# UNIT 1 ELECTRIC CHARGE, FORCE AND FIELD

**Structure**



## 1.1 INTRODUCTION

During hot humid days, you might have observed that rain is often accompanied by lightning and thunder. Do you know the cause of the lightning and thunder? Benjamin Franklin was the first to prove through his experiment that roaring clouds possess electric charge. These charged clouds, when discharged in the atmosphere, give rise to a giant spark. This spark is called lightning. Can you imagine that the amount of current during discharge of the cloud is about 20,000 amperes!

When there is lightning due to an electric discharge, a great amount of heat is produced. During one millionth of a second, the temperature rises to 15000°C which is about two and a half times the temperature of the sun. The lightning flash develops in a zone 20cm wide. Due to excessive heat produced in this zone, the air molecules move very fast and cause sound. This is called thunder. When this sound is reflected by clouds or hills or any other obstacle you hear the roaring of clouds.

This spectacular event of nature – which has drawn the attention of most ancients – is associated with charging and discharging phenomena. The most noticeable thing about electric charges is that the forces between them are extremely large. This force, known as electrostatic force (or electric force), is responsible for holding electrons to nuclei to form atoms and for holding the groups of atoms together to form molecules, solids and liquids. In this unit, you will learn about the nature of charges and the electrostatic force between them. Around every charged body, there is a region where the electric force can be detected. This region is called electric field. You will learn to calculate the electric field due to different charge configurations. In the next unit, a mare easy and elegant method will be used to determine the electric field due to various charge distributions.

## Objectives

After studying this unit, you should be able to:

- distinguish between the two types of electric charge,
- show that the total electric charge in an isolated system is conserved,
- infer that any electric charge is always an integral multiple of the charge on the electron,
- use Coulomb's law to find the electrostatic force between two charges,
- state the principle of superposition of forces and calculate the resultant force due to more than two charges,
- calculate the resultant electric field due to an arbitrary distribution of charges, and
- draw the electric lines of force.

## 1.2 PROPERTIES OF ELECTRIC CHARGE

The word 'electricity' or 'electric' is derived from the Greek word **'elecktron'** which means amber. In about 600 B.C., the Greek philosopher Thales discowered **that** when amber (a natural resin) is rubbed with fur, the amber becomes capable of attracting small particles of matter. This discovery did not attract much attention until 1600 A.D. William Gilbert showed that many substances, such as glass, ebonite **and resin,** when rubbed with silk, flannel or other suitable materials acquire similar property as the rubbed amber. The substances in such a state are said to be electrified or to **have** acquired electric charges or are simply referred to as charged bodies. This section will be a quick recapitulation of what you have learnt so far in your school.

### 1.2.1 Types of Charges

Consider two pith balls suspended by a metal thread from metallic supports at a, short distance. The metal wire connects the two supports. A piece of rubber is taken and it is rubbed with a piece of glass rod so that both of them get charged. When the charged rubber is touched to the metal support, the charge distributes itself over the pith balls. As a result, the two balls move apart as shown in Fig. **1.1a.** Next, when the charged glass rod is touched to the metal support the balls **again** move apart as in Fig. 1.1b. But when the metallic connection between the pith balls is removed as shown in Fig. 1.1c, and if one of the supports is touched. with charged glass rod while the other with charged rubber, then the two balls move towards each other.

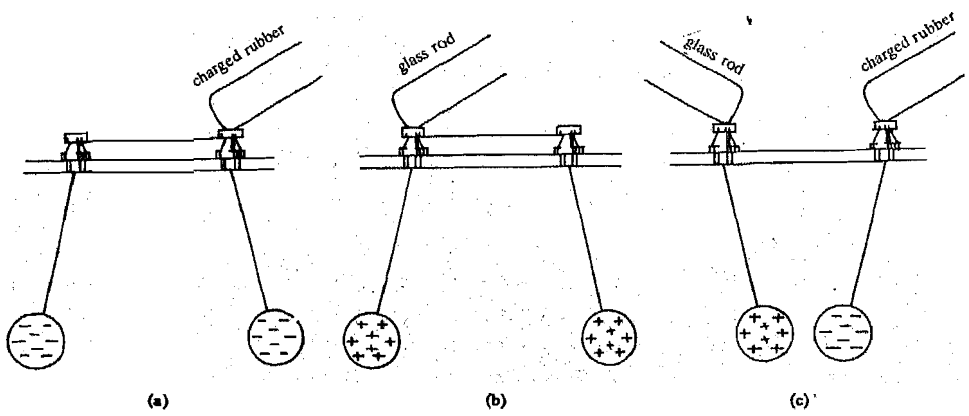


**Fig. 1.1: Pith ball demonstration of like charges repelling and unlike charges attracting.**
**(a) Repulsion produced between two balls when charged rubber is touched to the metal support.**
**(b) Repulsion produced between two balls when charged glass rod is touched to the metal support. In (a) and (b) a metal wire connects the two supports.**
**(c) Attraction produced between two balls when the metallic connection between the pith balls is removed and one is touched with charged rubber and the other with charged glass rod.**

What do you infer from these observations? In case (a), both the balls acquire the same kind of charge and move apart. In case (b), the two balls again acquire the same kind of charge and move apart.

In case (c), the two balls do not acquire the same kind of charge otherwise the balls would have repelled as happened in the earlier two cases. It means that the charge on the ball suspended from that support which is being touched by glass rod is different from the charge on the ball suspended from the support which is being touched by rubber. In other words, there are only two types of electric charge. Bodies carrying the same kind of charge repel one another, whereas bodies carrying different types of charge attract one another. The origin of the two types of charges is explained as follows :

You all know from your previous classes that an atom consists of a positively charged nucleus with negatively charged electrons around it. The nucleus consists of protons and neutrons. The neutron is uncharged (neutral) while the electron and proton have equal but opposite charges (negative and positive respectively). As the prqtons and neutrons are in the nucleus, they are held together very tightly by a nuclear force. This force is so strong that protons are unable to move away from the atomic nucleus, whereas the force holding the electrons to the atomic nucleus is much weaker. So that the electrons are more free to move away from the atom as compared to protons.

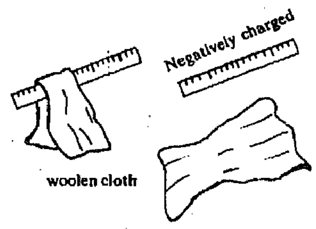


**Fig. 1.2: Charging by friction. When a plastic ruler is rubbed with a woollen cloth, electrons flow from the wool to the plastic. This process will leave an excess of electrons on the plastic, so that it carries a net negative charge, whereas the wool, with a deficit of electrons, carries a positive charge of equal magnitude.**

When two different materials are brought into contact and rubbed together, the electrons (being more free) get transferred from one material to the other. Since some materials tend to hold their electrons more strongly than others, the direction of transfer of electrons depends on the materials concerned and is always the same for any two materials. For example, when a plastic ruler is rubbed with a woollen cloth, as shown in Fig. 1.2, electrons flow from wool to plastic. This process will leave an excess of electrons on the plastic, so that it carries a net negative charge, whereas the wool, with a deficit of electrons, carries a positive charge of equal magnitude. In a similar way, amber or ebonite or rubber rod when rubbed with wool or fur acquire negative charge whereas wool or fur becomes positively charged. This method of charging the bodies by means of rubbing them together is called **charging** by friction (though friction actually has nothing to do with the charging process). Table 1.1 lists some materials in the triboelectric series (tribo means friction), which ranks materials according to their tendency to give up their electrons. Materials towards the top of the list become positively charged when placed in contact and rubbed with those lower on the list:

**Table 1.1**

| | |
|---|---|
| | rabit fur |
| | glass |
| | wool |
| | cat's fur |
| The triboelectric series: | silk |
| | felt |
| | cotton |
| | wood |
| | cork |
| | rubber |
| | celluloid |

**SAQ 1**

We have two charged bodies X and Y which attract each other. X repels a third charged body *Z*. Will *Z* attract or repel Y?

### 1.2.2 Unit of Charge

In the Systeme *Internationale* (SI), the unit of charge is Coulomb (abbreviated C) which is defined in terms of ampere. You all must be familiar with the definition of the ampere which is as follows:

"An ampere is the current, which when maintained in two straight parallel wires placed one metre apart in vacuum, would produce between these wires a force equal to $2 \times 10^{-7}$ N per metre of length."

The definition of ampere involves force between currents, which we shall discuss later in Block 3.

Using ampere, the unit of charge is defined as:

"A Coulomb is the amount of charge that flows through any cross-section of a wire in one second if there is a steady current of one ampere in the wire." In symbols,

$$q = It \qquad ...(1.1)$$

where q is in Coulombs, if $I$ is in amperes and t is in seconds.

The reason for defining Coulomb in terms of ampere is that it is easy to maintain; control and measure a current through a conductor rather than the amount of charge.

### 1.2.3 Conservation of Charge

In the rubbing process of a plastic ruler by woollen cloth as shown in Fig. 1.2, no new charges are created. The algebraic sum of the individual charges, i.e., net charge always remains constant. Let us see how? Before the process of rubbing, both the plastic and wool are neutral (having no charge). So the net charge is zero. After rubbing, the plastic ruler gets negatively charged and the woollen cloth acquires positive charge of equal magnitude. Now the algebraic sum of the equal and opposite charge on the plastic and wool is zero. So the net charge is again zero.

Here isolated means that no matter is allowed to cross the boundary of the system.

This shows that electric charge is a conserved quantity. Conservation of charge implies that the total charge in an isolated system never changes. It does not mean that the total amount of positive or negative charge in a system is fixed, rather it implies that for every additional positive charge created, there is always an equal amount of negative charge created. An example of conservation of electric charge is beautifully illustrated in the process known as 'pair production' as shown in Fig. 1.3. Here, a thin-walled box in a vacuum is an isolated system. When this box gets exposed to a gamma ray photon (carrying no charge), we find the existence of an electron and positron (a particle having same mass as electron but having a charge equal and opposite to that of an electron) inside the box. Although, two electrically charged particles have been newly created, but the net change in the total charge in the box is zero.

The charge conservation law may be stated as follow:

The total electric charge in an isolated system, that is, the algebraic sum of the positive and negative charge present at any time, never changes.

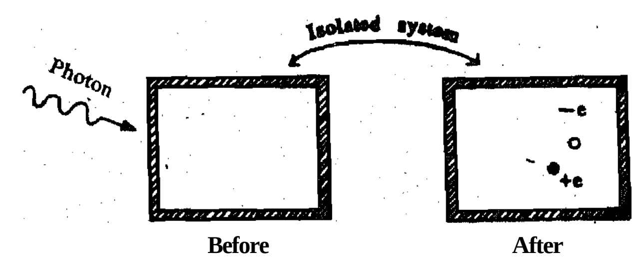


**Fig. 1.3: "Par production" exhibiting conservation of charge. In this process, a gamma ray photon is converted into an electron (with a negative charge denoted as -e) and a positron (a particle with the same mass as an electron but with charge +e).**

**SAQ 2**

Complete the following equations using the principle of conservation of charge. The notation for writing these equations is : $_ZX^A$ where X represents the chemical symbol of an element, Z is the atomic number (number of electrons) and A is the mass number (number of protons + number of neutrons).

(i) $_{92}U^{238} \rightarrow {}_{?}Th^{234} + {}_{2}He^{4}$ (radioactive decay)

(ii) $_{20}Ca^{44} + {}_{1}p^{1} \rightarrow {}_{21}Sc^{44} + {}_{?}n^{1}$ (nuclear reaction)

### 1.2.4 Quantization of Charge

The **smallest** charge that is **possible** to obtain is that of an electron or proton. (Both electron and proton have the same magnitude of charge but electron is negatively charged while proton is positively charged). The magnitude of this charge is denoted by '$e$'. It was first measured by **Millikan** in his famous **oil-drop** experiment by observing the motion of a charged oil drop under the combined influence of **gravitational** field and externally applied electric field.

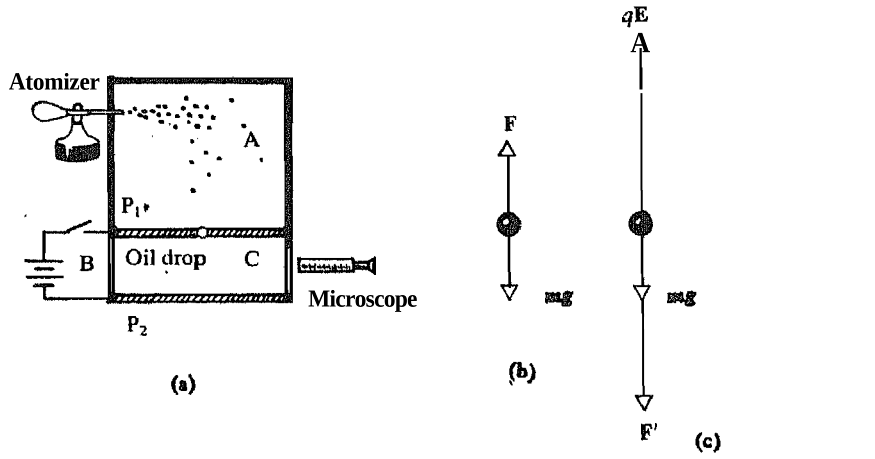


**Fig. 1.4: (a) The Millikan oil-drop apparatus for measuring the elementary charge e. (b) An oil drop falls at terminal speed v in a field-free region. Its weight is balanced by an upward drag force. (c) An electric field force acts upward on the drop, which now rises with a terminal speed $v'$. The drag force, which always opposes the velocity. now acts downward.**

Fig. **1.4a** shows the apparatus for the measurement. The oil droplets introduced by the atomizer in the chamber A are either positively or negatively charged. Let us consider a drop in the chamber $C$ which has got into it through a small hole in plate $P_1$.

In-the absence of the electric field, two forces act on the drop: its weight $mg$ and an upwardly directed viscous force **F** as shown in Fig. **1.4b**. The magnitude of **F** is proportional to the speed of the falling drop. The drop acquires a constant terminal speed $v$ when the gravitational force gets just balanced by the viscous force. In the **presence** of electric field, a third force $q\mathbf{E}$ acts on the **drop**. IT q is negative, this force will act upward and the drop will now move upward. The **new** drag force will act downward **as** it has to point in the direction opposite to **that in** which the drop is moving. As shown in Fig. **1.4c,** when the upward electric force $q\mathbf{E}$ is just balanced by the weight mg and the new drag force $\mathbf{F}'$, the drop acquires a new terminal speed $v'$. By measuring $v$ **and** $v'$, the charge $q$ is found. Millikan made observations on a large number of **drops** and found that charges on different drops were an integral multiple of a number which is $1.6 \times 10^{-19}$**C, i.e., electronic** charge. In fact, a charge smaller than e has not been found (**see** margin remark on quarks). **If** one determines the amount of charge on any charged body (**like** a charged sphere or charged drop) or any charged particle (like positron, **α-particle)** **or** any ion, then its charge is always found to be an integral multiple sf $e$, **i.e.,** e, **2$e$, 3e**; $4e$,...... No Charge will be fractional multiple of e like $0.7e$ or $2.5e$. This is not only true for negative charges but also for positive charges. Mathematically, it can be expressed as:

$$q = ne \quad \text{...(1.23}$$

where n is an positive or negative integer. Thus, the charge exists in discrete packets rather than in continuous amounts, Whenever a physical quantity possesses discrete values **instead** of continuous values, then that physical quantity is said to be quantized. Hence, charge is 'quantized'.

The existence of charged **particles called**, quarks, whose **electric** charges **come in multiples of $e/3$,** would **not alter the fact that charge is** quantized– **it would** merely **reduce the size of the** basic unit **from** $e$ **to $e/3$.**

**SAQ 3**

A conductor possesses $3.2 \times 10^{-17}$ Coulomb positive charge. How many electrons does it have in excess or deficit?

## 1.3 COULOMB'S LAW

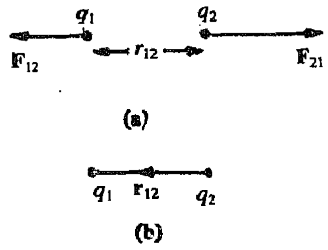


**Fig.** 1.5: Quantities and forces involved in Coulomb's Law.
(a) $\mathbf{F}_{12}$ is the electrostatic force on charge $q_1$ due to $q_2$. The separation between the charges is $r_{12}$. The unit vector $\hat{\mathbf{r}}_{12}$ conveys directional information only, allowing the law to be written in vector notation. $\mathbf{F}_{21}$ is the force that charge $q_1$ exerts on $q_2$.
(b) $\mathbf{r}_{12}$ is a vector originating from the position of $q_2$ and ending at the position of $q_1$. The distance between the positions of these two charges represents its magnitude.

The unit vectors along the positive $x$, y and z-axes are denoted by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ respectively.

We have already seen in the last section that like charges repel while unlike charges attract one another. The quantitative study of electrostatic force of attraction or repulsion between two point charges at rest was first done by a French physicist Charles **Augustin** de Coulomb in 1785. He observed that electrostatic force depends on the magnitude of charges. Specifically, it is proportional to their product. The force also depends on the separation between the charges. It is inversely proportional to the square of the separation between them. Here the distance between the charges is large compared to their dimensions so that the charges are treated as point charges.

To express this mathematically, suppose we have two point charges $q_1$ and $q_2$ placed at a distance $r_{12}$ as shown in Fig. 1.5. According to Coulomb's law, the force, $\mathbf{F}_{12}$, acting on the charge $q_1$ due to the presence of $q_2$ can be written as

$$|\mathbf{F}_{12}| \propto \frac{q_1 q_2}{r_{12}^2} \qquad \text{...(1.3)}$$

where $|\mathbf{F}_{12}|$ represents the magnitude of force $\mathbf{F}_{12}$ and $r_{12}$ denotes the distance between the point charges. We **should** be able to write Eq. (1.3) as vector equation, since it involves force, which is a vector quantity. In order to indicate that the right hand side of the equation is also a vector quantity, we introduce the unit vector $\hat{\mathbf{r}}_{12}$ (read as $\mathbf{r}_{12}$ cap) which is a vector of unit magnitude and has the direction of the vector joining the position of charge 2 to charge 1.

**Mathematically,** if $\mathbf{r}_{12}$ denotes the vector originating from position of charge 2 and ending at the position of charge 1, then,

$$\hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|}$$

where $|\mathbf{r}_{12}|$ is the magnitude of the vector $\mathbf{r}_{12}$ or the distance between the charges. With this unit vector notation, we rewrite Eq. (1.3) as follows:

$$\boxed{\mathbf{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{\mathbf{r}}_{12}} \qquad \text{...(1.4)}$$

The constant of proportionality is normally written as $1/4\pi\epsilon_0$ where $\epsilon_0$ is called the permittivity of free space. The value of $1/4\pi\epsilon_0$ in SI system is $9 \times 10^9$ N $m^2$ $C^{-2}$. The constant of proportionality depends upon the system of units. In another commonly used system called CGS, the constant is set equal to unity. However, we **shall** not employ the CGS system here.

**SAQ 4**

Write down the equation similar to Eq. (1.4) for the force $\mathbf{F}_{21}$.

The proportionality constant introduced as $1/4\pi\epsilon_0$ has important physical significance. If the charges are placed in different medium, it is found that Eq. (1.4) always holds except that the constant of proportionality (permittivity) varies from medium to **medium.** It is found that the maximum electrostatic force between two charges separated by a fixed distance is obtained when two charges are placed in **vacuum** and decreases when the charges are placed in any other medium, We can infer that the permittivity of free space is minimum. The ratio ,of **permittivities** for a medium to that of vacuum is known as **dielectric constant** or **specific inductive capacitance.** This ratio for air is about 1.005. For electrostatic experiment done in a medium, Coulomb's law may be written as

$$\mathbf{F}_{12} = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r_{12}^2} \hat{\mathbf{r}}_{12} \qquad \text{...(1.5)}$$

where $\epsilon$ is the permittivity of the medium, Let us now solve a problem using Coulomb's law.

Example 1

A charge $q_1 = 5.0\mu C$ is placed 30cm to the west of another charge $q_2 = -12\mu C$. What is the force exerted by the positive charge on the negative charge? Also calculate the force experienced by the positive charge due to negative charge.

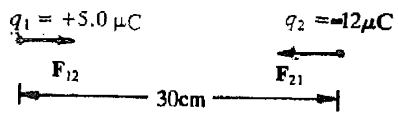


Fig. 1.6: Example 1

**Solution**

Refer to Fig. 1.6, Coulomb's law gives force on negative charge due to positive charge as follows :

$$\mathbf{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{21}$$

$$= \frac{(9\times 10^9\ \mathrm{N\,m^2\,C^{-2}})\ (5\times 10^{-6}\ \mathrm{C})\ (-12\times 10^{-6}\ \mathrm{C})}{(0.30\mathrm{m})^2}\ \hat{\mathbf{i}}$$

$$= -\ 6\,\hat{\mathbf{i}}\mathrm{N},$$

where we write $\hat{\mathbf{i}}$ for $\hat{\mathbf{r}}_{21}$ because a unit vector pointing from the positive charge $q_1$ towards the negative charge $q_2$ is in the positive x-direction. The minus sign shows that the force is actually in the negative x–direction or towards west, that is, attractive.

The force on positive charge due to the negative charge is:

$$\mathbf{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{12}$$

$$= \frac{(9\times 10^9\ \mathrm{N\,m^2\,C^{-2}})\ (5\times 10^{-6}\ \mathrm{C})\ (-12\times 10^{-6}\ \mathrm{C})}{(0.30\mathrm{m})^2}\ (-\hat{\mathbf{i}})$$

$$= 6\hat{\mathbf{i}}\mathrm{N}.$$

Here the unit vector $\hat{\mathbf{r}}_{12}$ becomes $-\hat{\mathbf{i}}$ because a unit vector from negative charge $q_2$ towards positive charge $q_1$ is in the negative $x$-direction. The two minus signs multiply to a plus sign showing that force is in positive x-direction or towards east, that is, attractive.

Thus Newton's third law is explicitly satisfied, i.e., the two charges exert equal but opposite forces on each other.

**SAQ 5**

Hydrogen atom consists of an electron and a proton separated by an average distance of $5.3 \times 10^{-11}$m. Find the electrical force between the electron and proton and compare it with gravitational force acting between them. (Charge of electron $= 1.6 \times 10^{-19}$ C, mass of electron $= 9.1 \times 10^{-31}$ kg, mass of proton $= 1.7 \times 10^{-27}$ kg, G $= 6.7 \times 10^{-11}$ N m$^2$ kg$^{-2}$, and $1/4\pi\epsilon_0 = 9 \times 10^9$ N m$^2$ C$^{-2}$.)

**SAQ 6**

Two point charges $Q_1$ and $Q_2$ are 3m apart and their combined charge is $20\mu$C. If one repels the other with a force of 0.075N, what are the two charges?

## 1.4 PRINCIPLE OF SUPERPOSITION

In Section 1.3, we had considered the forces (electrostatic) between two point charges. Suppose we have more than two charges, say three charges $q_1$, $q_2$ and $q_3$, placed as shown in Fig. 1.7.

Then how do we calculate the electrostatic force on any charge say $q_1$ due to the presence of other two charges. We can still calculate the force between different pair of charges by making use of Coulomb's law, The total force on $q_1$ will be the vector sum of forces on $q_1$ due to $q_2$ and $q_3$ independently. This is the principal of

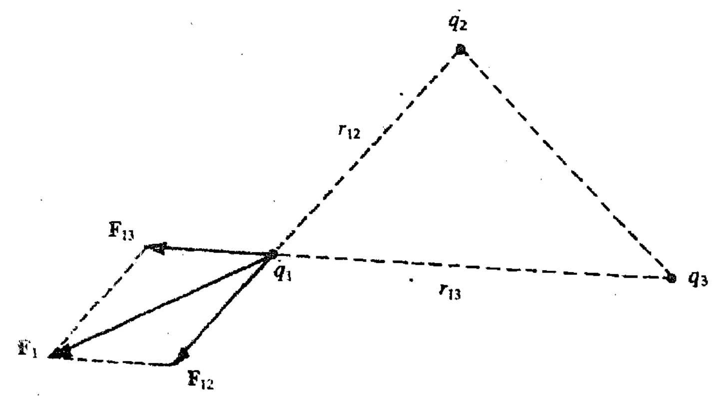


Fig. 7.7: Illustrating the principle of superposition. Charges $q_2$ and $q_3$ are at a distance of $r_{12}$ and $r_{13}$ respectively from the charge $q_1$. $\mathbf{F}_{12}$ is the force on $q_1$ due to the charge $q_2$. Because $q_1$ and $q_2$ are like charges, the force will be repulsive and it will act away from $q_2$ along the line joining $q_2$ and $q_1$. Similarly, the force $\mathbf{F}_{13}$ on $q_1$ due to $q_3$ will act along the line joining $q_3$ and $q_1$ and away from $q_3$. According to principle of superposition, the resultant force $\mathbf{F}_1$ on $q_1$ is the vector sum of the forces $\mathbf{F}_{12}$ and $\mathbf{F}_{13}$ which may be found by drawing a parallelogram of forces.

superposition. The fact that electric forces add vectorially is known as **superposition principle.**

Thus, the total force $\mathbf{F}_1$ on charge $q_1$, due to charges $q_2$ and $q_3$ at distances $r_{12}$ and $r_{13}$ respectively, will be given by $\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13}$

$$\therefore \mathbf{F}_1 = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\,\hat{\mathbf{r}}_{12} + \frac{q_1 q_3}{r_{13}^2}\,\hat{\mathbf{r}}_{13}\right] \qquad ...(1.6)$$

Here $\mathbf{F}_{12}$ is the force acting on $q_1$ due to $q_2$ and $\mathbf{F}_{13}$ is the force acting on $q_1$ due to $q_3$. The unit vector $\hat{\mathbf{r}}_{12}$ and $\hat{\mathbf{r}}_{13}$ have the directions of the lines from $q_2$ to $q_1$ and $q_3$ to $q_1$ respectively. To illustrate this principle let us find the solution of the following example.

**Example 2**

In Fig. 1.8, $q_1 = -1.0\mu C$, $q_2 = 2.0\mu C$ and $q_3 = 4.0\mu C$. Find the electrostatic force on $q_1$ owing to other two charges. Here $r_{12} = 10cm$ and $r_{13} = 20cm$. Express your result both in unit vector notation and as a magnitude and direction.

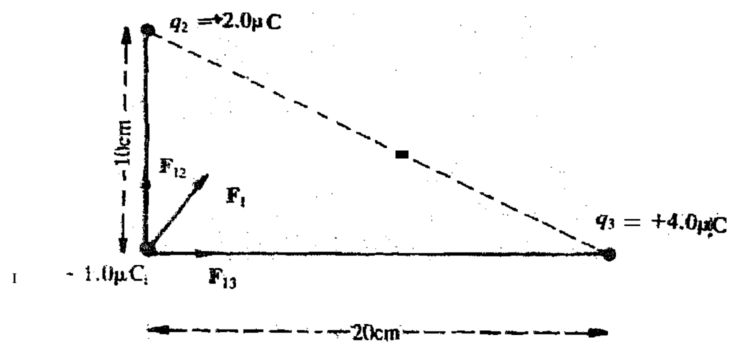


**Fig. 1.8** Charges of -1.0μC, + 2.0μC and + 4.0μC are located at the corners of a rightangle triangle.

**Solution**

This problem can be solved using the superposition principle. The force on $q_1$ due to the charge $q_2$ is given by:

$$\mathbf{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\,\hat{\mathbf{r}}_{12}$$

$$= \frac{(9\times10^{9}\ \mathrm{N\,m^2\,C^{-2}})\ (-1\times10^{-6}\mathrm{C})\ (2\times10^{-6}\mathrm{C})}{(0.10\mathrm{m})^2}\ (-\hat{\mathbf{j}})$$

$$= +\ 1.8\hat{\mathbf{j}}\ \mathrm{N}.$$

The unit vector 3 becomes $(-\hat{\mathbf{j}})$ because it points from $q_2$ to $q_1$ in the negative $y$-direction. Positive sign shows that force $\mathbf{F}_{12}$ is in positive y–direction, that is, attractive. Similarly, the force on $q_1$ due to $q_3$ is :

$$\mathbf{F}_{13} = \frac{1}{4\pi\epsilon_0}\ \frac{q_1\ q_3}{r_{13}^2}\ \hat{\mathbf{r}}_{13}$$

$$= \frac{(9\times10^{9}\ \mathrm{N\,m^2\,C^{-2}})\ (-1\times10^{-6}\mathrm{C})\ (4\times10^{-6}\mathrm{C})}{(0.20\mathrm{m})^2}\ (-\hat{\mathbf{i}})$$

$$= 0.90\hat{\mathbf{i}}\ \mathrm{N}.$$

Force $\mathbf{F}_{13}$ **is** in positive x–direction, that is, attractive.

According to the superposition principle, the force $\mathbf{F}_1$ acting on $q_1$ is the vector **sum** of the forces due to $q_2$ and $q_3$. That is,

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13}$$

$$\mathbf{F}_1 = (0.90\hat{\mathbf{i}} + 1.8\hat{\mathbf{j}})\ \mathrm{N}.$$

Magnitude of the force $\mathbf{F}_1$ is

$$\sqrt{(0.90)^2 + (1.8)^2} = 2.01\mathrm{N}.$$

and it makes an angle $\theta = \tan^{-1}\frac{1.8}{0.9} = \tan^{-1} 2 \approx 63.5°$ with positive **x-axis.**

**Answer**

$\mathbf{F}_1 = (0.90\hat{\mathbf{i}} + 1.8\hat{\mathbf{j}})\mathrm{N}$; $F_1 = 2.01\mathrm{N}$, making an angle of approx. 63.5° **with** positive x-axis.

Now you would like to work out an SAQ on superposition **principle.**

**SAQ 7**

If the individual forces acting on a given charge due to the presence of five charges are represented by the **sides** of a closed pentagon, what will be the **resultant** force on the test charge? (This is a tricky problem involving the principle of superposition; if the actual calculation **takes** more than 30 **seconds,** you are **doing wrong.)**

## 1.5 THE ELECTRIC FIELD

*An* electric field is a region in which the electric charges experience an electric force. By knowing the electric **field,** one can calculate the forces on electric charges and then, **via** Newton's law, one **can know** the motion of these charges. **Since all** matter contains **electrically charged** particles, an **understanding** of electric field **will** help to know the structure and **behaviour** of matter. Moreover, one **can** build devices in which electric **fields** accelerate charged **particles** in useful ways. For example, in your television **sets,** the electric fields in the TV tube accelerate electrons toward the front of the tube where their energy is **converted** to the light that we **see.**

In this **section,** you will **learn** to calculate **electric** field due to a single point charge and due to simple charge distribution including continuous charge distribution. In order to **picturise** the entire electric *fields,* the concept of electric lines of force **will also** be introduced.

### 1.5.1 Calculating the Electric Field

We have learnt in Section 1.3 about the electrostatic force acting between two charges, say $q_1$ and $q_2$. If one of the charges, say $q_2$, is a unit charge, then the force exerted on this unit charge due to the presence of charge $q_1$ is defined as the electric field at the location of unit charge.

In other words, if the force experienced by a test charge q located at a point in the electric field be **F** then, according to definition, the electric field (also called electric field intensity) E at that point is given by

$$\mathbf{E} = \frac{\mathbf{F}}{q} \text{ or } \mathbf{F} = q\,\mathbf{E} \qquad ...(1.7)$$

In fact, to measure electric field in a given region, one has to introduce a test charge and measure the force on it. However, test charge exerts forces on the charges that produce the field, so it may change the configuration of these charges. So, in principle, the test charge should be so small as to have **no** appreciable effect on the charge configuration that produces the field.

Eq. (1.7) shows that the electric field is measured in Newtons coulomb-' ($NC^{-1}$). Since **F** is a vector quantity, **E** will also be a vector. If $q$ is positive, the electric field E has the same direction as the force acting on the charge. If q is negative, the direction of E is opposite to that of the force F.

In the case of a point charge, calculation of the electric field is particularly simple. We already know from Coulomb's law **that** if we place a point charge $q_1$ at a distance $r$ from another point charge q, the force on $q_1$ will be

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{qq_1}{r^2}\,\hat{\mathbf{r}} \qquad ...(1.8)$$

with $\hat{\mathbf{r}}$ a unit vector pointing from $q$ towards..the location of $q_1$. Since the electric field is defined as the force per unit charge, we divide the force in **Eq.** (1.8) by the charge $q_1$ to obtain the field due to q at the location of $q_1$. That is

$$\boxed{\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{\mathbf{r}} \text{ (field of a point charge)}} \qquad ...(1.9)$$

**This** equation gives the field arising due to the charge q at any location **which** is at a distance of $r$ from q. In **Eq. (1.9),** the unit vector $\hat{\mathbf{r}}$ points from the charge q (due to which an electric field exist) to the location at which the electric field is being determined.

Now, what would be the electric field due to two or more point charges? Since the electric force obeys the superposition principle, so does the electric field (since it is the force per unit charge). Therefore, the field at **a given** point due to two or more charges is the vector **sum** of the fields of individual charges. The fields of individual point charges is given by Eq. (1.9). Hence, the electric field E due to $n$ charges may be written as:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + ..... + \mathbf{E}_n = \sum_{j=1}^{n} \mathbf{E}_j = \frac{1}{4\pi\epsilon_0}\frac{q_j}{r_j^2}\,\hat{\mathbf{r}}_j \qquad ...(1.10)$$

where $\mathbf{E}_j$s are the electric fields due to the point **charges** $q_j$ that are located at distances $r_j$s from the point where we are evaluating the field.

Example 3

An electric field is set up by two point charges $q_1$ and $q_2$ such that $q_1 = -q_2 = 12 \times 10^{-9}$C and separated by distance of 0.1m as shown in Fig. 1.9. Find the electric field at the points marked as ***A*** and ***B***.

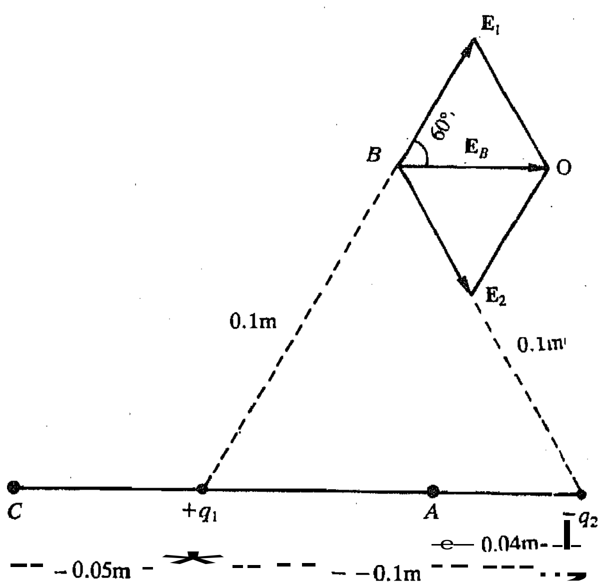


Fig. 1.9 : Example 3

i) At A, the electric field $\mathbf{E}_1$ due to $q_1$ is

$$\mathbf{E}_1 = (9\times10^9\ \mathrm{Nm^2\ C^{-2}})\ \frac{12\times10^{-9}\mathrm{C}}{(0.06\mathrm{m})^2}\ \hat{i}$$

$$= \frac{(9\times10^9\ \mathrm{N\,m^2\ C^{-2}})\times12\times10^{-9}\ \mathrm{C}}{36\times10^{-4}\ \mathrm{m^2}}\ \hat{i}$$

$$= 3\times10^4\ \hat{i}\,\mathrm{NC^{-1}}$$

At A, the electric field $\mathbf{E}_2$ due to $q_2$ is

$$\mathbf{E}_2 = (9\times10^9\ \mathrm{N\,m^2\ C^{-2}})\ \frac{(-12\times10^{-9}\ \mathrm{C})}{(0.04\mathrm{m})^2}\ (-\hat{i})$$

$$= \frac{(9\times10^9\ \mathrm{N\,m^2\ C^{-2}})\times12\times10^{-9}\ \mathrm{C}}{4\times10^{-4}\ \mathrm{m^2}}\ \hat{i}$$

$$= 6.75\times10^4\ \hat{i}\ \mathrm{NC^{-1}}$$

Therefore, net electric field $(\mathbf{E}_A)$ at $A$

$$\mathbf{E}_A = \mathbf{E}_1 + \mathbf{E}_2$$

$$= (3 + 6.75)\times10^4\ \hat{i}\ \mathrm{NC^{-1}}$$

$$= 9.75\times10^4\ \hat{i}\,\mathrm{NC^{-1}}.$$

ii) At B, the electric field $\mathbf{E}_1$ due to $q_1$ is

$$\mathbf{E}_1 = (9\times10^9\ \mathrm{N\,m^2C^{-2}})\ \frac{12\times10^{-9}\ \mathrm{C}}{(0.1\mathrm{m})^2}\ \hat{\mathbf{r}}_+.$$

(Here $\hat{\mathbf{r}}_+$ points diagonally upward to the right)

$$= 1.08\times10^4\ \hat{\mathbf{r}}_+\ \mathrm{NC^{-1}}$$

$\mathbf{E}_1$ is directed in the same direction as $\hat{\mathbf{r}}_+$, as shown in the diagram.

Also the field $\mathbf{E}_2$ due to $q_2$ is

$$\mathbf{E}_2 = (9\times10^9\ \mathrm{N\,m^2\ C^{-2}})\ \frac{(-12\times10^{-9}\ \mathrm{C})}{(0.1\mathrm{m})^2}\ (\hat{\mathbf{r}}_-)$$

(Here $\hat{\mathbf{r}}_-$ points diagonally upward to the left)

$$\mathbf{E}_2 = -1.08\times10^4\ \hat{\mathbf{r}}_-\ \mathrm{NC^{-1}}$$

The minus sign shows that the electric field points diagonally downward to the light.

Now we must add the two forces vectorially. If we resolve $\mathbf{E}_1$ and $\mathbf{E}_2$ into components along x-axis and y-axis, it is clear from the figure that y-components of vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ cancel out and those along x-axis, i.e., BO, add. The angle between either vector and the x-direction is 60° because the triangle formed by R, $q_1$ and $-q_2$ is an equilateral triangle. The direction of the resultant field is, therefore, along BO and its magnitude is given by

$$\mathbf{E}_B = (1.08\times10^4 \cos 60^\circ + 1.08\times10^4 \cos 60^\circ)$$
$$= 1.08\times10^4\ \mathrm{NC^{-1}}.$$

Try to solve the following SAQ so that you can work out the problems on your own.

**SAQ 8**

See Fig, 1.9. Find the electric field at the point $C$ which is at a distance of 0.05m from $q_1$.

Till now we have considered electric field due to simple kinds of charge distribution, viz., an isolated point charge and an arrangement of two or more point charges. Now suppose that the charge is continuously distributed over a region as shown in Fig. 1.10. Such situations occur frequently, for example, when we give some charge to a metallic body, it gets distributed over its surface.

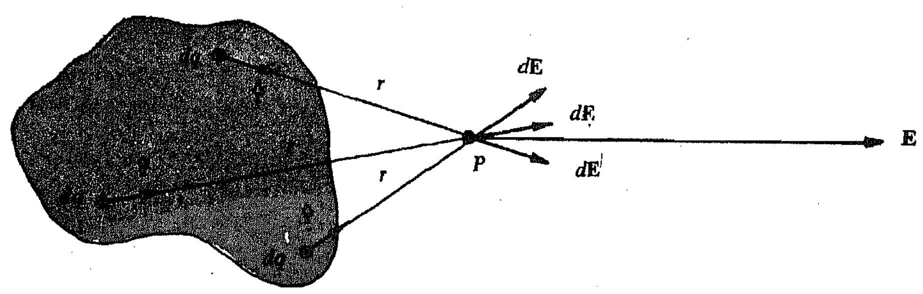


**Fig. 1.10: The electric field at piont $P$ is the sum of the vectors $d\mathbf{E}$ arising from all the individual charge elements $dq$ in the entire charge distribution. The appropriate distance $r$ and unit vector $\hat{\mathbf{r}}$ both vary from one charge element to another.**

To calculate the electric field at point P due to this continuous distribution of charge, we consider the charged region to consist of many small charge elements dq. The charge element is chosen small enough so that every point in the element can be treated as equidistant from P. Now each dq will produce a small electric field $d\mathbf{E}$ in accordance with Eq. (1.9) as follows:

$$d\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}} \qquad \text{...(1.11)}$$

Then, in analogy with Eq. (1.10), the vector sum of all the $d\mathbf{E}$'s will give the total electric field E due to the whole charged region. Here, we have made an approximation that, by placing together all the charge elements. we get the charged region. However, in order to improve the approximation, we have to make the charge elements dq infinitesimally small and for the continuous distribution, the vector sum will become an integral. Thus

$$\mathbf{E} = \int d\mathbf{E} = \int \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}} \qquad \text{...(1.12)}$$

The limits of this integral are so chosen that it includes the entire region over which charge is distributed.

The region over which the charge is continuously distributed may be a line, an area or a volume as shown in Fig. 1.11.

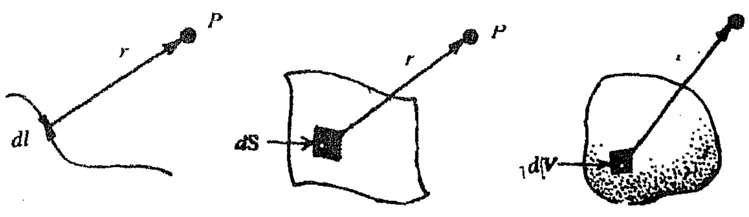


Fig. 1.11: (a) Line charge (b) Surface charge (c) Volume charge

In such distributions, instead of charges, we speak of the density of charges, viz., line charge density denoted by λ, surface charge density σ and volume charge density ρ. These quantities describe the amount of charge per unit length, per unit area and per unit volume respectively. They have units $Cm^{-1}$, $Cm^{-2}$ and $Cm^{-3}$.

Consider Fig. 1.11a. Here the charges are distributed along a line. Consider a small element of length dl. The charge element $dq$ will be equal to λ dl, where λ is line charge density. Replace $dq$ in Eq. (1.12) by $dq = \lambda$ dl. Thus, the electric field of a line charge is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\hat{\mathbf{r}}}{r^2}\lambda dl \qquad ..(1.13)$$

See Fig. 1.11b. Here, the charges are spread over a surface. If $dS$ is a surface element, then the charge element $dq$ will be $\sigma dS$ where σ is surface charge density. By replacing $dq$ by $dq = \sigma dS$ in Eq. (1.12), the electric field for a surface charge is:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\hat{\mathbf{r}}}{r^2}\sigma dS \qquad ...(1.14)$$

Similarly, in Fig. 1.11c, the charges are distributed over a volume having volume charge density ρ. If $dV$ be a volume element then $dq = \rho\, dV$. Hence, the electric field due to volume charge is:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\hat{\mathbf{r}}}{r^2}\rho dV \qquad ...(1.15)$$

If you consider Cartesian co-ordinate, then in Eq. (1.13) the integration is performed with respect to single variable either x- or y- or z-axis depending on whether the line charge distribution is along x- or y- or z- axis. Since a surface is defined in two dimensions, the integration in Eq. (1.14) is performed with respect to any two variables x and y; or y and z; or z and x. Similarly, a volume is defined in three dimensions and hence, in Eq. (1.15), the integration is performed with respect to three variables x, y and z.

The vector integrals in Eq. (1.13), Eq. (1.14) and Eq. (1.15) are called line integral, surface integral and volume integral respectively. If you have offered the course on Mathematical Methods in Physics-I (PHE-04), these terms will not be new for you and you would be knowing their meaning. However, within this course, we have defined and explained these terms as and when you encounter them in detail. Surface and volume integral has been explained in Unit 2, whereas line integral has been explained in Unit 3. Note that the charge densities, viz., λ, σ or ρ need not be constant. We can allow these charge densities to be function of position. Thus, in Eq. (1.13) to Eq. (1.15), λ, σ and ρ can themselves depend upon **r**. For example, in the case of a volume charge distribution $\rho(\mathbf{r}')$ over a volume $V'$ (Fig. 1.12), the expression for electric field at the point $P$ at **r** is

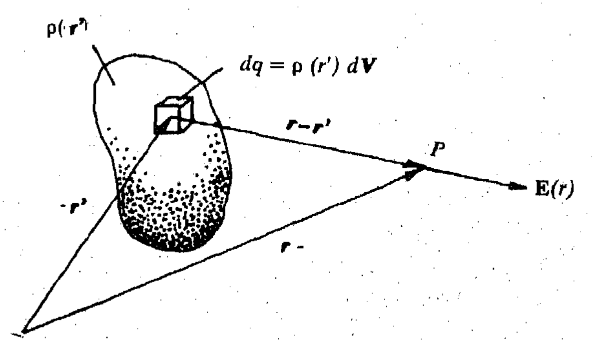


Fig. 1.12: Electric field at $P$ due to a volume charge density $\rho(\mathbf{r}')$. The vector $\mathbf{r}'$ varies during integration, but **r** is fixed.

Another common way of representing the vector nature of equations like Eq. (1.4) is to write it as

$$\mathbf{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2 \mathbf{r}_{12}}{r_{12}^3}$$

where $\mathbf{r}_{12}$ is now a vector with the magnitude of the distance $r_{12}$. The above equation has been obtained by using the relation

$$\hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|}.$$

The $r_{12}^2$ in the denominater of Eq. (1.4) has been changed to $r_{12}^3$ to compensate for this. Hence the meaning is identical with that of Eq. (1.4). However we shall use the Unit-vector notation.

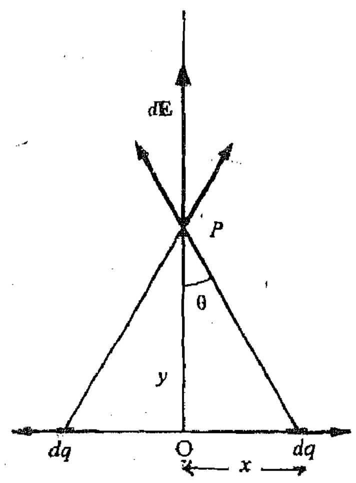


Fig. 1.13: A pair of charge elements dq on either side of the origin contributes to a net field $d\mathbf{E}$ in the y-direction.

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')\,dV'}{|\mathbf{r}-\mathbf{r}'|^3} \quad ...(1.16)$$

Here, the dashed (or primed) coordinates refer to the source point and **r** is the point where field **E** is evaluated.

**Example 4**

An infinitely long uniformly charged rod shown in Fig. 1.13 coincides with the x-axis and carries a line charge density $\lambda\,\mathrm{C\,m^{-1}}$. What is the electric field at a point P on the y-axis?

**Solution**

Let the point P be at a distance of y along the perpendicular bisector of the rod. Consider a small length $dx$ of rod containing charge dq located at a distance $x$ to the right of the origin. Then

dq = Xdx

The magnitude of the electric field at P due to this element of charge $dq$ will be given by:

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

To determine the net field at P, we would write this $dE$ in terms of its $x$- and y-components, anu then integrate each component over the entire line. Remember that for each dq to the right of the origin, there is a corresponding $dq$ the same distance to the left. So, the x-components of the fields from such a pair cancel, while the y-components are the same. y-component of $dE$ will be

$$dE_y = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\cos\theta$$

$$= \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\frac{y}{r} \qquad \left(\because \cos\theta = \frac{y}{r}\right)$$

So that $dE_y = \frac{1}{4\pi\epsilon_0}\frac{y\,dq}{(x^2+y^2)^{3/2}}$

Addition of the two equal y-components then gives the net electric field $d\mathbf{E}_{net}$:

$$d\mathbf{E}_{net} = \frac{1}{4\pi\epsilon_0}\frac{2y\,dq}{(x^2+y^2)^{3/2}}\,\hat{\mathbf{j}}$$

The net electric field due to the whole rod will be:

$$\mathbf{E}_{net} = \int d\mathbf{E}_{net} = \int_{x=0}^{x=\infty}\frac{1}{4\pi\epsilon_0}\frac{2y\,dq}{(x^2+y^2)^{3/2}}\,\hat{\mathbf{j}}.$$

Although the line extends from $-a$ to $+a$, we integrate over only half the line because the expression we are integrating is already the field of a charge pair $dq$. Substituting the expression for dq ($dq$ = X dx) and bringing constant out of the integral, we get

$$\mathbf{E}_{net} = \frac{\lambda y}{2\pi\epsilon_0}\int_0^{\infty}\frac{dx}{(x^2+y^2)^{3/2}}\,\hat{\mathbf{j}}.$$

Put $x = y\tan\theta$ so that $dx = y\sec^2\theta\, d\theta$. Since tho rod is very long, as $x$ ranges from 0 to $\alpha$, $\theta$ ranges from 0 to $\frac{\pi}{2}$. Then we have

$$\mathbf{E}_{net} = \frac{\lambda y}{2\pi\epsilon_0} \int_0^{\pi/2} \frac{y \sec^2\theta \, d\theta}{(y^2 \tan^2\theta + y^2)^{3/2}} \hat{\mathbf{j}}.$$

$$= \frac{\lambda y}{2\pi\epsilon_0} \int_0^{\pi/2} \frac{y \sec^2\theta \, d\theta}{y^3 \sec^3\theta} \hat{\mathbf{j}} \quad (\because \tan^2\theta + 1 = \sec^2\theta)$$

$$= \frac{\lambda y}{2\pi\epsilon_0} \frac{1}{y^2} \int_0^{\pi/2} \cos d\theta \, \hat{\mathbf{j}}$$

$$= \frac{\lambda}{2\pi\epsilon_0 y} [\sin\theta]_0^{\pi/2} \hat{\mathbf{j}} = \frac{\lambda}{2\pi\epsilon_0} \hat{\mathbf{j}}.$$

Thus, the electric field due to infinitely long positively charged rod points radially outward'from the rod and its magnitude decreases inversely with distance.

As you have seen, in practice, the determination of electric field due to continuous charge distribution by above method requires more difficult calculations. There is a much easier and more elegant way to determine the electric field for such distribution which will be discussed in the next unit.

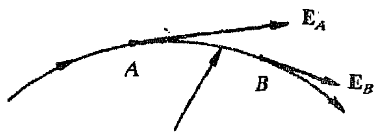


Fig. 1.14: An electric line of force.

Line of force is also defined as a path along which a free, positive, point charge would travel in an electric field. Hence a line of force is always provided with an arrowhead indicating the direction of travel of the positive charge.

## 1.5.2 Electric Lines of Force

In the last subsection, you learnt how to calculate the electric field (both magnitude and direction) at any point due to various charge distributions. In fact, the electric field extends throughout space. There is a useful method for representing the entire electric field visually by means of **lines of force (also** called electric **field line** or **electric lines of force).** This representation, though not good for quantitative purposes, serves a very useful purpose by allowing us to know the general features of the electric field in the entire region at a glance. The line of force is a line drawn in such a way that the tangent to it at any point shows the direction of electric field at that point as shown in Fig. 1.14. These lines are continuous and extends throughout space depicting the electric field.

Let us see how the lines of force provide information regarding the strength of electric field. Fig. 1.15 shows the lines of force due to a positive point charge. For a positive charge, the field at any point is directed away from the charge because a positive test charge would be repelled in that direction. So the lines of force are straight line pointing radially outward from the point charge. The lines'start on the charge and extend outward up to infinity. You would observe that the lines of force spread apart as they extend farther from the charge. Now you know that, according to Coulomb's law, the electric field decreases as you move away from the charge. So in Fig. 1.16, the electric field must be stronger at region A than at B meaning thereby that the electric field is stronger where lines of force are close together and weaker where they are farther apart.

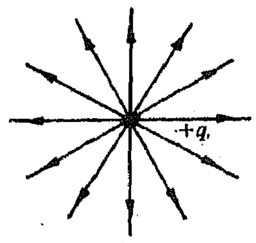


Fig. 1.15: Lines of force due to a positive charge.

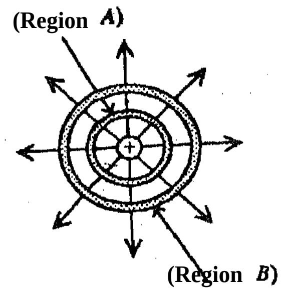


Fig. 1.16: The field is greater at region A than at B because in the region A, the lines of force are close together whereas in the region B they are farther apart.

Suppose, you have a charge q and another charge $2q$ and if you are told to draw the lines of force of these two charges, then can you draw as many lines of force as you want. No, it is not so. To make'the lines of force picture useful, we associate a fixed number of lines of force with a charge of given magnitude. So, if 6 lines of force emanate from a charge $q$, then a charge $2q$ will be represented by 12 lines of force. The above statement is consistent with the Coulomb's law. Because, from Coulomb's law we know that electric, field is proportional to the magnitude of charge ($E \propto q$). Therefore, the number of lines of force originating or ending (in case of negative charge) on charges is proportional to the magnitude of eachcharge. Fig. 1.17 shows the lines of force for two equal unlike charges and two equal like charges. **It should be remembered that lines of force are not real and they do not actually exist as threads in space; they are simply a device to** help **our thinking about the field.**

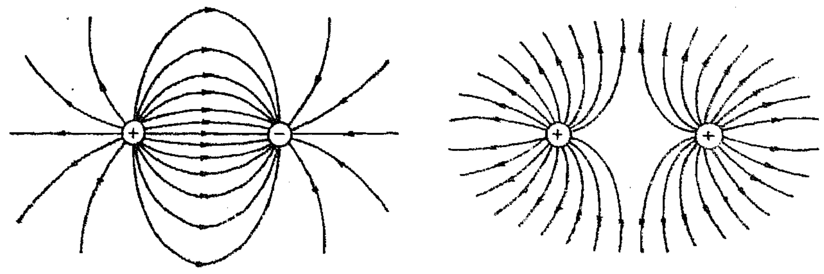


**Fig. 1.17 : The nature of lines of force.** We have adopted the convention of drawing 18 lines per charge.
(a) two unlike charges.
(b) two like positive charges.

**SAQ 9**

A charge of $+3\mu C$ and $-1\mu C$ are fixed at a distance of 2cm from each other. Sketch the lines of force.

Let us now sum up what we have learnt in this unit.

## 1.6 SUMMARY

- Only two types of electric charge exist and they are arbitrarily called positive and negative. Like charges repel and unlike charges attract each other.
- In SI system, unit of charge is Coulomb (C).
- **Charge is always conserved.** That is, the algebraic sum of charges in a closed region never changes.
- Electric charge is quantized, **occurring only in discrete amounts.**
- The **force between two** charges is **proportional** to the product of their charges **and inversely** proportional to the square of the distance between them. The force acts along the line joining the two charges.

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{(q_1 q_2)}{r^2} \hat{\mathbf{r}}.$$

The value of $1/4\pi\epsilon_0$ is $9\times10^9 \mathrm{N\,m^2\,C^{-2}}$.

- The electric force on a charge due to the presence of two or more charges is **simply the vector sum of the forces caused** by the individual charges. This important property of the electric force is known as superposition principle.
- **The electric field** at a point in space is defied as the electric force exerted on a test charge placed at that point.

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

- The **electric field** of **a point** charge q is given by

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

**where $\hat{\mathbf{r}}$ is a unit** vector pointing from the point charge q to the location at which the **electric field is being calculated.**

- **The electric field due** to a distribution of charges, **according to superposition principle,** is the vector sum of the fields of the individual charges **making up the distribution:**

$$\mathbf{E} = \sum \frac{1}{4\pi\epsilon_0} \frac{q_n}{r_n^2} \hat{\mathbf{r}}_n$$

With continuous distributions of charge, the sum becomes an integral over the entire charge distribution as follows :

$$\mathbf{E} = \int \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \hat{\mathbf{r}}.$$

As an example, for volume charge distribution $\rho(\mathbf{r}')$ over a volume $V'$ :

$$\mathbf{E(r)} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} (\mathbf{r} - \mathbf{r}')\, dV'.$$

- Electric lines of force are visual way of representing an electric field. It is a line in an electric field such that the tangent to it at any point shows the direction of the electric field at that point. The lines are close together where the field is strong. Lines of force.always begin or end on electric charges.

## 1.7 TERMINAL QUESTIONS

1) Two point charges $+4e$ and $+e$ are fixed at a distance of 'a'. A third charge $q$ is placed on a straight line joining these two charges so that q is in equilibrium. Find the position of $q$. Under what circumstances will Phis equilibrium be 'stable' and 'unstable'?

2) ABCD is a square of 0.04 metre side, charges of $16 \times 10^{-9}$, $-16 \times 10^{-9}$ and $32 \times 10^{-9}$ Coulomb are placed at the points $A$, $C$ and $D$ respectively. Find the intensity of the electric field at point $B$.

3) A small object carrying a charge of $-5 \times 10^{-9}$C experiences a force of $20 \times 10^{-9}$N in the negative x-direction when placed at a certain point in an electric field. (a) What is the electric field at this point? (b) What would be the magnitude and direction of the force acting on a proton placed at this point?

4) Two identical balloons Piled with helium are tied with a load of 0.005 kg and are suspended in air in equilibrium. If each balloon carries a charge of q Coulomb, calculate the value of $q$. The length of each thread is 1m and the distance between their centres is 0.5m.

(Hint :

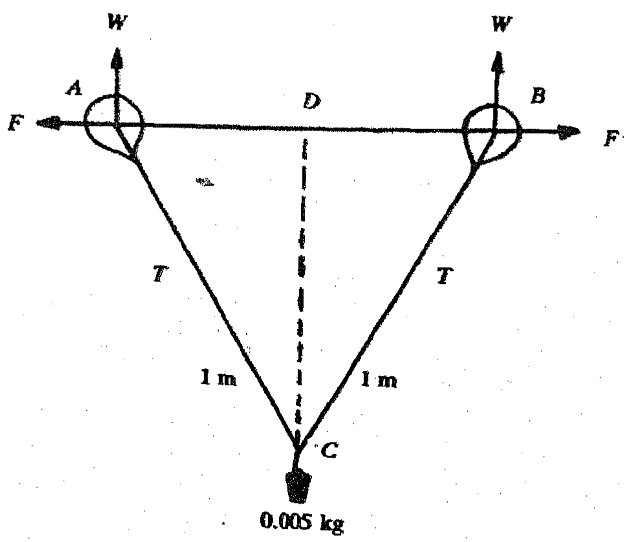


Fig. 1.18

As shown in Fig. 1.18, following forces act on the system in equilibrium

i) upward thrust, W
ii) electrostatic force, $F$
iii) tension $T$
iv) load

In equilibrium, the moment of forces about the point $C$ will be zero.)

## 1.8 SOLUTIONS AND ANSWERS

**SAQ 1**

$Z$ will attract $Y$.

**SAQ 2**

According to the principle of conservation of charge, the amount of charge present before the radioactive decay (or any nuclear reaction) is equal to the amount of charge present after the decay.

In (i) the charge present before the decay is 92e and the charge present after the decay is $2e$. Hence Eq. (i) will be

$${}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}$$

Eq. (ii) is

$${}_{20}Ca^{44} + {}_{1}P^{1} \longrightarrow {}_{21}Sc^{44} + {}_{0}n^{1}$$

${}_{0}n^{1}$ is neutron.

SAQ 3

Since the conductor is positively charged, it is short of electrons.

Let it be short of $n$ electrons, then from the relation $q = ne$, the number of electrons can be found out. Here $q = 3.2 \times 10^{-17}$C

and $e = 1.6 \times 10^{-19}$C

$$\therefore n = \frac{q}{e} = \frac{3.2 \times 10^{-17}\text{C}}{1.6 \times 10^{-19}\text{C}} = 200$$

The conductor is short of 200 electrons.

**SAQ 4**

If the force on $q_2$ is to be found, it is only necessary to change every subscript 1 to 2 and every 2 to 1.

$$\text{Force } \mathbf{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_2 q_1}{r_{21}^2} \hat{\mathbf{r}}_{21}$$

where $\hat{\mathbf{r}}_{21}$ is the unit vector from $q_1$ to $q_2$.

**SAQ 5**

According to Coulomb's law, the magnitude of force acting between two charges at a distance $r$ apart in air is given by

$$F_{\text{electrical}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \text{ N}$$

Here $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$.

$q_1 = q_2 = 1.6 \times 10^{-19}$C (Since the magnitude of charges on electron and proton are equal)

$r = 5.3 \times 10^{-11}$m

$$F_{\text{electrical}} = 9\times10^9\ \text{N m}^2\ \text{C}^{-2}\times\frac{(1.6\times10^{-9}\text{C})\times(1.6\times10^{-9}\text{C})}{(15.3\times10^{-11}\ \text{m})^2}$$

$$= 8.2\times10^{-8}\ \text{N}$$

According to Newton's law, the gravitational force between electron and proton is given by

$$F_{\text{gravitational}} = G\,\frac{m_1\,m_2}{r^2}\ \text{N}$$

Here $m_1$ = mass of electron = $9.1\times10^{-31}$ kg

$m_2$ = mass of proton = $1.7\times10^{-27}$ kg

$r$ = $5.3\times10^{-11}$ m

G = $6.6\times10^{-11}$ N m$^2$ kg$^{-2}$

$$F_{\text{gravitational}} = \frac{(6.7\times10^{-11}\ \text{N m}^2\ \text{kg}^{-2})\times(9.1\times10^{-31}\text{kg})\times(1.7\times10^{-27}\text{kg})}{(5.3\times10^{-11}\ \text{m})^2}$$

$$= 3.7\times10^{-47}\ \text{N}$$

$$\therefore\ \frac{F_{\text{electrical}}}{F_{\text{gravitational}}} = \frac{8.2\times10^{-8}}{3.7\times10^{-47}} = 2.2\times10^{-39}$$

Electric forces dominate on a small-scale structure whereas gravitational forces dominate on a large-scale structure.

You may observe that electrical force is $10^{39}$ times stronger than gravitational force. Therefore in such problems gravitational forces could be neglected.

**SAQ 6**

$Q_1 + Q_2 = 20\mu$C or $Q_2 = (20 - Q_1)\ \mu$C

Since force is repulsive, the two charges are of same type.

According to Coulomb's law

$$0.075\ \text{N} = (9\times10^9\ \text{N m}^2\ \text{C}^{-2})\times\frac{Q_1\,Q_2}{(3\text{m})^2}$$

or $Q_1\,Q_2 = 75\times10^{-12}\ \text{C}^2$

or $Q_1\,Q_2 = 75\times(10^{-6}\ \text{C})^2$

or $Q_1\,Q_2 = 75\mu\text{C}^2$

Substituting the value of $Q_2$ we get

$Q_1\,(20 - Q_1) = 75$

or $Q_1 - 20\,Q_1 + 75 = 0$

or $Q_1 = 5$ and **15.**

Therefore, the charges are 5 and 15 $\mu$C.

**SAQ 7**

The solution to this problem is obtained by using the principle of superposition. This means that to calculate the force on the test charge, you can first calculate the force $\mathbf{F}_1$, due to the first charge alone (ignoring all the others); then you calculate the force $\mathbf{F}_2$ due to the second charge alone; and so on. Finally, you take the vector sum of all the individual forces, viz. $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ ....... which gives the resultant force on the test charge. It is given that these individual forces are represented by the sides of a closed pentagon. You know that if the vectors representing various forces form a closed geometrical figure then the resultant of these vectors is zero. This implies that resultant of these forces is zero. Therefore, the resultant force on the test charge is zero.

SAQ 8

The electric field at $C$ due to $q_1$ is

$\mathbf{E}_1 = 4.03\times10^4\ (-\hat{\mathbf{i}})\ \text{NC}^{-1}$

The electric Weld at $C$ due to $q_2$ is

$\mathbf{E}_2 = 0.48 \times 10^4\ \hat{\mathbf{i}}\ NC^{-1}$

The electric fields are oppositely directed so the magnitude of the net electric field $\mathbf{E}_c$ at $C$ is $E_c = E_1 - E_2$, where $E_1$ and $E_2$ are the magnitudes of the respective electric fields.

$\therefore E_c = (4.03 - 0.48) \times 10^4\ NC^{-1}$

$= 3.55 \times 10^4\ NC^{-1}$ and it is directed towards negative x-direction.

**SAQ 9**

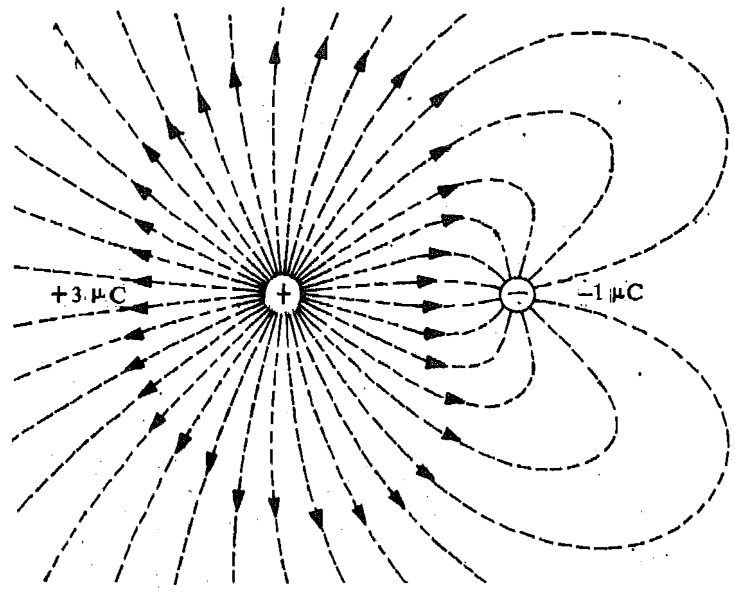


**Fig. 1.19**

Fig. **1.19** shows the lines of force due to two charges: $+3\mu C$ and $-1\mu C$. We have adopted the convention that **12** lines of force emanate from $1\mu C$ charge. Then, a total of 36 lines must **emanate** from the $+3\mu C$ charge and **12** of these must terminate on' the $-1\mu C$ charge. Very near each of these point charges, the lines of force are radially directed, pointing outward from the positive charge and inward to the negative. To complete the pattern, 12 of the lines emanating from the positive charge are connected to the 12 lines of force terminating on the negative charge in such a way that no lines intersect. The connecting links are shown as dashed line. **24** lines of forced extend to infinity. Hence at a sufficiently large distance from this charge **distribution, the** field pattern is the same as that due to a net charge of $+2\mu C$.

## Terminal Questions

**1)**

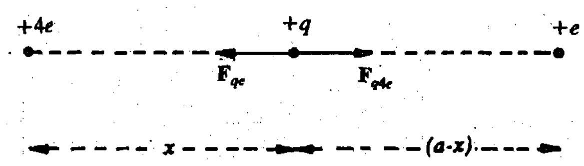


**Fig. 1.20**

Let the charge q be positive, As shown in Fig. **1.20,** let us locate it at some point x to the right of $+4e$, so that $x < a$. At this point, the charge q is at a **distance** $(a - x)$ from $+e$, so that the force on q due to $+e$ is

$$\mathbf{F}_{qe} = \frac{1}{4\pi\epsilon_0} \frac{e \times q}{(a-x)^2} (-\hat{\mathbf{i}})$$

where $(-\hat{\mathbf{i}})$ is a unit vector from $+e$ to $q$ and is in the negative x-direction.

Tie charge q lies at a distance $x$ from $+4e$ so that the force on q due to $+4e$ is :

$$\mathbf{F}_{q4e} = \frac{1}{4\pi\epsilon_0}\frac{4e\times q}{x^2}\,\hat{\mathbf{i}}$$

where $\hat{\mathbf{i}}$ is a unit vector from $+4e$ to q and is in the positive x-direction. For $q$ to be in equilibrium, the two forces must cancel, that is,

$$\mathbf{F}_{qe} + \mathbf{F}_{q4e} = \frac{1}{4\pi\epsilon_0}\frac{e\times q}{(a-x)^2}(-\hat{\mathbf{i}}) + \frac{1}{4\pi\epsilon_0}\frac{4e\times q}{x^2}\,\hat{\mathbf{i}} = 0$$

we then have

$$\frac{4}{x^2} = \frac{1}{(a-x)^2}$$

or $4(a-x)^2 = x^2$

or $2(a-x) = \pm\, x.$

For the $+$ sign we have

$$x = \frac{2a}{3}.$$

For the $-$ sign we have

$x = 2a.$

Here, $2a/3$ is the only possible value of $x$ because the charge g is placed to the right of $+4e$ and $x < \alpha$. Therefore, for the equilibrium, the charge $q$ is to be placed at a distance of $2a/3$ from $+4e.$

We have assumed that $q$ is positive. If q is slightly displaced (say towards right) from its equilibrium position, then the value of $\mathbf{F}_{q4e}$ will decrease and $\mathbf{F}_{qe}$ will increase, This is because of the fact that the electrostatic force between two charges is inversely proportional to the square of the distance between them. Hence, a net force $(\mathbf{F}_{qe} - \mathbf{F}_{q4e})$ will act on $q$ towards left due to which the charge will again return to its equilibrium position. Hence, it is clear that the equilibrium of q is stable.

If q is negative, then $\mathbf{F}_{q4e}$ and $\mathbf{F}_{qe}$ will be attractive forces and their direction will be as shown in Fig. 1.21.

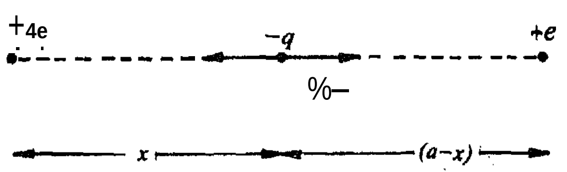


Fig. 1.21

The charge q will also be at equilibrium at a distance of $2a/3$ from $+4e$. If $-q$ is slightly displaced (say towards right), then $\mathbf{F}_{q4e}$ will decrease while $\mathbf{F}_{qe}$ will increase. Hence, a net force $(\mathbf{F}_{qe} - \mathbf{F}_{q4e})$ directed towards right will act on $-q$. Therefore, $-q$ will move more. towards right. Hence, it is clear that equilibrium of $-q$ is unstable.

2)

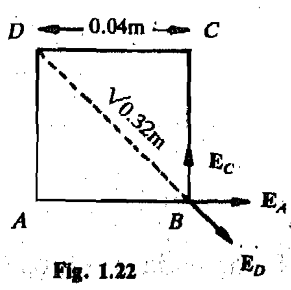


Fig. 1.22

Refer to Fig. 1.22. The electric field at the point $B$ due to the charges at A, $C$ and D are referred to as $\mathbf{E}_A$, $\mathbf{E}_C$, and $\mathbf{E}_D$ respectively. Therefore, the electric field E at B will be vector sum of $\mathbf{E}_A$, $\mathbf{E}_C$ and $\mathbf{E}_D$.

Here $E_A = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

$$= (9\times10^9\ \mathrm{Nm^2\,C^{-2}})\ \frac{(16\times10^{-9}\ \mathrm{C})}{(0.04\mathrm{m})^2}$$

$$= 9\times10^4\ \mathrm{N\,C^{-1}}$$

$E_A$ is directed along AB.

Similarly $E_C = (9\times10^9\ \mathrm{Nm^2\,C^{-2}})\ \frac{(-16\times10^{-9}\ \mathrm{C})}{(0.04\mathrm{m})^2}$

$$= 9\times10^4\ \mathrm{NC^{-1}}.$$

$E_C$ is directed along BC.

$$E_D = (9\times10^9\ \mathrm{Nm^2\,C^{-2}})\ \frac{(32\times10^{-9}\ \mathrm{C})}{32\times10^{-4}\ \mathrm{m^2}}$$

$$= 9\times10^4\ \mathrm{NC^{-1}}.$$

$E_D$ is directed 'along DB.

Now resolving $E_D$ along $x$– and y–axes, we get

component of $E_D$ along x–axis $= E_D \cos 45°$

component of $E_D$ along y-axis $= E_D \sin 45°$

Net component to $E$ along x-axis $= E_A + E_D \cos 45°$ and

Net component to $E$ along y-axis $= E_C - E_D \sin 45°$.

$\therefore$ Magnitude of resultant electric field $E$ at B is given by

$$E = [(E_A + E_D \cos 45°)^2 + (E_C - E_D \sin 45°)^2]^{1/2}$$

$$= [(E_A^2 + E_D^2 + E_C^2 + 2E_D\,(E_A \cos 45° - E_C \sin 45°)]^{1/2}$$

$$= \sqrt{3}\,E_A \text{ (because } \cos 45° = \sin 45° = 1/\sqrt{2})$$

$$= 9\sqrt{3}\times10^4\ \mathrm{NC^{-1}}.$$

If $\theta$ is the angle which $E$ makes with the horizontal then,

$$\tan\theta = \frac{E_C - E_D \sin 45°}{E_A + E_D \cos 45°}$$

$$= \frac{1 - 1/\sqrt{2}}{1 + 1/\sqrt{2}} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$$

$$= 0.171$$

$$\theta = \tan^{-1}(0.171)$$

$$\therefore \theta = 9°\ 45'$$

Hence, the resultant electric field, $E$ makes an angle of 9° 45' with the horizontal.

3) **Electric** field E is given by

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

Here F $= -20\times10^{-9}\,\hat{i}$ N and q $= -5\times10^{-9}$ C.

$$\therefore \mathbf{E} = \frac{-20\times10^{-9}\hat{i}\ \mathrm{N}}{-5\times10^{-9}\ \mathrm{C}} = 4\hat{i}\ \mathrm{NC^{-1}}$$

**The** direction of E is in the positive $x$-**direction** – **opposite** to the **force** on the negative charge. Force F acting on **any** charge q placed in this field is given by the **relation**

$$\mathbf{F} = q\mathbf{E}$$

Here $q$ = charge on a proton = $1.6 \times 10^{-19}$ $C$.

and E = $4\hat{i}$ $NC^{-1}$

$\therefore F = (1.6\times10^{-19}\ C) \times (4\hat{i}\ NC^{-1}) = 6.4\times10^{-19}\ \hat{i}\ N$

The force on proton is in the same **direction as** the field, **i.e.,** towards positive ***x*-direction.**

4) A and $B$ are'two balloons and each is acted upon by an upward thrust W

For equilibrium

$2W$ = load

or $2W = 0.005\ kg$

or $2W = 0.0025\ kg = (0.0025\times 9.81)\ N$.

if $F$ be the force of repulsion between the balloons then

$$F = 9\times10^9\ Nm^2\ C^{-2} \times \frac{q^2}{(0.5m)^2} \quad \text{...(i)}$$

As is clear from Fig. ***1.18,*** following forces act on the system in the equilibrium position.

(i) Upthrust W, (ii) Force of repulsion $F$, (iii) Tension $T$ in thread, and (iv) load.

In equilibrium, the moment of forces about C will be zero. So, taking moment of forces about C, we **get**

$F\times CD - W\times AD + T\times 0 - \text{load}\times 0 = 0$

or $F \times CD - (0.0025\times 9.81)N\times AD = 0$

or $F = 0.0025\times 9.81\ N\times\frac{AD}{CD}$

But from Eq. (i), we have $F = 9\times10^9\ Nm^2\ C^{-2} \times \frac{q^2}{(0.5m)^2}$

$$\therefore 9\times10^9\ Nm^2\ C^{-2} \times \frac{q^2}{(0.5m)^2} = (0.0025\times9.81)\ N \times \frac{AD}{CD}$$

$$\text{or } q^2 = \frac{0.0025\times9.81\times0.5\times0.5}{9\times10^9} \times \frac{AD}{CD}\ C^2$$

$$\text{or } q^2 = \frac{0.0025\times9.81\times0.5\times0.5\times0.25}{[(1)^2-(0.25)^2]^{1/2}\ 9\times10^9}\ C^2 \left[\because \begin{array}{l} AD = 0.25m \text{ and} \\ CD = \{1^2-(0.25)^2\}^{1/2}\ m \end{array}\right]$$

$$\text{or } q = \left[\frac{0.0025\times9.81\times0.5\times0.5\times0.25}{(0.9375)^{1/2}\times9\times10^9}\right]^{1/2} C$$

$\therefore q = 4.19\times10^{-7}$ C.