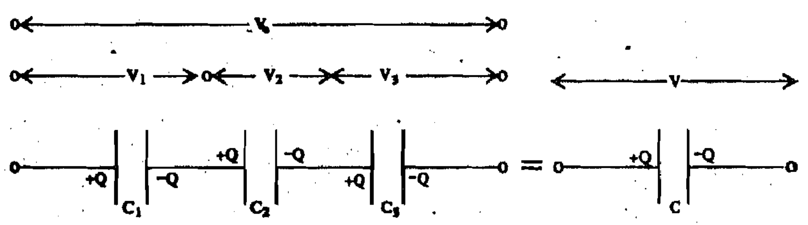


Fig.6.12

SAQ 8

Calculate the effective capacitance of three capacitors arranged in such a way that two of them C_1 and C_2 are in series and the third C_3 is in parallel with this series combination.

6.7 STORED ENERGY IN A DIELECTRIC MEDIUM

In Section 6.31, we have studied that the energy stored in a parallel plate capacitor is given by

$$V = \frac{1}{2} C V^2$$

We know that

$$C = \frac{\varepsilon_0 A}{d}$$

and

$$V = E \cdot d$$

Pitting these values in the above Eq. we get

$$U = \frac{1}{2} \frac{\varepsilon_0 A}{d} E_2 \cdot d^2$$

$$= \frac{1}{2} \varepsilon_0 (Ad) \cdot E^2$$

or

$$\frac{U}{V} = \frac{1}{2} \varepsilon_0 E^2 \; (Ad = V)$$

This is energy per unit volume.

When a dielectric of relative permittivity ε_r fills the space between the plate of the capacitor, then the effective capacitance is given by

$$C_{die} = \frac{\varepsilon_0 \varepsilon_1 A}{d}$$

The energy stored in a capacitor with a dielectric material is given by

$$U = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \frac{\varepsilon_0 \varepsilon_1 A}{d} (d \cdot E)^2$$

$$= \frac{1}{2} \varepsilon_0 \varepsilon_1 E^2 \; (A \cdot d)$$

$$\frac{U}{u} = \frac{1}{2} \varepsilon_0 \varepsilon_1 E^2 \; (A \cdot d = u)$$

In case of a parallel plate condensor, the energy stored per unit volume is $1/2 \; \varepsilon_0 E^2$ which become $1/2 \; \varepsilon_0 \varepsilon_r E^2 = 1/2$ E . D with the dielectric material. Where D is the electric displacement in the dielectric. We have considered here the case of a linear dielectric where E and D are in the same direction. However, there are dielectrics in which E and D are not in the same direction. Thus the energy stored per unit volume in a dielectric medium is given by

$$1/2 \; E \cdot D \; \text{Joules/m}^2 \qquad (6.26)$$

6.8 PRACTICAL CAPACITORS

We shall now study some of the capacitors that are commonly in use. Capacitors may be broadly classified into two groups i.e., fixed and variable capacitors. They may be further classified according to their construction and use. Following are the classifications of the capacitor.

Types of Capacitor

- **Fixed Capacitor**
  - Paper Capacitor
  - Mica capacitor
  - Ceramic capacitor
  - Electrolytic capacitor
- **Variable Capacitor**
  - Gauge capacitor of tube type receivers
  - Gauge type capacitor of transistor receivers
  - Trimmer capacitor

Now, we will discuss each type of the capacitor one by one.

6.8.1 Fixed Capacitors

These have fixed capacitance. These are essentially parallel plate capacitors, but compact enough to occupy less space. In their make they consist of two very thin layers of metal coated on the surface of mica or paper having a uniform coating of paraffin. The mica or paper having a uniform coating of paraffin. The mica or paper forms the dielectric between the conductors. They are shown in Fig. 6.13.

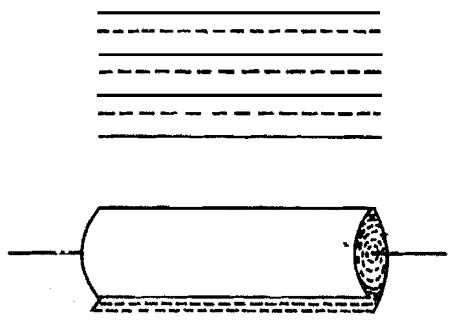


Fig. 6.13: Fixed capacitors

This arrangement is rolled up to the compact form. Usually they are piled up in parallel to give a large capacitance. Though paraffin-waxed paper capacitors are cheaper, they absorb a good amount of power. For this reason these capacitors are used in alternating current circuits, radio-sets, etc.

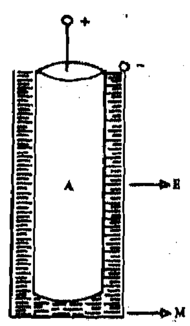
6.8.2 Ceramic Capacitors

These are low loss capacitors at all frequencies. Ceramic materials can be made to have very high relative permittivity. For example, teflon has $\varepsilon = 8$ but by the addition of titanim the value of ε_r becomes 100 and on adding barium titanate the value of ε_r may be increased to 5,000. Each piece of such dielectric is coated with silver on the two sides to form a capacitor of large capacitance. Yet another advantage with these ceramic dielectrics is that they have negative temperature coefficient. Ceramic capacitors arc widely used in transistor circuits.

6.8.3 Electrolytic Capacitors

An electrolytic capacitor consists of two electrodes of aluminium, called the positive and the negative plates. The positive plate is electrolytically coated with a thin layer of aluminium oxide. This coating serves as the dielectric. The two electrodes are in contact through the electrolyte which is a solution of glycerine and sodium (or a paste of borates, for example, amonium borate), There are two types of electrolytic capacitors—the wet type and the dry type.

In the wet type the positive plate (A) is in the form of cylinder to present a large surface area. This is immersed in the electrolyte (E) contained in a metal can (M). This can act as a negative plate. It is shown in Fig. 6.14.



Flg.6.14: Wet type capacitor (electrolytic).

In the dry type both plates are in the form of long strips of aluminium foils. Aluminium oxide is deposited electrically on one (A) of the foils. This is kept separated from the other (B) by cotton gauze (C) soaked in the electrolyte. It is then rolled up to a cylindrical form. The oxide films on aluminium offer a low resistance to current in one direction and a very high resistance in the other direction. Hence an electrolytic capacitor must be placed in a DC circuit such that the potential of the oxide plate is always positive relative to the other plate. It is shown in Fig. 6.15

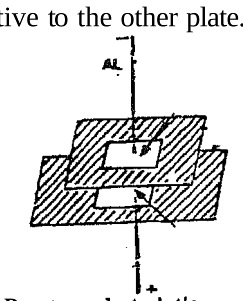
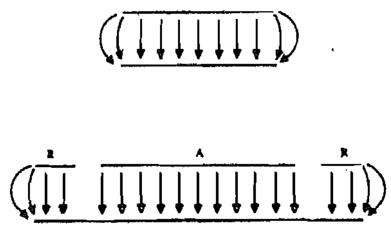


Fig. 6.15: Dry type electrolytic capncitor

6.8.4 Variable Air Capacitor/Gang Capacitor

A very common capacitor whose capacitance can be varied continuously is used for tuning in a radio station. The capacity of this capacitor can be uniformly varied by rotating a knob. (different forms of such a type of capacitor is shown in Fig. (6.16)).



Flg. 6.16: Variable air rnpncltor

The capacitor consists of two sets of semicircular aluminium plates. One set of plates is fixed and the other set of plates can be rotated by the knob. As it is rotated, the moving set of plates gradually gets into (or comes out of)the interspace between the fixed set. The area of overlap between the two sets of plates can thus be uniformly varied. This changes the capacitance of the capacitor. The air between the plates acts as the dielectric. Usually it consists of two condensers attached to the same knob (ganged). When the knob is rotated the variation of C in both the plate takes place simultaneously. This is widely used in wireless sets and electronic circuits. See Table 6.1 for a comparative range of voltages for different types of. condensers.

SAQ 9

What'is a variable capacitor? Give an example of a variable capacitor'with a solid dielectric.

6.8.5 Guard Ring Capacitor

In Section *6.2* we calculated the cpacitance of a parallel plate condenser. We neglected the nonuniformity of electric field at the edges. It is possible to get over the problem of edge effects by using a guard ring capacitor. In this capacitor a ring R is used around the upper plates of the parallel plate capacitor. This is shown in Fig. 6.17.

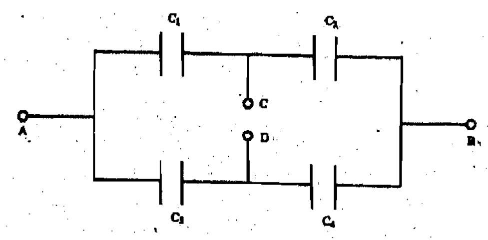


Fig. 6.17: Guard ring capacitor.

The inner diameter of the ring R is slightly larger than the diameter of the capacitor plate A. The diameter of the other capacitor plate B is equal to the outher diameter of the ring. Now the edge effects are absent as far as the plates A and B are concerned. In estimating the capacitance of the guard ring capacitor, we take the effective area of the plates as equal to the sum of the area of the plate A and half the area of the gap between A and R.

In Table 6.1, the capacity range, max. rating voltage and use of different types of capacitors are shown.

Table 6.1

Type of Dielectric	Capacitance Range	Max. Rating Voltage	Remarks
Paper	. 250PF-10 μF	150 KV	Cheap, used in circuits where losses are not important.
Mica	25PF-.25 μ.F	2 KV	High quality, used in low circuit
Ceramic	0.5PF-0.01 μF	500 KV	High quality used in low loss precision circuit where miniaturisation is important.
Electrolytic (Aluminium Oxide) .	1μ.F-1000 μF	600 V atsmall capacitance	Used where large capacitance is needed.

6.9 SUMMARY

- Any device which can store charges is a capacitor. The capacity of capacitor is given by

$$C = \frac{Q}{\phi} = \frac{\varepsilon_0 A}{d}$$

Where the symbols have their usual meaning.

- The energy stored in a capacitor is given by

$$W = \frac{1}{2} C\phi^2 = Q^2/2C \text{ Joules}$$

The symbols have their usual meanings.

- If you introduce an insulator of thickness 't' between the two plates of a capacitor, then the resultant capacity is given by

$$C = \varepsilon_0 A/(d - t + t/\varepsilon_0)$$

- The maximum'safe voltage is called rating voltage of a capacitor.
- The capacitance of a cylindrical capacitor, per unit length is given by

$$= 2\pi\varepsilon_0\varepsilon_r/\ln(b/a)$$

- If two capacitors C_1 and C_2 are connected in series, then the resultant capacity is given by

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

- The resultant capacity of two capacitors C_1 and C_2, when connected in parallel is given by

- The energy stored in a dielectric medium is given by

$$\frac{1}{2}\,\mathbf{E}.\mathbf{D}.$$

- Practical capacitors are made in different ways, to suit the particular applications. Layers of conducting foil and paper rolled up give a cheap form of capacitor. Mica and metal foil stands high electric field but are more expensive. Electrolytic capacitors, in which the dielectric is a very thin oxide film deposited electrolytically, give very large capacitance. Ceramic capacitors are useful in transistor circuits where voltages are low but small size and compactness are very desirable.

Terminal Questions

1) A capacitor has n similar plates at equal spacing, with the alternate plates connected together. Show that its capacitance is equal to $(n-1)\,\varepsilon_0\,\varepsilon_r\,A/D$.

2) What potential would be necessary between the parallel plates of a capacitor separated by a distance of 0.5cm in order that the gravitational force on a proton would be balanced by the electric field? Mass of proton = 1.67×10^{-27} kg.

3) A capacitor is made of two hollow concentric metal spheres of radii a and b (b>a). The outer sphere is earthed. See Fig. 6.18. Find the capacity.

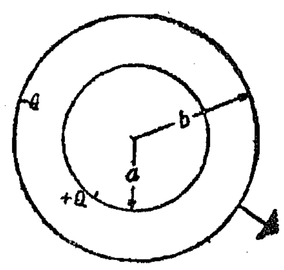


Fig. 6.18

4) In the arrangement shown in Fig. 6.19, find the values of the capacitances such that when a voltage is applied between the terminals A and B no voltage difference is set up between terminals C and D.

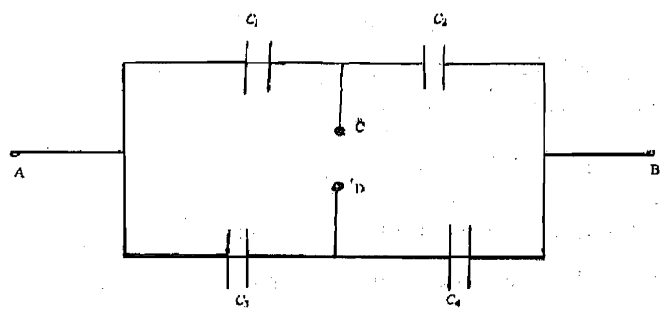


Fig. 6.19

5) Two capacitors one charged and the other uncharged are joined in parallel. Show that the final energy is less than the initial energy and derive the formula for the loss of energy in terms of the initial charges and the capacitances of the two capacitors.

Answer's of SAQ's

1) The potential difference (V) between the plates is not changed. But the electric field between the plates is $V/(d/2) = 2(V/d)$ = twice the value of the electric field E. The doubling of the electric field doubles the charge on each plate. Therefore, $C = (Q/V)$ also doubles. Thus if we halve the distance of separation between the plates, the capacitance doubles.

2) We know that

$$C = Q/V$$

$$C = 1000\ \mu F$$

$$= \frac{1000}{10^6} F = 1000 \times 10^{-6} F$$

$$= .001\ F$$

and V = 24 V

$$Q = CV = .001 \times 24 \text{ Coul} = .024 \text{ Coul.}$$

3) The energy stored in a capacitor is

$$W = \frac{1}{2} C\phi^2$$

It can be written

$$W = \frac{1}{2} C\phi \,.\, \phi \qquad \text{(i)}$$

We know that

$$Q = C\phi \qquad \text{(ii)}$$

Using Eq. (ii) in Eq. (i), we get

$$W = \frac{1}{2} Q\phi$$

Hence prove, the result.

4) $C = \varepsilon_0 A/d$

Here

$$\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$$

$$A = 4 \times 10^{-2} m^2$$

$$d = 10^3 m$$

Therefore

$$C = \frac{8.85 \times 10^{-12} \times 4 \times 10^{-2}}{10^{-3}} = 3.54 \times 10^{-10}\ F$$

Here C is the charge that raises the potential by unity or the charge holding capacity.

5) We have

$$\varepsilon_r = \frac{\text{Capacitance with the dielectric}}{\text{capacitance with free space}}$$

Here $\varepsilon_r = 3$. Thus the capacitance of the capacitor will get trebled when the dielectric ($\varepsilon_r = 3$) is filled up in all the air space.

Now a dielectric material is introduced. Let its thickness be t. The capacity of the capacitance is

$$C_{air} = \frac{\varepsilon_r A}{d},\ C_{dielectric} = \frac{\varepsilon_0 A}{(d - t + t/\varepsilon_r)}$$

$$\frac{C_{dielectric}}{C_{air}} = \frac{[\varepsilon_0 A/(d - t + t/\varepsilon_r)] \times d}{\varepsilon_0 A}$$

$$= \frac{d}{(d - t + t / \varepsilon_r)}$$

Here $t = \frac{3}{4}d$ and $\varepsilon_r = 3$

Therefore,

$$d - t + t / \varepsilon_r = d - \frac{3}{4}d + \frac{3d}{4 \times 3} = \frac{d}{2}$$

Therefore

$$\frac{C_{dielectric}}{C_{air}} = \frac{d}{\frac{d}{2}} = 2$$

That is, the capacitance will get doubled.

6) $C_1 = 2\pi\varepsilon_0\varepsilon_r/\ln(10/8)$

and

$C_2 = 2\pi\varepsilon_0\varepsilon_r/\ln(5/4)$

$C_1/C_2 = \ln(5/4)/(10/8) = 1$

or $C_1 = C_2$

7) When the capacitor are connected in series, the equilent is given by

$$\frac{1}{C_T} = \frac{1}{.05} + \frac{1}{.02} + \frac{1}{.10} + \frac{1}{.05}$$

$C_T = .01\mu F$

$Q = C_T V$

$= 220 \times .01 \times 10^{-6}$

$= 2.2 \times 10^{-6}$ Coul

$$V_1 = \frac{Q}{C_1} = \frac{2.2 \times 10^{-2}}{.05 \times 10^{-2}} = 44\text{V}$$

$$V_2 = \frac{Q}{C_1} = \frac{2.2 \times 10^{-6}}{.02 \times 10^{-6}} = 110\text{ V}$$

$$V_3 = \frac{Q}{C_3} = \frac{2.2 \times 10^{-6}}{.01 \times 10^{-6}} = 22\text{V}$$

8) The arrangement is shown in Fig. *6.20*. Let C_4 *be* the effective capacitance of C_1 and C_2. Using series law of capacitors

$$\frac{1}{C_4} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

$$C_4 = C_1 C_2 / (C_1 + C_2)$$

This capacitance C_4 then adds to C_3 to give the total capacitance C of the combination i.e.,

$$C = C_4 + C_3$$

or

$$C = C_3 + \frac{C_1C_2}{(C_1 + C_2)}$$

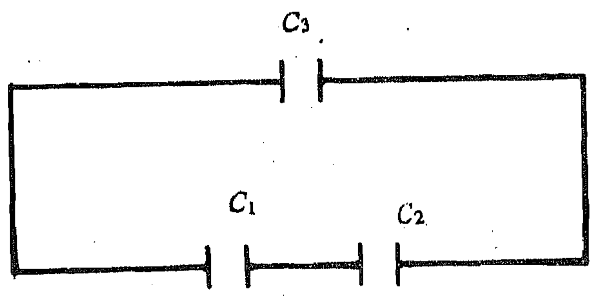


Fig. 6.20

Answer's of TQ's

1) As seen from Figure 6.21, n plates provide $(n\text{-}1)$ capacitors connected in parallel. The effective capacitance of $(n\text{-}1)$ capacitors, of equal capacitance in parallel is

= sum of the individual capacitance

= (n-1) * capacitance of a single unit

= $(n-1)\ \varepsilon_r\ \varepsilon_0\ A/d$.

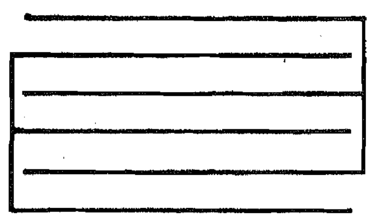


Fig. 6.21

For example in Fig. 6.21 the first 3 plates A, B, C give two capacitors AB & CB and so on.

2) Let the required potential be equal to ϕ. Then E = ϕ/d = $15 x 10^{-3} volt/m.

Electrostatic force on proton = qE

$= 2 \times 10^2\ \phi/q$

$= 2 \times 10^2 \times 1.6 \times 10^{-19}\ \phi$ Newtons.

gravitational force = $1.67 \times 10^{-27} \times 9.8$ Newtons. Equating the two we get

$$\phi = \frac{1.67 \times 10^{-27} \times 9.8}{2 \times 10^{-2} \times 1.6 \times 10^{-19}}$$

$$\phi = 5 \times 10^{-10} \text{ volts.}$$

3) If a charge q is placed on the inner sphere of radius 'a' an equal and opposite amount of charge appears on the inner side of the outer sphere, The electric field gets confined to the space between the concentric spheres. To evaluate E consider a Gaussian surface. The symmetry of the problem suggests a concentric sphere of radius r as Gaussian surface. The electric field, E, is normal to this surface and so the flux of E is given by

$$4\pi r^2 E = \frac{q}{\varepsilon_p}$$

The potential of the inner sphere with reference to the outer sphere at zero potential is equal to

$$\phi_a - \phi_b = -\int_b^a \frac{q\,dr}{4\pi r^2 \varepsilon_0}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_b}\right]_b^a$$

$$\phi_a = \frac{4}{4\pi\varepsilon_p}\left(\frac{1}{a} - \frac{1}{b}\right) \quad \text{since } \phi_b = 0$$

Hence the capacitance is given by

$$\mathbf{?} = \frac{q}{\phi} = \frac{4\pi\varepsilon_0 ba}{(b-a)}$$

4) The potentials of the two plates connected to the point C are the same. Hence if a charge q_1 is placed on one of these plates the other plate will have an equal and opposite charge. When a voltage is applied between A and B let a charge q_1 accumulate on C_1 and a charge q_2 on C_2. Then the potential difference (p.d) between the plates of various condensers are given by

$$\frac{q_1}{C_1}, \frac{q_1}{C_2}, \frac{q_2}{C_3} \text{ and } \frac{q_2}{C_4}$$

Now

$$\frac{q_1}{C_1} + \frac{q_1}{C_2} = \frac{q_2}{C3} + \frac{42}{C_4} = \text{p.d across AB}$$

It the p.d between C and D is equal to zero

$$\frac{41}{C_2} = \frac{q_2}{C_4} \text{ and } \frac{q_1}{C1} = \frac{q_2}{C_3}$$

which on elimination gives

$$\frac{C_2}{C_4} = \frac{C_1}{C_3}$$

is the required condition for zero potential difference between C and D.

5) Let the initial charge on the capacitor of capacitance C_1 be equal to q. When this capacitor is joined to the uncharged capacitor of capacitance C_2 then the charge q distributes in such a way that the potentials are equal as the combination is a parallel one. Let a charge q_2 flow from the charged capacitor to the uncharged one. The charge which remains on the initially charged capacitor as a result of sharing of charges is then equal to $q - q_2$. As the potentials are equal

$$\frac{q - q_2}{C_1} = \frac{q_2}{C_2}$$

$$\therefore \; q_2 = \frac{C_2 q}{(C_1 + C_2)}, \; q - q_2 = \frac{C_1 q}{C_1 + C_2} = q_1$$

Initial energy E of the charged capacitor (before sharing of charges) is

$$E_{\text{initial}} = \frac{1}{2}\frac{q^2}{C_1} = E_1$$

Final energy E_f of the 'two capacitors is given by

$$E_f = \frac{1}{2}\,\frac{(q - q_2)}{C_1.} + \frac{1}{2}\,\frac{q_2}{C_2}$$

Substituting for q_2 from **above**

$$= \frac{1}{2}\,\frac{C_1 q^2}{(C_1 + C_2)^2} + \frac{1}{2}\,\frac{C_2 q^2}{(C_1 + C_2)^2}$$

$$= \frac{q^2}{2(C_1 + C_2)}$$

Hence the loss in energy = $\frac{q^2}{2}\,\frac{1}{C_1} - \frac{1}{C_1 + C_2} = \frac{q^2 C_2}{2C_1(C_1 + C_2)}$