UNIT 25 FACTOR ANALYSIS

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25.1 INTRODUCTION

Factor analysis is a generic name given to a class of multivariate statistical methods whose primary purpose is to define the underlying structure in a data matrix and achieve the objectives of summarization and reduction of data. The essential purpose of factor analysis is to describe, if possible, the covariance relationships among many variables in terms of a few underlying but unobservable random quantities called *factors*. Factor analysis has originated in the field of psychology 100 years ago by Charles Spearman to define the concepts like intelligence, attitude, measures of mathematical skill, vocabulary, other verbal skills, artistic skills, logical reasoning ability, etc.

A frequent source of confusion in the field of factor analysis is the term factor. It sometimes refers to a hypothetical, unobservable variable as in the phrase common factor. In this sense, factor analysis must be distinguished from component analysis since a component is an observable linear combination. Factor is also used in the sense of matrix factor, in that one matrix is a factor of second matrix if the first matrix multiplied by its transpose equals the second matrix. In this sense, factor analysis refers to all methods of data analysis using matrix factors, including component analysis and common factor analysis, which is discussed in Section 25.2.

A common factor is an unobservable hypothetical variable that contributes to the variance of at least two of the observed variables. The unqualified term "factor" often refers to a common factor. A unique factor is an unobservable hypothetical variable that contributes to the variance of only one of the observed variables. The model for common factor analysis posits one unique factor for each observed variable is discussed in Section 25.3. The common factors generate the covariance among the observable responses while the specific factor contribute only to the variances of their particular response. Basically the factor model is motivated by the following argument - suppose variables can be grouped by their correlations, i.e., all variables within a particular group are highly correlated among themselves but have relatively small correlations with variables in a different group. It is conceivable that each group of variables represents a single underlying construct, or factor, that is responsible for the observed correlations. For example, for an individual, marks in different subjects may be governed by aptitudes (common factors) and individual variations (specific factors) and interest may lie in obtaining scores on unobservable aptitudes from observable data on marks in different subjects. The methods of estimation are discussed in Section 25.4 and the factor analysis versus clustering and multidimensional scaling is discussed in Section 25.5.

Objectives

After reading this unit, you should be able to:

- have an overview of factor analysis and meaning of common factors.
- derive factors.
- define factor loadings.
- identify the number of factors.
- apply factor analysis to data.

25.2 ABSOLUTE VERSUS HEURISTIC USES OF FACTOR ANALYSIS

A heuristic is a way of thinking about a topic which is convenient even if not absolutely true. We use a heuristic approach when we talk about the sun rising and setting as if the sun moved around the earth, even though we know it doesn't. Spearman hypothesized that if g could be measured and one could select a subpopulation of people with the same score on *g*, in that subpopulation one would find no correlations among any tests of mental ability. In other words, he hypothesized that *g* was the only factor common to all those measures. Spearman's *g* theory of intelligence, and the activation theory of autonomic functioning, can be thought of as absolute theories which are or were hypothesized to give complete descriptions of the pattern of relationships among variables.

It is well known that Rubenstein (1986) studied the nature of curiosity by analyzing the agreements of junior-high-school students with a large battery of statements such as “I like to figure out how machinery works” or “I like to try new kinds of food.” A factor analysis identified seven factors: three measuring enjoyment of problem-solving, learning, and reading; three measuring interests in natural sciences, art and music, and new experiences in general; and one indicates a relatively low interest in money. Rubenstein never claimed that her list of the seven major factors of curiosity offered a complete description of curiosity. Rather those factors merely appear to be the most important seven factors - the best way of summarizing a body of data. Factor analysis can suggest either absolute or heuristic models; the distinction is in how you interpret the output.

25.3 THE FACTOR MODEL

Suppose observations are made on p variables, $x_1, x_2, \ldots x_p$. The factor analysis model assumes that there are m underlying factors (m<p) $f_1, f_2, \ldots, f_m$ and each observed variables is a linear function of these factors together with a residual variate, so that

$$x_j = a_{j1} f_1 + a_{j2} f_2 + \ldots + a_{jm} f_m \qquad \text{where} \qquad j = 1,2, \ldots, p$$

where $a_{j1}, a_{j2}, \ldots, a_{jm}$ are called factor loadings i.e. a_{ji} is loading of j-th variable on i-th factor. y_j 's are called specific factors.

The proportion of the variance of the j-th variable contributed by the *m* common factors is called the j-th communality and the proportion due to the specific factor is called the uniqueness, or specific variance.

In matrix notation, we can write the model as

$$X = \mathbf{a}F + Y$$

The covariance matrix of X if the population means of X_i are zero, is given as

$$\boldsymbol{\Sigma} = \text{cov}\,(X) = E(X-\mu)(X-\mu')$$
$$= \mathbf{aa}' + \boldsymbol{\psi}$$

where $\psi = \begin{bmatrix} \psi_1 & \cdots & 0 \\ \vdots & \ddots & \\ 0 & & \psi_p \end{bmatrix}$ and $\psi_i = \text{var}(y_i)$

Also, $\text{var}(X_i) = \sigma_{ii} = a_{i1}^2 + a_{i2}^2 + \cdots + a_{im}^2 + \psi_i$,

$\text{cov}(X_i, X_j) = \sigma_{ij} = a_{i1}a_{j1} + a_{i2}a_{j2} + \cdots + a_{im}a_{jm}$

and $\text{cov}(\mathbf{X}, \mathbf{F}) = \mathbf{a}$

or $\text{cov}(X_i, F_j) = a_{ij}$.

where we have assumed that y_j are uncorrelated with each other and with factors $f_1, f_2, \cdots, f_m$. Also f_j are standardised variables and are uncorrelated.

Example 1: Consider the three random variables X_1, X_2 and X_3 having the following the covariance matrix.

$$\Sigma = \begin{bmatrix} 1 & 0.63 & 0.45 \\ 0.63 & 1 & 0.35 \\ 0.45 & 0.35 & 1 \end{bmatrix}$$

for p = 3 and m = 1, write its factor model.

Solution: Comparing the given covariance matrix with

$\boldsymbol{\Sigma} = \mathbf{aa}' + \boldsymbol{\psi}$, we get

$$a_{11}^2 + \psi_1 = 1$$
$$a_{11}a_{21} = 0.63$$
$$a_{11}a_{31} = 0.45$$
$$a_{21}^2 + \psi_2 = 1$$
$$a_{21}a_{31} = 0.35$$
$$a_{31}^2 + \psi_3 = 1$$

from these we get

$$|a_{11}| = 0.9,\ |a_{21}| = 0.7,\ |a_{31}| = 0.5 \text{ and } \psi_1 = 0.19,\ \psi_2 = 0.51,\ \psi_3 = 0.75$$

Now try the following exercise.

E1) Let p = 3 and m = 1 and suppose the random variables X_1, X_2, and X_3 have the positive definite covariance matrix

$$\Sigma = \begin{bmatrix} 1 & 0.9 & 0.7 \\ 0.9 & 1 & 0.4 \\ 0.7 & 0.4 & 1 \end{bmatrix}$$

Write its factor model.

Factor analysis involves:

- Deciding number of common factors (m)
- Estimating factor loadings (a_{ji})
- Calculating factor scores (f_i)

Now let us discuss methods of estimation.

25.4 METHODS OF ESTIMATION

Factor analysis is done in two parts, first solution is obtained by placing some restrictions and then final solution is obtained by rotating this solution. There are two most popular methods available in literature for parameter estimation, the principal component (and the related principal factor) method and the maximum likelihood method. After the factors are estimated, it is necessary to interpret them, i.e., to assign a name to each common factor that represents the importance of the factor in predicting each of the observed variables. The interpretation is a subjective process. To simplify the interpretation of factors and to make it less subjective, the solution from either method can be rotated in order i.e. either factor loadings are close to unity or close to zero.

The most popular method for orthogonal rotation is Varimax Rotation method. Varimax rotation method emphasizes on detection of the factors each of which is related to few variables rather than detection of factors influencing all variables. In some specific situations, oblique rotations are also used. It is always prudent to try more than one method of solution. If the factor model is appropriate for the problem at hand, the solutions should be consistent with one another. The estimation and rotation methods require iterative calculations that must be done on a computer. If variables are uncorrelated factor analysis will not be useful. In these circumstances, the specific factors play the dominant role, whereas the major aim of the factor analysis is to determine a few important common factors.

After the initial factor extraction, the common factors are uncorrelated with each other. If the factors are rotated by an orthogonal transformation, the rotated factors are uncorrelated. If the factors are rotated by an oblique transformation, the rotated factors become correlated. Oblique rotations often produce more useful patterns than do orthogonal rotations. However, a consequence of correlated factors is that there is no single unambiguous measure of the importance of a factor in explaining a variable. Thus, for oblique rotations, the pattern matrix doesn't provide all the necessary information for interpreting the factors.

Number of factors is theoretically given by rank of population variance covariance matrix. However, in practice, number of common factors retained in the model is increased until a suitable proportion of total variance is explained. Another convention, frequently encountered in packaged computer programs is to set m equal to the number of eigenvalues greater than one. In fact determining the optimal number of factors to extract is not straightforward. Besides, the criteria of eigenvalue more than one, there are several criteria for the number o factors to be extracted, but these are just empirical guidelines rather than exact solution. Some of the most commonly used guidelines are the Kaiser- Guttman rule (eigenvalues more than the average of initial communality estimates, which is always less than one); percentage of variance (the pre fixed percentage or proportion of the common variance that is explained by successive factors obtained by the sum of communality estimates); the scree test (plot the eigenvalues against the corresponding factor numbers and take maximum number of factors to be extracted as the one less than the factor number at which the curve bends or make an "elbow"), the size of residuals and interpretability.

As in principal component analysis, principal factor method for factor analysis depends upon unit of measurement. If units are changed, the solution will change. However, in this approach estimated factor loadings for a given factor do not change as the number of factors is increased. In contrast to this, in maximum likelihood method, the solution does not change if units of measurements are changed. However, in this method the solution changes if number of common factors is changed.

Example 2: What underlying attitudes lead people to respond to the questions on a political survey as they do? Examining the correlations among the survey items reveals that there is significant overlap among various subgroups of items--questions about taxes tend to correlate with each other, questions about military issues correlate with each other, and so on. With factor analysis, you can investigate the number of underlying factors and, in many cases; you can identify what the factors represent conceptually. Additionally, you can compute factor scores for each respondent, which can then be used in subsequent analyses. For example, you might build a logistic regression model to predict voting behavior based on factor scores.

Example 3: A manufacturer of fabricating parts is interested in identifying the determinants of a successful salesperson. The manufacturer has on file the information shown in the following table. He is wondering whether he could reduce these seven variables to two or three factors, for a meaningful appreciation of the problem.

Data Matrix for Factor Analysis of seven variables (14 sales people)

Sales Person	Height (x_1)	Weight (x_2)	Education (x_3)	Age (x_4)	No. of Children (x_5)	Size of Household (x_6)	IQ (x_7)
1	67	155	12	27	0	2	102
2	69	175	11	35	3	6	92
3	71	170	14	32	1	3	111
4	70	160	16	25	0	1	115
5	72	180	12	30	2	4	108
6	69	170	11	41	3	5	90
7	74	195	13	36	1	2	114
8	68	160	16	32	1	3	118
9	70	175	12	45	4	6	121
10	71	180	13	24	0	2	92
11	66	145	10	39	2	4	100
12	75	210	16	26	0	1	109
13	70	160	12	31	0	3	102
14	71	175	13	43	3	5	112

Can we now collapse the seven variables into three factors? Intuition might suggest the presence of three primary factors: maturity revealed in age/children/size of household, physical size as shown by height and weight, and intelligence or training as revealed by education and IQ.

The sales people data have been analyzed in the following table. For this the data is given in the original units, and transformed into standard scores. The three factors derived from the sales people data by principal component analysis are presented below:

Three-factor results with seven variables

	Sales People Characteristics			
Variable	**Factor I**	**Factor II**	**Factor III**	**Communality**
Height	0.59038	0.72170	-0.30331	0.96140 (sum sq I, II and III)
Weight	0.45256	0.75932	-0.44273	0.97738
Education	0.80252	0.18513	0.42631	0.86006
Age	-0.86689	0.41116	0.18733	0.95564
No. of Children	-0.84930	0.49247	0.05883	0.96730
Size of Household	-0.92582	0.30007	-0.01953	0.94756
IQ	0.28761	0.46696	0.80524	0.94918
Sum of squares	**3.61007**	**1.85136**	**1.15709**	
Variance summarized	**0.51572**	**0.26448**	**0.16530**	**Average=0.94550**

Factor Loadings

The coefficients in the factor equations are called "factor loadings". They appear above in each factor column, corresponding to each variable. The equations are:

$$\mathbf{F}_1 = 0.59038x_1 + 0.45256x_2 + 0.80252x_3 - 0.86689x_4 - 0.84930x_5 - 0.92582x_6 + 0.28$$

$$\mathbf{F}_2 = 0.72170x_1 + 0.75932x_2 + 0.18513x_3 + 0.41116x_4 + 0.49247x_5 + 0.30007x_6 + 0.46$$

$$\mathbf{F}_3 = -0.30331x_1 - 0.44273x_2 + 0.80252x_3 + 0.18733x_4 + 0.58830x_5 - 0.01953x_6 + 0.8$$

The factor loadings depict the relative importance of each variable with respect to a particular factor. In all the three equations, education (x_3) and IQ (x_7) have got positive loading factor indicating that they are variables of importance in determining the success of sales person.

Variance summarized

Factor analysis employs the criterion of maximum reduction of variance - variance found in the initial set of variables. Each factor contributes to reduction. In our example Factor I accounts for 51.6% of the total variance. Factor II for 26.4% and Factor III for 16.5%. Together the three factors "explain" almost 95% of the variance.

Communality

In the ideal solution the factors derived will explain 100% of the variance in each of the original variables; "Communality" measures the percentage of the variance in the original variables that is captured by the combinations of factors in the solution. Thus communality is computed for each of the original variables. Each variables communality might be thought of as showing the extent to which it is revealed by the system of factors. In our example the communality is over 85% for every variable. Thus the three factors seem to capture the underlying dimensions involved in these variables.

Example 4: In a consumer - preference study, a random sample of customers were asked to rate several attributes of a new product. The response on a 5-point semantic differential scale were tabulated and the attribute correlation matrix constructed and is given below

Attribute		**Correlation matrix 1**	**2**	**3**	**4**	**5**
Taste	**1**	1	.02	.96	.42	.01
Good buy for money	**2**	.02	1	.13	.71	.85
Flavor	**3**	.96	.13	1	.5	.11
Suitable for snack	**4**	.42	.71	.50	1	.79
Provides energy	**5**	.01	.85	.11	.79	1

It is clear from the correlation matrix that variables 1 and 3 and variables 2 and 5 form groups. Variable 4 is "closer" to the (2,5) group than (1,3) group. Observing the results, one can expect that the apparent linear relationships between the variables can be explained in terms of, at most, two or three common factors.

Solution: Using the method of Principal Components we get

Initial Factor Method: Principal Components

Prior Communality Estimates: ONE

Eigenvalues of the	Correlation Matrix:	Total = 5	Average	= 1	
1	2	3	4	5	
Eigenvalue	2.8531	1.8063	0.2045	0.1024	0.0337
Difference	1.0468	1.6018	0.1021	0.0687	
Proportion	0.5706	0.3613	0.0409	0.0205	0.0067
Cumulative	0.5706	0.9319	0.9728	0.9933	1.0000

	FACTOR1	FACTOR2
TASTE	0.55986	0.81610
MONEY	0.77726	-0.52420
FLAVOR	0.64534	0.74795
SNACK	0.93911	-0.10492
ENERGY	0.79821	-0.54323

Variance explained by each factor

FACTOR1	FACTOR2
2.853090	1.806332

Final Communality Estimates: Total = 4.659423

TASTE	MONEY	FLAVOR	SNACK	ENERGY
0.979461	0.878920	0.975883	0.892928	0.932231

Residual Correlations with Uniqueness on the Diagonal

ENERGY	TASTE	MONEY	FLAVOR	SNACK
0.02054	0.01264	-0.01170	-0.02015	0.00644
0.01264	0.12108	0.02048	-0.07493	-0.05518
-0.01170	0.02048	0.02412	-0.02757	0.00119
-0.02015	-0.07493	-0.02757	0.10707	-0.01660
0.00644	-0.05518	0.00119	-0.01660	0.06777

Rotation Method: Varimax; Rotated Factor Pattern

	FACTOR1	FACTOR2
TASTE	0.02698	0.98545
MONEY	0.87337	0.00342
FLAVOR	0.13285	0.97054
SNACK	0.81781	0.40357
ENERGY	0.97337	-0.01782

Variance explained by each factor

	FACTOR1	FACTOR2
Weighted	25.790361	58.765959
Unweighted	2.397426	2.076256

	TASTE	MONEY	FLAVOR	SNACK	ENERGY

Communality	0.971832	0.762795	0.959603	0.831686	0.947767

It can be seen that two factor model with factor loadings shown above is providing a good fit to the data as the first two factors explains 93.2% of the total standardized sample variance, i.e., $\left(\frac{\lambda_1 + \lambda_2}{p}\right) x100$, where p is the number of variables. It can also be seen from the results that there is no clear-cut distinction between factor loadings for the two factors before rotation but after rotation the same is clear.

Now, in the following section we shall discuss factor analysis versus clustering and multidimensional scaling.

25.5 FACTOR ANALYSIS VERSUS CLUSTERING AND MULTIDIMENSIONAL SCALING

Factor analysis is typically applied to a correlation matrix, where as clustering and multidimensional scaling methods can be applied to any sort of matrix of similarity measures, such as ratings of the similarity of faces. But unlike factor analysis, those methods cannot cope with certain unique properties of correlation matrices, such as reflections of variables. For instance, if you reflect or reverse the scoring direction of a measure of "introversion", so that high scores indicate "extroversion" instead of introversion, then you reverse the signs of all that variable's correlations: -.36 becomes +.36, +.42 becomes -.42, and so on. Such reflections would completely change the output of a cluster analysis or multidimensional scaling, while factor analysis would recognize the reflections for what they are; the reflections would change the signs of the "factor loadings" of any reflected variables, but would not change anything else in the factor analysis output.

Another advantage of factor analysis over these other methods is that factor analysis can recognize certain properties of correlations. For instance, if variables A and B each correlate .7 with variable C, and correlate .49 with each other, factor analysis can recognize that A and B correlate zero when C is held constant because $.7^2 = .49$. Multidimensional scaling and cluster analysis have no ability to recognize such relationships, since the correlations are treated merely as generic "similarity measures" rather than as correlations.

We are not saying these other methods should never be applied to correlation matrices; sometimes they yield insights not available through factor analysis. But they have definitely not made factor analysis obsolete.

As discussed earlier, the primary purpose of factor analysis is data reduction and summarization. This technique has been widely used especially in behavioral sciences, to assess and construct the validity of a test or a scale. For example, a psychologist developed a new battery of 15 subtests to measure three distinct psychological constructs and wants to validate that battery. A sample of 300 subjects was drawn from the population and measured on the battery of 15 subsets. The 300×15 data matrix can be subjected to a factor analysis. The number of factors extracted and the pattern of relationships among the observed variables and the common factors can provide information on the construct validity of the test battery to the researcher.

Factor analysis is an interdependence technique in which all the variables are simultaneously considered each related to all others. It is a highly useful and powerful multivariate statistical technique for effectively extracting information from large databases and making sense of large bodies of interrelated data. Factor analysis has the ability to identify sets of related variables and even develop a single composite measure to represent the entire set of related variables. Factor analysis has a lot of potential applications in problem solving and decision making in business and research institutions and it will continue to grow as more and more researchers gets familiarized with the benefits of data summarization and data reduction.

Factor analysis is a much more complex and involved subject than given in this brief exposition. The major limitations of this technique are

- Existence of several techniques for performing factor analysis and no clear cut comparison of these techniques
- Subjective assessment of number of factors to be extract, rotation method to be used and judging the significance of factor loadings
- Problems with reliability with the results of a single-factor analytic solution which looks plausible but may change with the changes in sample, data gathering process or the numerous kinds of measurement errors. It is important to emphasize that the plausibility is no guarantee of validity or even stability.

Now try the following exercises.

E2) A large number of people were asked to rate their liking of each of the five ready to serve fruit beverages: lemon, aonla, grape, mango and pineapple. The factor loadings of first three factors obtained through factor analysis are

	Factors			
	I	II	III	Communality
Lemon	-0.219	0.363	-0.338	0.2939
Aonla	-0.137	0.682	0.307	0.5781
Grape	0.514	-0.213	-0.227	0.3611
Mango	0.485	-0.117	0.115	0.2621
Pineapple	-0.358	-0.635	0.534	0.8165
Sum of Squares	0.6943	1.0592	0.5584	
Variance Summarized	0.1389	0.2118	0.1117	0.4624

(i) Write the linear equations for all the three factors.
(ii) Interpret the loading coefficients, variance summarized and communality value of this Table.

E3) Department of transportation obtained the data on the following features of 50 bridges: Design time in man days (X_1); Deck area of bridge in square metre (X_2), Construction cost in rupees (X_3), number of structural drawing (X_4), length of bridge in metres (X_5), number of spans (X_6) and degree of difficulty in bridge design(X_7). The factor loadings of first two factors obtained through factor analysis using principal component estimation method with varimax rotation are

	Factor	
	I	II
X_1	0.69732	0.47572
X_2	0.74797	0.44545
X_3	0.83123	0.35001
X_4	0.59594	0.64808
X_5	0.93472	0.16039
X_6	0.86564	0.20127
X_7	0.16549	0.93573

(i) Compute the communality of each variable and the percentage of its variance that I explained by factors.

(ii) Compute the percentage of total variation explained by each factor separately and both factors jointly.

(iii) Interpret the loading coefficients, variance summarized and communality value obtained in (a) and (b).

Now, let us summarize the unit.

25.6 SUMMARY

In this unit, we have covered the following points.

1) Factor analysis is a dimensional reduction technique in which the covariance relationships among many variables is descried in terms of a smaller number of unobservable random quantities called factors.

2) Communality of a variable is the part of its variance that is explained by the common factors.

3) Most widely used method for determining a first set of loadings is the principal component method and gives the values of loadings that bring the estimate of total communality as close as possible to the total observed variance.

4) If the variables are measured in different units, then it is advisable to standardize the variables so that all have mean equal to zero and variance equal to one. This is achieved by subtracting the mean from each of the observed vales of the corresponding variable and dividing the difference by its corresponding standard deviation.

5) The initially extracted factors can be rotated either using orthogonal or oblique transformations.

27.7 SOLUTIONS/ANSWERS

E1) The covariance structure is

$$\mathbf{\Sigma} = \mathbf{aa}' + \boldsymbol{\psi}$$

or

$$1 = a_{11}^2 + \psi_1$$

$$0.90 = a_{11}a_{21}$$

$$0.70 = a_{11}a_{31}$$

$$1 = a_{21}^2 + \psi_2$$

$$0.40 = a_{21}a_{31}$$

$$1 = a_{31}^2 + \psi_3$$

The pair of equations

$$0.70 = a_{11}a_{31}$$

$$0.40 = a_{21}a_{31}$$

imply

$$a_{21} = \left(\frac{0.40}{0.70}\right)a_{11}$$

Substituting this result for a_{21} in the equation

$$.90 = a_{11}a_{21}$$

Yields $a_{11}^2 = 1.575$ or $a_{11} = \pm 1.255$. Since $\text{Var}(F_1) = 1$ (by assumption) and $\text{Var}(X_1) = 1$, $a_{11} = \text{Cov}(X_1, F_1) = \text{Corr}(X_1, F_1)$. A correlation coefficient cannot be greater than unity (in absolute value) so, from this point of view, $|a_{11}| = 1.255$ is "too large." Also the equation

$$1 = a_{11}^2 + \psi_1 \quad \text{or} \quad \psi_1 = 1 - a_{11}^2$$

given

$$\psi_1 = 1 - 1.575 = -.575$$

Which is unsatisfactory since it gives a negative value for $\text{Var}(y_1) = \psi_1$.
Thus, with m = 1,it is possible to get a unique numerical solution to the equations $\mathbf{\Sigma} = \mathbf{LL}' + \mathbf{\psi}$. However, the solution is not consistent with the statistical interpretation of the coefficients, so it is not a proper solution.

E2) (i) The linear equations of all the three factors are

$$F_1 = -0.219 * \text{lemon} - 0.137 * \text{aonla} + 0.514 * \text{grape} + 0.485 * \text{mango} - 0.358 * \text{pineapple}$$
$$F_2 = 0.363 * \text{lemon} + 0.682 * \text{aonla} - 0.213 * \text{grape} - 0.117 * \text{mango} - 0.635 * \text{pineapple}$$
$$F_3 = -0.338 * \text{lemon} + 0.307 * \text{aonla} - 0.227 * \text{grape} + 0.115 * \text{mango} + 0.534 * \text{pineapple}$$

(ii) Factor 1 accounts for 13.9 % of variance; Factor II accounts for 21.2% and Factor III accounts for 11.2% of variance. Taken together 3 factors account for 46.2% of variance.

Communality for most of the variables is less except pineapple. Therefore, one may conclude that the 3 factors are not able to capture the underlying dimensions in these variables.

E3) Communality of each variable are obtained by taking sum of squares of the loadings for that variable in all the factors. Average variance explained is obtained by taking the average of communalities. For obtaining the variance summarized by each factor, take sum of squares of the loadings of all the variables for that factor and divide it by the sum of such sum of squares for all the factors. Then this ratio is multiplied by the average variance explained. The results are given as

Variables	Factors		Communality (Sum squares I and II)
	I	II	
X_1	0.69732	0.47572	0.712565
X_2	0.74797	0.44545	0.757885
X_3	0.83123	0.35001	0.81345
X_4	0.59594	0.64808	0.775152
X_5	0.93472	0.16039	0.899426
X_6	0.86564	0.20127	0.789842
X_7	0.16549	0.93573	0.902978
Sum of Squares	3.742223	1.909075	
Variance summarized	0.534603	0.2772725	Average=0.807328

25.8 PRACTICAL ASSIGNMENTS

Session 11

Write a programme in C-language to write the factor model of a covariance matrix given in Example 1.