UNIT 3 WORK AND ENERGY

Structure

3.1 INTRODUCTION

In the previous unit we have studied the causes behind change of motion. We have also studied the important law of conservation of linear momentum and its applications. In our everyday experience we often feel that when we execute a motion some energy is expended. Some times we say that work is done at the expense of some energy. However, the word 'work' has a special meaning in physics. For instance, if a lecturer stands near a table and delivers a lecture for one hour, then no work is done according to the principles of physics. In this unit you will learn about the work done by various forces and also different kinds of energies. We will go into the details of the very important principle of conservation of energy. This principle has very wide applications and will be used very often in your physics courses. In the next unit we will apply some of the concepts of motion developed in the first three units to the study of angular motion.

Objectives

After studying this unit you should be able to:

- compute work of constant and variable forces
- apply work-energy theorem
- distinguish between conservative and non-conservative forces
- solve problems based on the principle of conservation of energy
- interpret energy diagrams
- solve problems based on elastic and inelastic collisions
- compute power in mechanical systems.

3.2 WORK

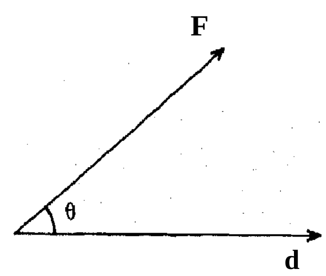


Fig. 3.1

You have studied in Unit 1 that the **work done** by a force **F** during a displacement **d** of its point of application is given according to Eq.1.11b as **F.d**. So if **there** is an angle θ between F and **d** as shown in Fig. 3.1 then work done is $F\cos\theta d$. The unit of work is newton-metre which is named as joule. If θ is acute ($\cos\theta$ is positive), then work is said to be done **by** the force and if θ is obtuse ($\cos\theta$ is negative), then work is said to be done **against** the force. If $\theta = 90°$ ($\cos\theta = 0$), then it is a **no-work** force. **For** example, when a **man** walks on the ground, the reaction force experienced by him is always normal to his displacement. Hence, reaction is a no-work force.

SAQ 1

Give an example (other than the above) of a no-work force.

Now, for a small displacement $\Delta\mathbf{l}$ the work done by a force F is given by

$$W = \mathbf{F} \cdot \Delta\mathbf{l}. \tag{3.1}$$

3.2.1 Work Done by a Constant Force

Let a particle undergo a succession of displacements $\Delta \mathbf{l}_1, \Delta \mathbf{l}_2, \ldots, \Delta \mathbf{l}_n$ under the action of a constant force. Then the net displacement is $\mathbf{\Delta l} = \Delta \mathbf{l}_1 + \Delta \mathbf{l}_2 + \ldots + \Delta \mathbf{l}_n$ (see Fig. 3.2), and the work done is

$$W = \mathbf{F} \cdot \Delta \mathbf{l} = \mathbf{F} \cdot (\Delta \mathbf{l}_1 + \Delta \mathbf{l}_2 + \ldots + \Delta \mathbf{l}_n)$$

We can use the distributive law of scalar products (Eq. 1.14) and write

$$W = \mathbf{F} \cdot \Delta \mathbf{l}_1 + \mathbf{F} \cdot \Delta \mathbf{l}_2 + \ldots + \mathbf{F} \cdot \Delta \mathbf{l}_n. \qquad (3.2)$$

Thus, the work done by a constant force for a succession of displacenients is the sum of the work done by that force for individual displacements.

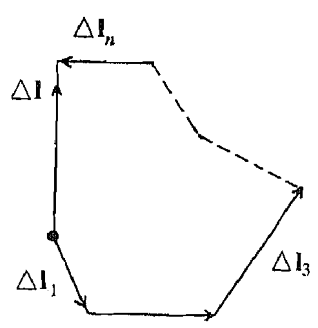


Fig. 3.2

However, in nature we come across many forces that vary with position. For example, let a unit positive charge be taken from point A to point B in the electrostatic field of a charge + q (Fig. 3.3). q is located at the origin of a two-dimensional rectangular Cartesian coordinate system having x-and y-axes. The force experienced by a unit positive charge when placed at P is given by

$$\mathbf{F} = \frac{kq}{r^2}\hat{\mathbf{r}}, \qquad (3.3)$$

where k is a constant dependent on the nature of the intervening medium. OP = **r** and $\hat{\mathbf{r}}$ is the unit vector in the direction of **r**. As the unit positive charge moves, the magnitude as well as the direction of **r** change. So **F** is a force which varies with position. How do we calculate the work done for such forces?

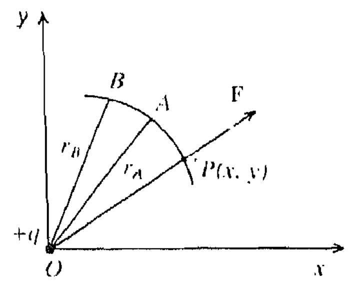


Fig. 3.3

3.2.2 Work Done by a Variable Force

A force of the type given by Eq. 3.3 can be expressed in general as

$$\mathbf{F} = \mathbf{F}(\mathbf{r}). \qquad (3.4)$$

Let us now calculate the work done when a particle moves under the influence of this force from point A to B (Fig. 3.4).

The section of the path from A to B can be approximated by a zigzag polygon consisting of successive displacements $\Delta \mathbf{l}_1, \Delta \mathbf{l}_2, \ldots \Delta \mathbf{l}_n$, where the path AB is shown exaggerated. We can take each successive displacement $\Delta \mathbf{l}_i$ to be very smell. Then $\mathbf{r}_i$ corresponding to each $\Delta \mathbf{l}_i$ will be effectively constant, so that $\mathbf{F}(\mathbf{r}_i)$ is also constant over that displacement. The work done for this displacement is given from Eq. 3.1 as $W_i = \mathbf{F}(\mathbf{r}_i) . \Delta \mathbf{l}_i$.

Since work is a scalar quantity, the work done in going from A to B is the sum of the work done for each successive displacement, i.e.

$$W = \mathbf{F}(\mathbf{r}_1) \cdot \Delta \mathbf{l}_1 + \mathbf{F}(\mathbf{r}_2) \cdot \Delta \mathbf{l}_2 + \cdots + \mathbf{F}(\mathbf{r}_n) \cdot \Delta \mathbf{l}_n$$

$$= \sum_{i=1}^{n} \mathbf{F}(\mathbf{r}_i) \cdot \Delta \mathbf{l}_i. \qquad (3.5)$$

where the symbol Σ stands for the above summation.

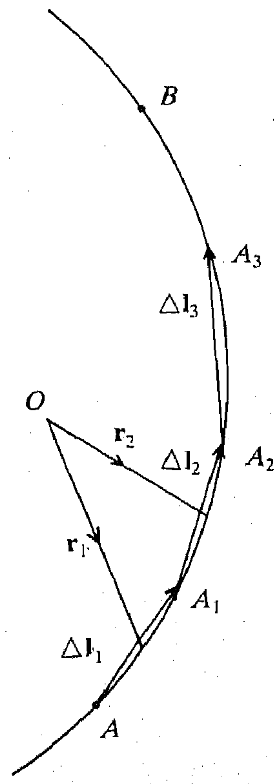


Fig. 3.4

We should expect to get a more significant value for zigzag paths if we fit the curve more closely. Therefore, it is natural to define work in the general case as the limit of such approximations as the lengths of the $\Delta \mathbf{l}_i$'s are made smaller and smaller, i.e.

$$W = \lim_{\Delta \mathbf{l}_i \to 0} \left[\sum_{i=1}^{n} \mathbf{F}(\mathbf{r}_i) \cdot \Delta \mathbf{l}_i \right] \qquad (3.6)$$

From the concept of definite integration, the above limit is given by

$$W = \int_A^B \mathbf{F}(\mathbf{r}) \cdot d\mathbf{l}, \qquad (3.7)$$

where A and B denote the initial and the final positions. This integral is called the line integral of F from A to B. So the integral of force with respect to the position variable over a certain path is the work done by the force over that path, provided the force depends on the position variable only.

For a two-dimensional system $\mathbf{F} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}}$ and $d\mathbf{l} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}}$

$$\mathbf{F}.d\mathbf{l} = (F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}}).(dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}}),$$

or

$$\mathbf{F}.d\mathbf{l} = F_x dx + F_y dy \qquad (3.8)$$

Let us now evaluate the work done by a variable force.

Example 1

Let us determine W along the path *AB* for the electrostatic force F given by Eq. **3.3.**

$$\mathbf{F} = \frac{kq}{r^2}\hat{\mathbf{r}} = \frac{kq}{r^2}\frac{\mathbf{r}}{r} \quad \left(\because \hat{\mathbf{r}} = \frac{\mathbf{r}}{r}\right)$$

$$\text{or } \mathbf{F} = \frac{kq}{r^3}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}}),$$

$$\text{or } F_x = \frac{kqx}{r^3},\; F_y = \frac{kqy}{r^3}.$$

Hence, from Eq. 3.8, we get

$$\mathbf{F}.\mathbf{dl} = \frac{kq}{r^3}(xdx + ydy)$$

$$= \frac{kq}{r^3}\left\{d\left(\frac{x^2}{2}\right) + d\left(\frac{y^2}{2}\right)\right\}$$

$$= \frac{k}{2}\frac{q}{r^3}d(x^2 + y^2)$$

$$= \frac{kq}{2r^3}d(r^2) = \frac{kq}{2r^3}2r\,dr = \frac{kq}{r^2}dr$$

From Eq. **3.7,** we get

$$W = \int_{r_A}^{r_B}\frac{kq}{r^2}dr = kq\left(\frac{1}{r_A} - \frac{1}{r_B}\right). \tag{3.9}$$

You can now try the following SAQ based on the ideas discussed so far.

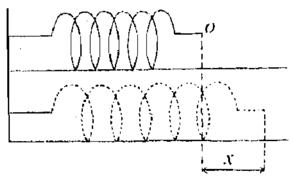


Fig. 3.5

SAQ 2

Suppose the equilibrium position of an end of a spring is O, (Fig. 3.5) and it is stretched through a length x. Due to elasticity a restoring force comes into play, which is proportional to the displacement x from the equilibrium position, i.e.

$$\mathbf{F} = -k_0\,\mathbf{x}.$$

k_0 is a constant and the negative sign appears because the restoring force is directed opposite to the displacement. How much work is done in stretching the spring from the position $x = x_1$ to $x = x_2$?

We have thus discussed the meaning of the line integral of a force when it is a function of the position variable. We shall seek a more general meaning of the line integral of a force in the next section.

3.3 ENERGY

We have defined work. The capacity of a body to do work is called its **energy** and is always measured by the work the body is capable of doing. So the unit of energy is the same as that of work, i.e. joule. In nature, energy manifests itself in different forms — mechanical, heat, electrical, chemical, sound, light, etc. You have read about energy in general in FST-1, Block-4 (Sec. 17.3). In this unit we shall concentrate on mechanical energy. It can be of two kinds — kinetic energy and potential energy.

3.3.1 Kinetic Energy and Work-Energy Theorem

Kinetic energy (K.E.) is possessed by a body by virtue of its motion. For example, a moving car or a ball in motion has kinetic energy. We shall arrive at a quantitative measure of kinetic energy by applying Newton's second law of motion to Eq. 3.7. In the next few steps we shall be doing some algebra. It is necessary for obtaining the result which is physically very significant.

From Newton's second law, for a particle of mass m having a velocity v at a time *t* we have

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v}),$$

or $\mathbf{F} = m\dfrac{dv}{dt}$, for a system of constant mass.

Again $v = \dfrac{d\mathbf{l}}{dt}$

and $d\mathbf{l} = \dfrac{d\mathbf{l}}{dt}dt = v\,dt$

$$\mathbf{F}\cdot d\mathbf{l} = m\frac{d\mathbf{v}}{dt}\cdot\mathbf{v}\,dt. \tag{3.10}$$

We shall now apply some algebra to simplify Eq. 3.10.

Now, $\mathbf{v}\cdot\mathbf{v} = (v_x^2 + v_y^2 + v_z^2)$

$$\frac{d}{dt}(\mathbf{v}\cdot\mathbf{v}) = \frac{d}{dt}(v_x^2 + v_y^2 + v_z^2)$$

$$= \frac{d}{dt}(v_x^2) + \frac{d}{dt}(v_y^2) + \frac{d}{dt}(v_z^2)$$

$$\frac{d}{dt}(v_x^2) = \frac{d}{dv_x}(v_x^2)\frac{dv_x}{dt} = 2v_x\frac{dv_x}{dt}$$

Thus $\dfrac{d}{dt}(\mathbf{v}\cdot\mathbf{v}) = 2v_x\dfrac{dv_x}{dt} + 2v_y\dfrac{dv_y}{dt} + 2v_z\dfrac{dv_z}{dt}$

$$= 2(v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}})\cdot\left(\frac{dv_x}{dt}\hat{\mathbf{i}} + \frac{dv_y}{dt}\hat{\mathbf{j}} + \frac{dv_z}{dt}\hat{\mathbf{k}}\right), \text{ from Eq. 1.15a.}$$

$$\therefore \frac{d}{dt}(\mathbf{v}\cdot\mathbf{v}) = 2\mathbf{v}\cdot\frac{d\mathbf{v}}{dt}.$$

Since dot product is commutative, we get

$$\mathbf{v}\cdot\frac{d\mathbf{v}}{dt} = \frac{d\mathbf{v}}{dt}\cdot\mathbf{v} = \frac{d}{dt}\left(\frac{\mathbf{v}\cdot\mathbf{v}}{2}\right). \tag{3.11}$$

Thus, from Eqs. 3.10 and 3.11, we get

$$\mathbf{F}\cdot d\mathbf{l} = m\frac{d}{dt}\left(\frac{\mathbf{v}\cdot\mathbf{v}}{2}\right)dt = md\left(\frac{\mathbf{v}\cdot\mathbf{v}}{2}\right) = d\left(\frac{m}{2}\mathbf{v}\cdot\mathbf{v}\right).$$

$$\therefore \int_A^B \mathbf{F}\cdot d\mathbf{l} = \left|\tfrac{1}{2}m\,\mathbf{v}\cdot\mathbf{v}\right|_A^B \tag{3.12}$$

where A and B indicate the limits of position between which the definite integral has to be worked out. Since $\mathbf{v}.\mathbf{v} = v^2$, from Eq. 3.12, we get

$$\int_A^B \mathbf{F}\cdot d\mathbf{l} = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2. \tag{3.13}$$

We now want to interpret Eq.3.13. For the sake of simplicity let us consider $v_A = 0$. Then the right-hand of Eq. 3.13 becomes $\frac{1}{2}mv_B^2$. This in accordance with Eq. 3.7 represents the work done on the particle in its attaining a velocity v_B from rest. Thus, this work will be a measure of the energy the particle has acquired by virtue of its motion. So $\frac{1}{2}mv^2$ *is a measure of K.E. of a particle of mass m moving with a velocity v*. Now go back and look at the few steps worked out before Eq. 3.10. You will understand that the above analysis has been done for a system of constant mass. But $\frac{1}{2}mv^2$ is taken to be the measure of K.E. *also for systems having variable mass*. You have read about them in Sec. 2.3.3. Hence, the right-hand side of Eq. 3.13 represents the change in K.E. of the particle between the positions A and B and is expressed as

$$\int_A^B \mathbf{F}\cdot d\mathbf{l} = T_B - T_A. \tag{3.13a}$$

Thus, we have amved at the general meaning of the line integral of a force which can be stated as follows:

The line integral of a force between two positions is equal to the change in K.E. of the particle in coming from the initial to the final position. Moreover, if **F** is a function of position, the line-integral of the force is equal to the work done by the force between these points. So we can now state the **work-energy theorem.**

The work done on a particle by the resultant force acting on it is always equal to the change in K.E. of the particle.

Let us supplement the above statement with a very simple example.

Example 2

A body of mass 1 kg and initial velocity 10 m s^{-1} is sliding on a horizontal surface. If the coefficient of kinetic friction between the body and the surface is 0.5, then find the

a) work done by friction when the body has traversed a distance of 5m along the surface,

b) the initial and the final kinetic energies of the body.

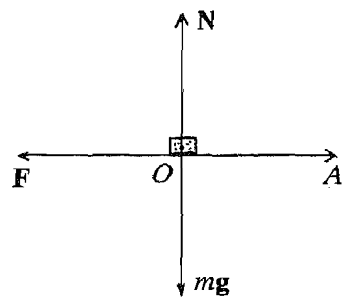


Fig. 3.6

(a) Refer to Fig. 3.6. Let the mass of the body be m. As we have seen in Sec. 2.2.2, the magnitude of the normal reaction $N = mg$ and that of the force of friction $= \mu_k N = \mu_k mg$. Let the displacement of the body be **d** in the direction OA. We shall assume that the force of friction is constant over that displacement. So Eq. 3.7 reduces to

$$W = \mathbf{F}.\mathbf{d}$$

Here **F**, being the force of kinetic friction, is opposite to **d**.

$$W = -Fd = -\mu_k mgd$$

$$W = -(0.5) \times (1\text{kg}) \times (9.8\text{m s}^{-2}) \times (\text{Sin}) = -24.5\text{J}$$

b) Initial K.E. $= \frac{1}{2}mv^2 = \frac{1}{2} \times (1\text{kg}) \times (10\text{ ms}^{-1})^2 = 50\text{J}$

We know from the work-energy theorem that

Work done = Final K.E. − Initial K.E.

∴ Final K.E. = Initial K.E. + W = 50J + (−24.53) = 25.53

SAQ 3

A truck and a car having equal K.E.s are travelling along a straight road. Equal braking forces are applied on them. Which one will travel farther before stopping?

We should realise that the work-energy theorem is not a new law. It is simply a relation between work and K.E. derived from Newton's second law of motion. In Example 2 and SAQ 3 we have used work-energy theorem. But for evaluating the left-hand side of Eq. 3.13 in these problems we have not effectively performed any integration. In general cases, we shall need to work out that integral for which the path of the particle must be known. If the path has a complicated geometry then it is not easy to determine W. However, there is a special kind of force for which W can be determined without the knowledge of the path of the particle. Only the initial and final positions need be known. Let us now discuss about this special kind of force, known as the conservative force. Through this discussion we shall also arrive at the concept of potential energy.

3.2.2 Conservative Force and Potential Energy

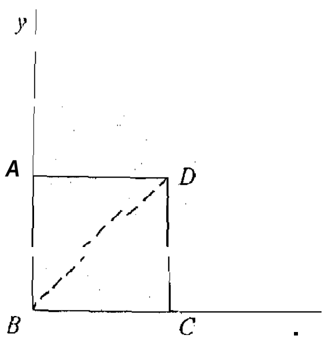


Fig. 3.7

Refer to Fig. 3.7. Let us consider four points A, B, C, D which are the vertices of a square of side L on a smooth vertical wall. A particle of mass m has *to* be taken from *A* to B in two ways

a) directly along the straight line AB,

b) along the path $ADCB$.

We shall calculate the work done by the force of gravity in these two cases. For this we shall take a two-dimensional rectangular Cartesian coordinate system with *x* and y-axes along BC and BA, respectively. As we shall use Eq. 3.7 let us write down the expression for **F** explicitly :

$$\mathbf{F} = -mg\,\hat{\mathbf{j}} \tag{3.14}$$

For case **(a),** the work done is given by

$$W_a = W_{AB} = \int_A^B \mathbf{F}\cdot d\mathbf{l} = \int_A^B -mg\hat{\mathbf{j}}\cdot(-dy\hat{\mathbf{j}}) = \int_A^B mg\,dy$$

$$\text{or } W_{AB} = mg(y_B - y_A) = -mgL. \tag{3.15a}$$

For case (b), the work done is given by

$$W_b = W_{ADCB} = W_{AD} + W_{DC} + W_{CB}$$

$$= \int_A^D \mathbf{F}\cdot d\mathbf{l} + \int_D^C \mathbf{F}\cdot d\mathbf{l} + \int_C^B \mathbf{F}\cdot d\mathbf{l}$$

$$= \int_A^D -mg\hat{\mathbf{j}}\cdot dx\hat{\mathbf{i}} + \int_D^C -mg\hat{\mathbf{j}}\cdot(-dy\hat{\mathbf{j}}) + \int_C^B -mg\hat{\mathbf{j}}\cdot(-dx\hat{\mathbf{i}})$$

$$= 0 + \int_D^C mg\,dy + 0 = mg(y_C - y_D) = -mgL. \qquad (3.15b)$$

So $W_a = W_b$. In other words, the work done in taking the particle from A to B along two different paths is the same, i.e. the work done is independent of the path followed. Such a force is called a **conservative force.** It is defined as that fotce for which the work done is *independent of the path followed and* depends *only* on the *initial and final positions* of the *particle*. Like the force of gravity, electrostatic force is also conservative. You can now work out an SAQ.

SAQ 4

a) Verify for Fig. 3.7 that $W_{AB} = W_{ADB}$

b) Prove that for **a** conservative force the work done around a closed path is zero.

There are forces for which the work done depends on the path followed, called **non-conservative forces.** Thus, for non-conservative forces, the work done around a closed path is non-zero. For example, friction is a non-conservative force.

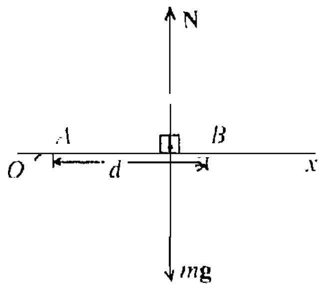


Fig. 3.8: When the motion is from A to B, the force of friction is along BA, i.e. opposite to $\hat{\mathbf{i}}$ and when the motion is from B to A, the force of friction is along $\hat{\mathbf{i}}$.

Refer to Fig. 3.8. Let us consider the motion of a particle over a fixed horizontal distanced, from A to B and back. This is a closed path. How much work is done by the frictional force acting over this path'? The force of friction has magnitude $\mu_k N (= \mu_k mg)$ and is opposed to the direction of motion. Let us choose x-axis to be along the direction of motion. Therefore,

$$W_{AB} = \int_A^B \mathbf{F} \cdot d\mathbf{l} = \int_A^B (-\mu_k m_g)\hat{\mathbf{i}} \cdot dx\hat{\mathbf{i}} = -\mu_k mgd.$$

and $$W_{BA} = \int_B^A (\mu_k mg)\hat{\mathbf{i}} \,.\, (-dx\hat{\mathbf{i}}) = -\mu k\, mgd.$$

So, the total work done around the closed path $ABA = -2\mu_k mgd$. It is not zero.

Let us now go back to the discussion of conservative forces. In fact, before coming to this section, we have dealt with three conservative forces. These were the force of gravity, the electrostatic force between two charges (Example 1) and that of the spring-mass system (SAQ 2). In the first case, the work done in taking a particle of mass m from A to B can be given from Eq. 3.15a by

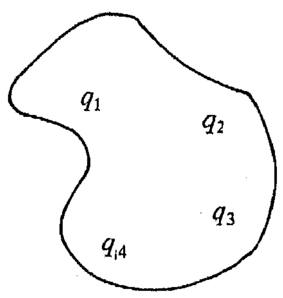
$$W = -(U_B - U_A), \text{ where } U = mgy. \qquad (3.16a)$$

Again from the results of Example 1 and SAQ 2, we get

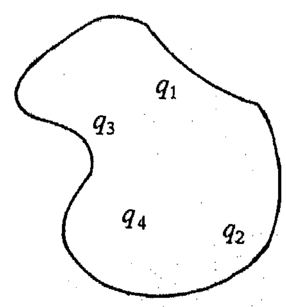
$$W = -(U_B - U_A), \text{ where } U = \frac{\quad}{r}, \qquad (3.16b)$$

and $$W = -(U_2 - U_1), \text{ where } U = \tfrac{1}{2}k_o x^2 \qquad (3.16c)$$

We can see the similarities between the Eqs. 3.16a, 3.16b and 3.16c. In each case we are able to associate a quantity U, the negative of whose change gives the work done, In the first case U is dependent on the position y of the particle and in the second, on the position r of the unit positive charge. In the third case U depends on x. This is a variable which gives the displacement of the free end of the spring from its unstretched position. Thus, the value of x is indicative of the extent of stretching of the spring. So instead of saying that x is the displacement of the free end of the spring from its normal position we say that x measures the configuration of the system. Change in the value of x is a change in configuration of the spring. Similarly, if we have a system of charges (q_1, q_2, q_3, q_4) in an enclosure (Fig. 3.9a) then a change in their relative positions (Fig. 3.9b) also amounts to changing the configuration of the system. Thus, the work done by a force $\mathbf{F}$ in taking a system from A to B can be expressed as



(a)



(b)

Fig. 3.9

$$W = \int_A^B \mathbf{F} \cdot d\mathbf{l} = -(U_B - U_A), \qquad (3.16d)$$

where U is a quantity depending on the configuration of the system. U is called **the potential energy** (or P.E.) of the system. It is defined as the energy a body possesses by virtue of its configuration. In order to measure P.E. we need to know the conservative force which gives rise to this P.E. We shall now see how P.E. is measured. From Eq. 3.16d, we get that if $U_A = 0$, then

$$U_B = -\int_A^B \mathbf{F} \cdot d\mathbf{l} = \int_B^A \mathbf{F} \cdot d\mathbf{l}. \qquad (3.16e)$$

Thus, the P.E. of a system in a certain configuration is measured by the work done by the concerned force in taking the system from that configuration to some standard configuration. Eq. 3.16e gives the P.E. at B with respect to the standard A. We have chosen A as the standard by putting $U_A = 0$. Let us now quickly go back to Eq. 3.15a. There we have $W_{AB} = -mgL$. Or the work done in taking the particle from B to A is mgL. So considering A as standard, the P.E. of the particle at B is mgL. You may now work out an SAQ to clarify the concepts about P.E.

SAQ 5

A peculiar spring is governed by a force **law:** $\mathbf{F} = -Cx^3\,\hat{\mathbf{i}}$ where C is a constant. What is the P.E. at x with the standard $U = 0$ at $x = 0$?

So, measurement of P.E. is never absolute. It is always determined with respect to some standard. You have also learnt that the knowledge of the corresponding force is essential for determining the P.E. Now let us try and see whether we can determine the concemed force or not if the P.E. is known.

An infinitesimal change in the value of P.E. is given in accordance with Eq. 3.16d as

$$dU = -\mathbf{F}.d\mathbf{l}. \tag{3.17}$$

Hence, for a simple case of one-dimensional force like in the case of spring, we have $dU = -F dx$. Thus,

$$F = -\frac{dU}{dx}. \tag{3.18}$$

Eq. 3.18 indicates that *the conservative force is the negative of the rate of change of P.E. with respect to the position vnriable.* Let us take up an interesting application of Eq. 3.18.

Eq.3.18 indicates the connection between P.E. and equilibrium. If the total force acting on a body is zero, then it is in equilibrium. For a conservative force, this equilibrium means

$$\frac{dU}{dx} = 0. \tag{3.19}$$

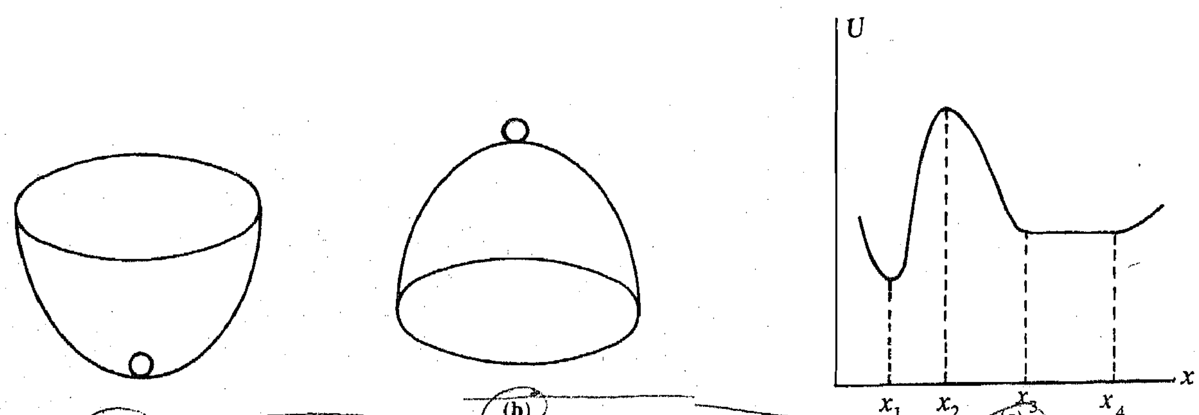
This can occur in three-ways:

i) U is a minimum

ii) U is a maximum

iii) U is a constant independent of x.

At this stage let us recall the very simple fact that a ball falls if it is released. It does so due to gravity. But have you observed that in the process of falling, the P.E. of the ball decreases? Again if you stretch a spring and release it subsequently, it soon returns to its normal length. Thus, its P.E. also decreases, In fact, in nature all processes proceed to that configuration for which the P.E. of the concemed system gets minimised.



(a) (b) (c)

Fig. 3.10

Now, let us consider the situation in case (i) above. If this kind of an equilibrium is disturbed, the system tends to regain its equilibrium configuration. So it is called **stable equilibrium.** A ball resting at the bottom of a bowl (Fig. 3.10 a) provides the example of stable equilibrium. If an equilibrium of type (ii) is disturbed, the system does not return to its equilibrium configuration as a process cannot take a system to a configuration for which **its**

P.E. increases. So it is called **unstable equilibrium. A** ball resting at the top of a bowl (Fig. 3.10b) provides an example of unstable equilibrium. Case (iii) refers to a situation where the system will continue to remain at equilibrium even if its equilibrium is disturbed. This is called **neutral equilibrium. A** book resting on your table provides an example of neutral equilibrium. If you move it by giving a slight push it occupies another position on your table and remains in equilibrium there without bothering to come back to the previous position. Thus, if the variation of P.E. of a system is plotted with position variable x (as in Fig. 3.10c), then the system is in stable and unstable equilibrium at $x = x_1$ and $x = x_2$, respectively. It is in neutral equilibrium over the range $x_3 < x < x_4$.

SAQ 6

What kind of equilibria are the following?

a) A simple pendulum bob at its mean position.

b) A stick held vertically on the fingertip by a juggler,

So, we have seen that if the P.E. of a conservative system is known as a function of position then the corresponding force can be derived from it. We have also seen the role of P.E. in determining the nature of equilibrium of a body.

We shall now combine Eq. 3.16c with the work-energy theorem and arrive at the very important principle of conservation of energy.

3.3.3 Principle of Conservation of Energy

From Eqs. 3.13a and 3.166, we get that

$$W = -(U_B - U_A) = T_B - T_A,$$

$$\text{or } T_A + U_A = T_B + U_B. \qquad (3.20)$$

Since the points A and B are arbitrary, we conclude that for a system being acted upon by a conservative force, the sum of the kinetic energy and the potential energy is always a constant. We denote this constant as the total mechanical energy E of the system, i.e.

$$T + U = E = \text{a constant}. \qquad (3.21)$$

Let us take up an example to illustrate Eq. 3.21.

Example 3

We had discussed projectile motion in Unit 2. Prove that in the absence of air-resistance, the sum total of K.E. and P.E. remains constant.

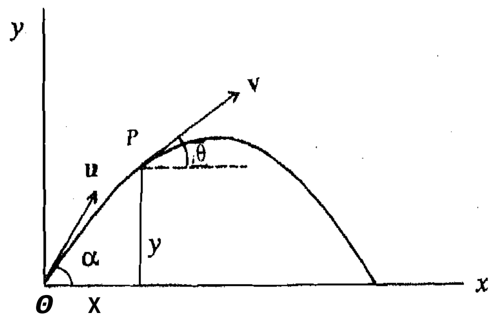


Fig. 3.11: Projectile Motion

Refer to Fig. 3.11. Since the magnitude of the velocity of projection is u, the K.E. of the projectile at O is $\frac{1}{2}mu^2$. Let us take the horizontal level through O as the reference for determining P.E. So, P.E. at O is zero.

$$\text{Hence at } O, \text{ K.E.} + \text{P.E.} = \frac{1}{2} mu^2 \qquad (3.22)$$

At the point P (x, y), the velocity vector makes an angle θ with the horizontal direction. There is no acceleration in the horizontal direction.

Hence, the horizontal component of velocity remains unchanged and the vertical component experiences an acceleration **g** downwards. So

$$v \cos\theta = u \cos\alpha, \qquad (3.23a)$$

$$v^2 \sin^2\theta = u^2 \sin^2\alpha - 2gy. \qquad (3.23b)$$

From Eq. 3.23a, we get

$$v^2 \cos^2\theta = u^2 \cos^2\alpha. \qquad (3.23c)$$

Adding Eqs. 3.23b and 3.23c, we have

$$v^2 = u^2 - 2gy. \tag{3.23d}$$

Now, K.E. at P $= \frac{1}{2} mv^2 = \frac{1}{2} m(u^2 - 2gy) = \frac{1}{2} mu^2 - mgy.$ (3.24a)

And P.E. at P $= mgy.$ (3.24b)

Hence, K.E. + P.E. $= \frac{1}{2} mu^2$, at P. (3.25)

Thus, from Eqs. 3.22 and 3.25, we get that the sum-total of K.E. and P.E. remains constant.

You must have noted that Eq. 3.23d appears to be the same as the equation we get in studying linear motion under gravity. But there is a difference. In linear motion, v and u are along the same direction, while in this case they are not. So you should not feel that proving this equation is a futile exercise.

At this stage let us go back to Example 2. There we had seen that the final K.E. is less than the initial K.E., whereas the initial and final P.E.s are the same, as the body was moving on a horizontal surface. So we cannot say that (K.E. + P.E.) is a constant. This means that Eq. 3.21 does not hold. Now let us try to find out what has happened to the loss in K.E. You must have noted that there is a non-conservative force in the form of friction and the work done by that force is –24.5 J. The negative sign indicates that the work done against the force of friction is 24.5 J which is exactly equal to the loss in K.E. Now, is the work done against the force of friction lost? Well, the answer is that it is not lost. It has been dissipated in the form of heat energy. This energy which amounts to heating the body and the surface is not useful at all. But still the energy is *not lost* in the true sense of the term. Through Example 2 we may conclude that

K.E. + P.E. + (work done against Friction) = a constant

or (Total mechanical energy) + (Energy dissipated) = a constant.

Thus, we arrive at the statement of the *principle of conservation of energy:*,

> *Energy cannot be created nor can it be destroyed but it can be transformed from one form to another, the total amount of energy in the universe remaining constant.*

Let us take up an application of this principle.

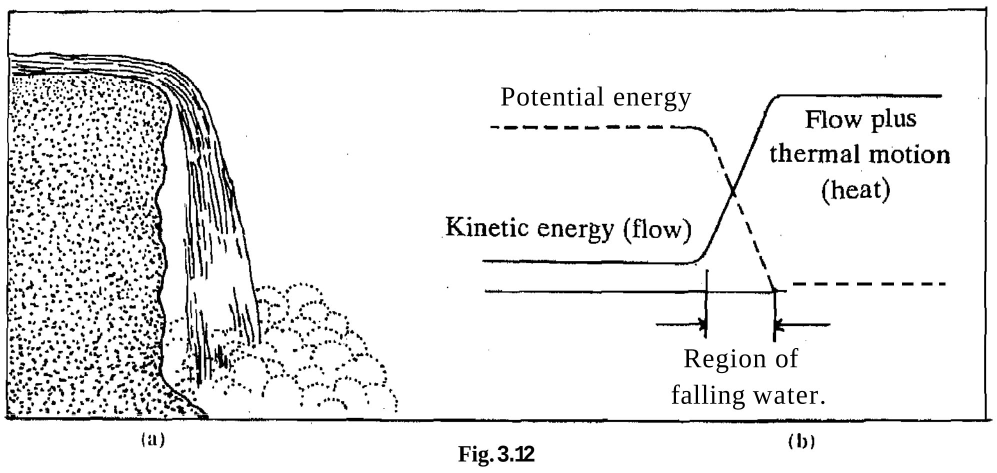


Fig. 3.12

The water at the top of the waterfall (Fig. 3.12 a) has gravitational P.E. which in falling is converted into K.E. So, P.E. decreases and at its cost the K.E. increases, maintaining the sum of K.E. and P.E. constant. Here we assume that the water particles do not experience any friction with the bed of the fall, However, on coming to the foot of the fall the P.E. becomes zero. The energy is solely kinetic. But what happens to this K.E. later? A part of K.E. remains with the water as it continues to flow and the rest can be used to drive the motor of a turbine and hydroelectricity can be generated. In any case some energy is always dissipated. The variation of different forms of energy is depicted in the graph in Fig, 3.12 b.

Whenever there is a conversion of energy from one form to another this principle is applicable. For example, in a lead acid cell chemical energy is converted to electrical energy whereas in a solar cell light energy is converted to electrical energy. The quantity of energy in one form is always equivalent to the quantity in the converted form.

In nature we always come across non-conservative forces. For example, in the case of a body falling freely under gravity there is some air resistance. When a body is sliding along an inclined plane there is ftiction. But if these forces are negligible then the principle of conservation of energy can be used in the form of Eq. 3.21 as a good approximation. We shall now see such an application.

3.3.4 Energy Diagrams

Fig. 3.13 shows the track of a car having negligible friction. How fast must the car be moving at point P if it is to reach point R? What happens if it is going slower than this?

As the friction is negligible the principle of conservation of energy will provide answers to our questions. Let us consider the lowest point on the track as our reference for determining P.E. The energy at P is ($\frac{1}{2} mv_P^2 + mgh_P$), and this is the total mechanical energy everywhere, as mechanical energy is conserved. To reach the point S, the car must clear the highest peak at R, where P.E. is mgh_R. If it is just able to do so, its K.E. is extremely close to zero on the peak. From the principle of conservation of energy we have

$$mgh_R = \frac{1}{2} mv_P^2 + mgh_P,$$

$$\text{or} \quad \frac{1}{2} mv_P^2 = mg(h_R - h_P). \tag{3.26}$$

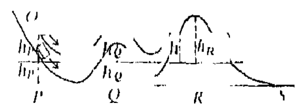


Fig. 3.13

In other words, the initial K.E. must be at least equal to the difference in P.E. between the highest and the initial point. Eq. 3.26 may be solved for v_P to get the minimum speed required at P to get to R. What happens if the car is moving a little slower? Then it would not be able to reach the top of the second peak but will reverse direction before that from a point T where its K.E becomes zero. If it occurs at a height h, then

$$mgh = \frac{1}{2} mv_P^2 + mgh_P. \tag{3.27}$$

It will head back clearing peak Q, then down and'up past point P to a point O where its K.E is again zero. You can now quickly work out an SAQ.

SAQ 7

What is the height of tho point O above the reference level in Fig. 3.13?

O and Tare called the turning points set by the value of total energy. With still lower speed, the car won't clear peak Q and its motion will be confined to the first valley alone.

Fig. 3.14 is a drawing of the actual track followed by the car. But, because gravitational potential energy near earth's surface is directly proportional to height, we can also regard it as a plot of potential energy versus position: a potential energy curve. We can understand the car's motion graphically by plotting the car's total energy on the same graph as the potential energy curve. Since total energy E is constant, the total energy curve is a straight, horizontal line. Fig. 3.14 shows the potential energy curve and the total energy curve for several values of the total energy. These graphs tell us immediately about the motion of the car. In Fig 3.14a, the total energy E_a exceeds the potential energy at peak R, therefore, the car will reach R with kinetic energy to spare, and will make it all the way to S. In Fig 3.14b, the total energy E_b is less than the potential energy at R. The car must stop when its total energy is entirely potential. This happens when the total energy curve intersects the potential energy curve; the points of intersection are the turning points that bound the car's motion. In Fig. 3.14c, the total energy E_c is still lower, and the turning points are closer together. We say that the car is trapped in the potential well between its turning points.

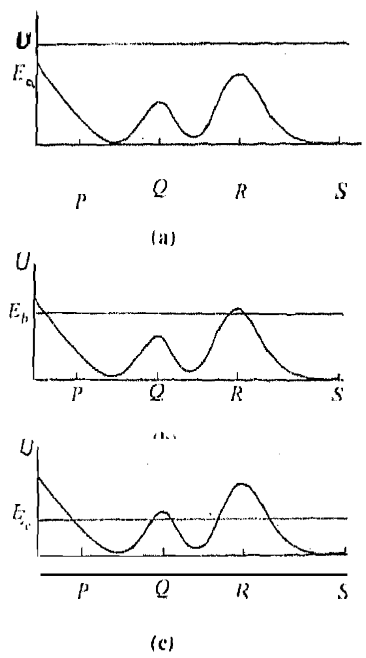


Fig. 3.14

We know that for a conservative system K.E. + P.E. = E, the total energy. i.e.

$$\frac{1}{2} mv^2 + U = E,$$

$$\text{or } v = \sqrt{\frac{2}{m}(E-U)}. \tag{3.28}$$

If $U > E$, then from Eq. 3.28, v is imaginary and motion is not possible. But if $U < E$ then motion is possible.

In both Figs. 3.14b and 3.14c the car's total energy E exceeds the potential energy in the rightmost region, so that motion in this region is possible. But starting at point P, the car is blocked from this region, because near peak R, $U > E$. In the case of Fig. 3.14c, peak Q also poses a situation (i.e. $U>E$) that keeps the car out of the valley between Q and R where motion is possible. Such a peak which does not allow motion by having $U>E$ is called a potential barrier. We shall use the terminologies 'potentia wells' and 'bamers' widely in our courses on Quantum, Atomic and Molecular Physics, Nuclear Physics and Solid State/ Materials Science.

We have thus studied a few applications of the principle of conservation of energy. We have studied another conservation principle, that of linear momentum, in Unit 2. We shall now take up the study of collisions which involve both these conservation principles

3.4 ELASTIC AND INELASTIC COLLISIONS

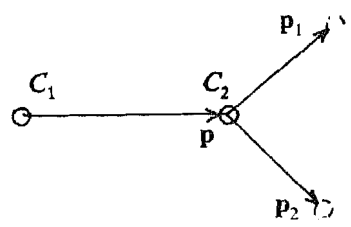


Fig. 3.15

We know that certain metallic surfaces emit electrons when ultra-violet light is made to fall upon it. This process can be explained through the study of collisions. It can also throw light upon several aspects of other processes like generation of X-rays and artificial radioactivity. However, let us begin our study on collisions by thinking of a coin hit by another on the surface of a table. You may try this yourself with the help of two coins. What do you observe? A situation of the type is shown in Fig. 3.15. The striking coin C_1 and the struck coin C_2 move on two sides of the original line of motion of C_1. Let the linear momentum of C_1 be **p** immediately before it hits C_2. Let the momenta of C_1 and C_2 be $\mathbf{p}_1$ and $\mathbf{p}_2$ respectively, after the collision. We shall assume that the surface of the table is reasonably smooth so that the frictional forces can be neglected. Then during the collision no external force acts on the system of coins. So its linear momentum is conserved, i.e.

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2. \qquad (3.29)$$

The principle of conservation of linear momentum holds for all sorts of collisions provided no external force acts. The principle of conservation of energy aiso holds. Since the coins are always on a table whose surface is horizontal, their P.E. is always zero with respect to the table top as standard. This means that the K.E. of C_1 before collision is equal to the sum total of the K.E.s of C_1 and C_2 after collision and any amount of energy in the form of heat or sound, which might have dissipated due to collision. We must remember that we have assumed that the surface of the table is frictionless. Now if, the kinetic energy of the system remains constant in a collision process, then it is said to be an elastic collision. In other words, the condition of an elastic collision of the coins is given by

$$T = T_1 + T_2, \qquad (3.30)$$

where T is the K.E. of C_1 before collision and T_1, T_2 are the K.E.s of C_1, C_2, respectively after collision. If the kinetic energy of the system does not remain constant then the collision is said to be inelastic. But the total energy is conserved in both the cases. Let us first discuss elastic collisions.

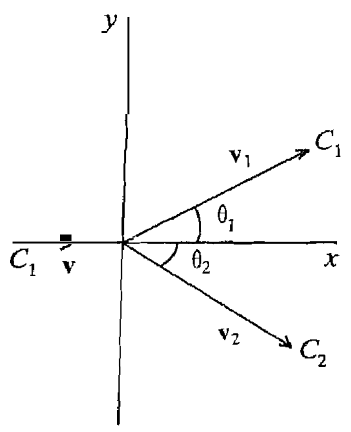


Fig. 3.16

Study of elastic collisions

We understand that in the case of an elastic collision both the equations 3.29 and 3.30 will hold. We shall now apply them to obtain the angular separation between the directions of motion of the coins after the collision. We shall primarily be applying some results of trigonometry. This kind of an analysis will be required for solving any problem on collision in two dimensions. For this we redraw Fig. 3.15 as shown in Fig. 3.16. We take the x-axis along the original direction of motion of C_1 and y-axis perpendicular to it. C_1 is referred to as the *projectile* and C_2 as the target. Let the masses of C_1 and C_2 be m_1 and m_2, respectively. Before collision, C_1 is moving with a velocity **v**. Let the velocities of C_1 and C_2 after collision be $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively, $\mathbf{v}_2$ is called the recoil velocity *and* θ_2, the angle of recoil. Eq. 3.29 can be written as,

$$m_1 \mathbf{v} = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2.$$

Using Eqs. 1.3a and 1.3d, we have

$$m_1 v\hat{\mathbf{i}} = m_1 (v_1 \cos\theta_1 \hat{\mathbf{i}} + v_1 \sin\theta_1 \hat{\mathbf{j}}) + m_2 (v_2 \cos\theta_2 \hat{\mathbf{i}} - v_2 \sin\theta_2 \hat{\mathbf{j}}),$$

or $$m_1 v\hat{\mathbf{i}} = (m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2)\hat{\mathbf{i}} + (m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2)\hat{\mathbf{j}}.$$

From Eqs. 1.5 and 1.6, we get

$$m_1 v = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \qquad (3.31a)$$

and $$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2. \qquad (3.31b)$$

Again from Eq. 3.30, we get

$$m_1 v^2 = m_1 v_1^2 + m_2 v_2^2 \qquad (3.32)$$

First we shall eliminate θ_1 between Eqs. 3.31a and 3.31b, i.e. we have

$$(m_1 v_1)^2 = (m_2 v_2 \sin\theta_2)^2 + (m_1 v - m_2 v_2 \cos\theta_2)^2$$

or $$m_1^2 v_1^2 = m_2^2 v_2^2 + m_1^2 v^2 - 2 m_1 m_2 v v_2 \cos\theta_2.$$

We shall now use Eq. **3.32** to get an expression of v_2 in terms of v and 8,. Using Eq. 3.32 we get,

$$0 = m_2^2 v_2^2 + m_1 m_2 v_2^2 - 2m_1 m_2 v v_2 \cos\theta_2 .$$

This is effectively a quadratic in v_2 of which $v_2 = 0$ is a trivial solution. We disregard that and write only the acceptable solution

$$v_2 = \frac{2m_1 m_2 v \cos 8,}{m_2^2 + m_1 m_2} = \frac{2\alpha v \cos\theta_2}{1+\alpha} \qquad (3.33)$$

where $\alpha = \dfrac{m_1}{m_2}$.

Again, from Eqs. 3.31a and 3.31b, we get

$$\frac{m_1 v, \sin\theta_1}{m_1 v_1 \cos\theta_1} = \frac{m_2 v_2 \sin\theta_2}{m_1 v - m_2 v_2 \cos\theta_2}$$

Multiplying the numerator and denominator of the right-hand side by 2 cos θ_2, we get,

$$\tan\theta_1 = \frac{m_2 v_2 \sin 2\theta_2}{2m_1 v \cos\theta_2 - 2m_2 v_2 \cos^2\theta_2}.$$

Using Eq. 3.33, we get

$$\tan\theta_1 = \frac{m_2 v_2 \sin 2\theta_2}{(m_1 + m_2)v_2 - 2m_2 v_2 \cos^2\theta_2} = \frac{\sin 28,}{a - \cos 2\theta_2} \qquad (3.34)$$

Eq. 3.34 indicates that the relation between θ_1 and θ, is strongly dependent on **a.** We shall discuss the extreme case when $a \gg 1$, (i.e. $m_1 \gg m_2$).

Since sin $2\theta_2$ and cos 28, lie between -1 and $+1$, in this case we have tan $\theta_1 \to 0$ or 8, $\to$ 0. And as $\theta_1 \to 0$ we get from Eq. **3.3**1b that 8, $\to$ 0 also. ***So*** when the projectile is much heavier than the target then both move along the same straight line as that of the initial direction of the projectile.

SAQ 8

Prove that a) θ_1+ 28, = 180° when $a << 1$

and b) θ_1 + 8, = 90° when $\alpha = 1$

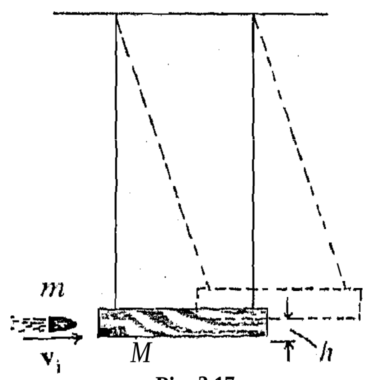
We shall now study an application of inelastic collision.

Ballistic pendulum

The ballistic pendulum is a device for measuring the velocity of a bullet. The pendulum is a large wooden block of mass M hanging vertically by two cords. A bullet of mass m strikes the pendulum with an initial velocity $\mathbf{v}_i$ and gels embedded in it. The final velocity **v,** of the system after collision is much less than that of the bullet before collision. This final velocity can easily be determined, so that the initial velocity of the bullet can be computed by applying the law of conservation of linear momentum, Initial linear momentum of the bullet = m v,. Linear momentum of the system after collision = $(m + M)\,\mathbf{v}_f$

So from conservation of linear momentum,

$$m\mathbf{v}_i = (m + M)\,\mathbf{v}_f. \qquad (3.35)$$



Pig. 3.17

The K.E. before collision is $\frac{1}{2} m v_i^2$ and the total K.E. after collision is

$\frac{1}{2}(m + M)v_f^2 = \frac{1}{2}\dfrac{m^2 v_i^2}{m + M}$ from Eq. 3.35. Now, we have

$$\text{(K.E. before collision)} - \text{(K.E. after collision)} = \tfrac{1}{2} m v_i^2\left(1 - \frac{m}{m+M}\right) = \tfrac{1}{2}\,\frac{mM}{m+M}\,v_i^2,$$

which is a positive quantity. This means that there is a loss of K.E. Hence, the collision is inelastic. However, the K.E. after collision makes the wooden block swing up to a maximum height, h as shown in Fig. 3.17. Our task is to determine v_i in terms of the known parameters ***m***, M and ***h.*** The K.E. of the bullet and the block is used up in raising the block through a height h. So the block and bullet acquire a P.E. equal to $(M + m)\,gh$. The K.E. of the block and bullet $= \frac{1}{2}(M + m)v_f^2 = (M + m)gh$, from the principle of conservation of energy. So $v_f^2 = 2gh$ or $v_f = \sqrt{2gh}$. Hence from **Eq.** 3.35, we get

$$v_i = \frac{M + m}{m}\sqrt{2gh}. \qquad (3.36)$$

You may like to make an estimate of v_i in the form of a numerical example in the following SAQ,

SAQ 9

In a ballistic pendulum, the masses of the bullet and the block are 5 g and 2 kg, respectively. After being struck by the bullet the block along with the bullet is raised through 0.5 cm. Find the velocity of the bullet. ($g = 9.8\ m\ s^{-2}$)

How long has it taken you so far to go through Unit 3? May be something like 3 hours. Some of your friends might have taken 4 hours to complete the same matter and some may again have taken only 2 hours. So all of you have covered the study material to the same extent, but your rates of working are different. This throws some light on the following question! Why do you feel more exhausted when you run up the staircase at constant speed than when you walk up the same at a constant speed? In each case you exert an average force exactly equal to your weight and do so over a fixed distance. And as the product of force and distance is the work done, the same amount of work is done in each case. But what matters is the rate at which work is done. We shall discuss this aspect now.

3.5 POWER

Power is defined as the rate of doing work. If an amount of work AW is done in a time At, then the **average power** $\bar{P}$ **is**

$$\bar{p} = \frac{AW}{At}. \tag{3.37}$$

If the rate varies with time, we define **instantaneous power** as the average power taken in the limit of arbitrarily small time interval At:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}. \tag{3.38}$$

Eqs. **3.37** and 3.38 show that the unit of power is joule s^{-1}, whose other name is watt. You may now try a very simple SAQ.

SAQ 10

A man ascends to Badrinath Temple from Joshimath, a vertical rise of 1,500 m. His mass is 60 kg. He takes 5 h. A 1,500 kg car drives up the motorable road for the same vertical rise in 1h. What is the average power exerted in each case if we neglect friction for the sake of simplicity? Assume that the man and the car maintain constant speed.

We have not seen how P depends on the applied force F. For this we go back to Eq. 3.1. Using Eqs. 3.1 and 3.38 we get

$$P = \lim_{\Delta t \to 0} \frac{\mathbf{F} \cdot \mathbf{Ai}}{\mathbf{At}} = \mathbf{F} \cdot \lim_{\Delta t \to 0} \frac{\Delta \mathbf{l}}{\Delta t},$$ (as the process of taking dot product with F is independent of At).

$$\text{or} \quad P = \mathbf{F} \cdot \frac{d\mathbf{l}}{dt} = \mathbf{F} \cdot \mathbf{v} \tag{3.39}$$

So we have obtained the expression of P in the form of a scalar product. You can apply your knowledge of scalar product to work out the following SAQ.

SAQ 1%

A man weighing 60 kg is riding his 15 kg bicycle. What power must he supply to maintain a steady speed of 20 kmh^{-1} (a) on a level ground and (b) while going up a 5° incline if the frictional force is 30N in each case?

Let us now summarise what we have learnt in this unit.

3.6 SUMMARY

- The work done by a force over a path from A to B is given by

$$W = \int_A^B \mathbf{F} \cdot d\mathbf{l}.$$

- Work-Energy theorem:
  The work done on a particle by the resultant force acting on it is always equal to the change in kinetic energy of the particle.
- If the work done by a force on a particle in taking it from one point to another is independent of the path followed and dependent only on the initial and final positions of the path then the force is said to be conservative.

- The work done by a conservative force **F** in taking a particle from A to B is given by

$$\int_A^B \mathbf{F} \cdot d\mathbf{l} = -(U_B - U_A)$$

where U_A and U_B are the potential energies of the particle at the positions A and A, respectively.
- In the absence of non-conservative forces the total mechanical energy of a system is conserved.
- Principle of conservation of energy: Energy cannot be created, nor can it be destroyed but it can be transformed from one form to another, the total amount of energy in the universe remaining constant.
- In any collision process, the linear momentum and the total energy are conserved. In an elastic collision kinetic energy is conserved, whereas in an inelastic collision it is not.
- Power P is defined as $P = \frac{dW}{dt}$

  For a constant force **F**, power is $P = \mathbf{F} \cdot \mathbf{v}$.

3.7 TERMINAL QUESTIONS

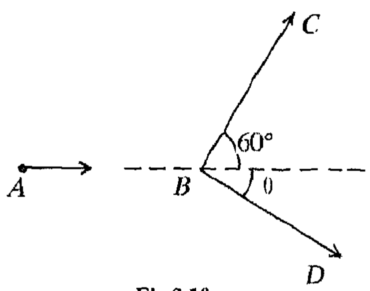


Fig 3.18

1. Cite two examples in which you might think you are doing work but from the point of view of physics you are not doing so.
2. An electron is projected with an initial speed of 3.24×10^5 m s^{-1} directly towards a proton which is at rest. The electron is initially at a very large distance from the proton. At what distance from the proton does the electron's speed become instantaneously equal to twice its initial value? [Hint: Use Eq. 3.9 along with the work-energy theorem. Take the value of k in Eq. 3.9 equal to 9×10^9 Nm^{2}C^{-2}.]
3. In a nuclear collision, an alpha-particle A of mass 4 units is incident with speed v on a stationary helium nucleus B of mass 4 units (Fig. 3.18). After collision A moves in the direction BC with a speed $v/2$ at an angle of 60° with the initial direction AB, and the helium nucleus moves along BD. Calculate the speed of the He nucleus along BD and the angle 8.
4. Refer to Fig. 3.19. The variations of potential and total energy with position are shown for a particle executing simple harmonic oscillation. Indicate the turning points. Also indicate the point where the velocity of the oscillator is maximum.

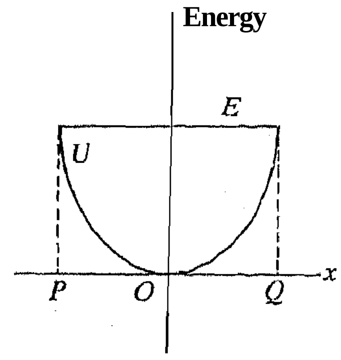


Fig. 3.19

3.8 ANSWERS

SAQs

1. Tension in the string of a simple pendulum. See Fig. 3.20. It acts perpendicular to the displacement of the bob, which is always along the tangent to the circular path,
2. $\mathbf{F} = -k_0 x\hat{\mathbf{i}}$; $d\mathbf{l} = dx\hat{\mathbf{i}}$, $\mathbf{F} \cdot d\mathbf{l} = -k_0 x dx$.

   $$W = -\int_{x_1}^{x_2} k_0 x dx = -\frac{k_0}{2}(x_2^2 - x_1^2).$$
3. From work-energy theorem we know that work done =change in K.E. Let the distances travelled by the truck and car be x_1 and x_2, respectively. Their initial K.E.s are the same and final K.E.s are zero. So change in K.E. is same for both. Let the magnitude of the equal braking force be F. Thus, we have $Fx_1 = Fx_2$

   or $x_1 = x_2$. So they travel equal distances before stopping.

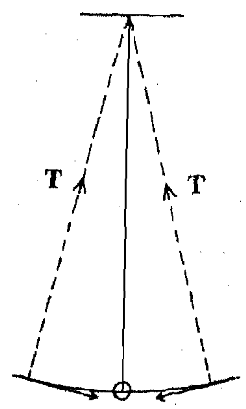


Fig. 3.20

4. a) Refer to Fig. 3.21.

   $$W_{ADB} = W_{AD} + W_{DB} = \int_A^D \mathbf{F} \cdot d\mathbf{l} + \int_D^B \mathbf{F} \cdot d\mathbf{l}$$

   Now,

   $$\int_A^D \mathbf{F} \cdot d\mathbf{l} = \int_A^D -mg\hat{\mathbf{j}} \cdot dx\hat{\mathbf{i}} = 0$$

   And

$\int_B^B \mathbf{F}.\mathbf{dl} = \int_D^B -mg\hat{\mathbf{j}}\cdot(-dx\hat{\mathbf{i}} - dy\hat{\mathbf{j}}) = \int_D^B mg\,dy = mg\int_L^0 dy = -mgL$ and we know from Eq. 3.15a that $W_{AB} = -mgL$

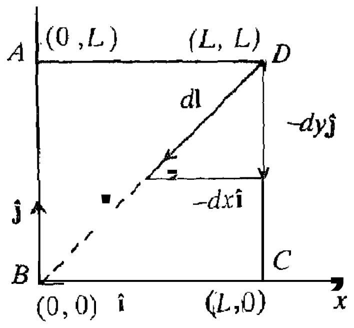


Fig 3.21

So $W_{AB} = W_{ADB}$.

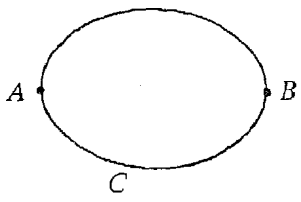
b) Refer to Fig. 3.22. Let us consider two points *A* and B in space. We now join A and *B* by two paths ADB and ACB, so that we get a closed path *ADBCA*. We know that for a conservative force, $W_{ADB} = W_{ACB}$

or $W_{ADB} = -W_{BCA}$ $\quad\therefore\quad W_{ADB} + W_{BCA} = 0$

or $W_{ADBCA} = 0$. Hence, the work round a closed path is zero.

5. We shall use Eq, 3.16 to determine the P.E. Here B corresponds to the point having *x*-coordinate equal to *x* and A to *x* = 0. $\mathbf{F} = -Cx^3\hat{\mathbf{i}}$, $dl = dx\hat{\mathbf{i}}$

So the required P.E. is $U = -\int_0^x (-Cx^3\hat{\mathbf{i}})\cdot(dx\hat{\mathbf{i}}) = \int_0^x Cx^3dx = \frac{1}{4}Cx^4$



Pig 3.22

6. a) Stable b) Unstable
7. It is again *h* as K.E. becomes zero when P.E. = *mgh*.
8. a) If $\alpha \ll 1$, then from Eq. 3.34, we get $\tan\theta_1 = -\tan 2\theta_2 = \tan(180" - 28,)$

   or $0, = 180° - 2\theta_2 \quad \therefore \theta_1 + 2\theta_2 = 180°$

   b) If $a = 1$, then from **Eq.** 3.34, we get $\tan 0, = \frac{\sin 28}{1 - \cos 2\theta_2}$

   or $\tan\theta_1 = \frac{2\sin\theta_2\cos\theta_2}{2\sin^2\theta_2} = \cot\theta_2 = \tan(90° - \theta_2)$

   $\therefore$ 8, $= 90° - \theta_2$, or $\theta_1 + 0, = 90°$.
9. We shall use Eq. 3.36. There we put M = *2* kg, ***m*** = 0.005 kg, *g* = 9.8 m s^{-2} and h = 5 x 10^{-3} m and get v_i = 125.5 m s^{-1}.
10. $P_{man} = \left(\frac{AW}{\Delta t}\right)_{man} = \frac{(60\text{ kg})(9.8\text{ m s}^{-2})(1500\text{ m})}{(5 \times 3600)\text{s}} = 49\text{ W}$.

    $P_{car} = \left(\frac{AW}{\Delta t}\right)_{car} = \frac{(1500\text{ kg})(9.8\text{ m s}^{-2})(1500\text{ m})}{(1 \times 3600)\text{s}} = 6125\text{W}$.
11. $20\text{ km h}^{-1} = \frac{20 \times 1000}{3600}\text{ m s}^{-1} = \frac{50}{9}\text{ m s}^{-1}$

    Now P = F **.** **v,** where F = force applied.

    a) On level ground F is equal and opposite to force of friction $\mathbf{F}_o$.

    So, P = F .v = $F_o v$ = (30 N) (50/9 m s^{-1}) = 166.7 W.

    b) On the slope F has to oppose $\mathbf{F}_o$ and the component of mg down the plane, (as shown in **Fig.** 3.23). So F must act up the plane and is equal in magnitude to

    $(F_o + mg\sin\theta)$

    $P = (F_o + mg\sin\theta)v$

    = [30 N + (75 kg) (9.8 m s^{-2}) (sin 5°)] (50/9 m s^{-1})

    = [30 N + 64 N] (50/9 m s^{-1}) = *522.2* W

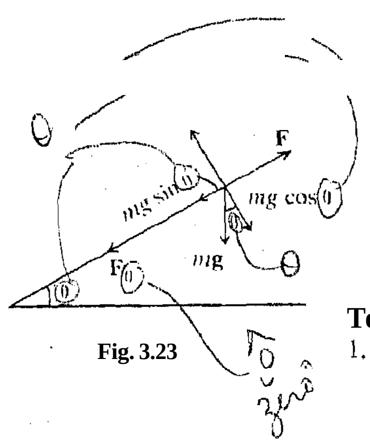


Fig. 3.23

Terminal Questions

1. Examplel. A boy reads a book continuously for a long time sitting on a chair.

   Example **2.** A cashier maintains the account of **total** transaction during the banking hours of a day.
2. Let the initial speed of the electron be *v*. Its change in K. E. = $\frac{1}{2}m_e\{(2v)^2 - v^2\} = \frac{3}{2}m_ev^2$, where m_e = mass of the electron. According to the work-energy theorem this change in K.E. is equal to the work done in bringing it from a large distance (which we shall assume to be infinite) to a point whose distance is *x* metres from the proton. Our task is to determine *x*. The above work done is **equal** to the product of the magnitude of charge

on an electron and the work done in bringing a unit positive charge from infinity to the said point. Let the magnitude of charges on a proton and an electron be ***e*.** Using Eq. 3.9 and putting $r_A = \infty$, $r_B = x$ and $q = e$ we get that the work done is equal to

$$(-e) \times ke\left(0 - \frac{1}{x}\right) = \frac{ke^2}{x}$$

From work-energy theorem,

$$\frac{3}{2} m_e v^2 = \frac{ke^2}{x} \quad or\ x = \frac{2ke^2}{3m_e v^2}. \qquad (3.40)$$

For our problem, $k = 9 \times 10^9$ N m^2 C^{-2} $v = 3.24 \times 10^5$ m s^{-1}. **Now,** putting the values of k, v, m_e and e in Eq. 3.40. we get $x = 1.6 \times 10^{-9}$ m.

3. Refer to Fig. 3.24. This is similar to Fig. 3.16. We shall write Eqs. 3.31a and 3.31b by putting $\theta_1 = 60°$, $\theta_2 = \theta$, $v = v$, $v_1 = v/2$, $v_2 = u$. We have

$$m_1 v = m_1 \frac{v}{2} \cos 60° + m_2 u \cos\theta. \qquad (3.41a)$$

$$0 = m_1 \frac{v}{2} \sin 60° - m_2 u \sin\theta, \qquad (3.41b)$$

Our task is to determine u in terms of v, and the value of θ. For our problem $m_1 = m_2 = 4$ units. So we have, $4v = v + 4u\cos\theta$ or $4u\cos\theta = 3v$. (3.41c)

and $0 = \sqrt{3}v - 4u\sin\theta$ or $4u\sin\theta = \sqrt{3}v$.

Squaring and adding Eqs. 3.41c and 3.41d, we get

$$16u^2 = 12v^2 \quad \text{or} \quad u = \frac{\sqrt{3}}{2} v. \qquad (3.41d)$$

Dividing Eq. 3.41d by Eq. 3.41c, we get $\tan\theta = 1/\sqrt{3}$ or $\theta = 30°$.

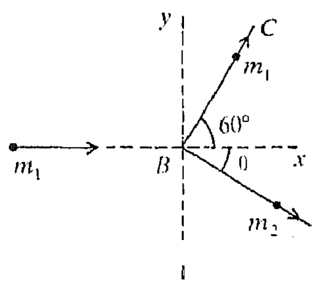


Fig. 3.24

4. Since $U = E$ at P and Q, the K.E.s of the particle at these points are zero, *So* P and Q are the turning points. For answering the second part we shall use Eq. **3.28.** Since E is a constant, v is maximum when U is minimum, i.e. zero. *So* the velocity of the particle is maximum at O.