UNIT 14 ECONOMICS OF REPAIR AND REPLACEMENT OF EQUIPMENT

Objectives

After going through this unit, the students shall be able to

- discuss the decisions of replacement, repair and overhaul.
- understand the economic consideration of repair and replacement of equipments.
- appreciate the fundamentals of replacement problems with regard to unit machines and components
- acquaint with mathematical models used for decision-making with regard to replacement of capital equipment, unit machines and components, and overhaul of systems

Structure

14.1 INTRODUCTION

The objective of preventive maintenance programmes is to prevent the inconvenience and high cost of breakdowns by keeping industrial plants, equipment and components in near-new, or as near as possible to the as-good-as-new, condition not only through routine or periodic preventive maintenance, but also through (i) replacement, and (ii) inspection to ascertain the state or condition of equipment, which, in turn, maybe followed by certain corrective actions. These corrective actions may include adjustments, minor repairs, and preventive replacements of some components, as required. A homely example is the inspection of spark plugs to determine the amount of carbon deposit and the plug gap. The finding of the inspection may call for either cleaning of the plug and adjustment of the gap, or its replacement. Industrial plants are also subjected to periodic overhauls. An overhaul of an equipment involves subjecting the equipment and its components to strict inspection and this is followed by readjustment and calibration of the equipment as required, some minor repairs of components as necessary, replacement of worn components and servicing of the equipment. Servicing of the equipment includes the cleaning of its components, greasing and replenishment of consumables, such as lubricating oil, fuel and compressed air etc. Overhaul of an equipment is also called capital repair and is a combination of repair replacement, and the overhaul action is taken to restore the equipment to a satisfactory working condition, or as nearly as possible to the 'as-good-as-new' condition.

In this unit, we will discuss replacement, and repair and overhaul decisions. The economic bases of these decisions will be discussed in detail and will also be illustrated through real-life examples.

14.2 REPLACEMENT DECISIONS

Replacement policies form an integral part of all industrial maintenance programmes. A replacement decision is only considered if the cost of an in-service failure exceeds the cost of preventive replacement action, that is, it must be more costly to replace an equipment, or component, after failure than before. Further there are three fundamental assumptions made in all replacement decisions. These are as follows:

1. The state of the equipment, whether good or failed, is always known with certainty. This is a reasonable assumption in most practical situations encountered in operating plants.

2. The replacement action returns the equipment to the as new condition so that the new equipment, or component, can continue to provide the same services as the equipment, or component, which it has replaced. This is also not an unreasonable assumption since the equipment which has been used to replace an operating equipment can be considered to be at least as good as the old one. There is of course an exception that of technological improvement, wherein the new equipment is considered to have lower operating/running and maintenance costs.

3. The operating/running and maintenance costs of the equipment subject to replacement is increasing, or alternatively, the hazard rate of the equipment, or component, must be increasing.

In practice, the replacement of capital equipment is considered from the engineering economics viewpoint and the model takes into account the capital, or first, cost, running and maintenance costs and the salvage value. In this case, there are two kinds of decisions, which are as follows:

i) Decision with regard to the replacement interval of a capital equipment, that is, determining the economic life of a capital equipment so as to minimise the total cost, which must include capital cost, maintenance cost and the salvage value as noted above.

ii) Decision with regard to replacement alternatives, which essentially involves the comparison of the defender, that is, the asset, which the organization presently owns, with the challenger, which is the equipment being considered as its replacement.

We shall initially take up these two types of replacement decisions under the heading of replacement of capital equipment. Thereafter, we shall discuss the quantitative procedures which are essentially used in replacement decisions involving unit machines, such as air pre-heaters, CW pumps etc. in thermal power plants, which are essentially considered, for the purpose of analysis, as single component equipment, and components, such as bearings, valves etc. under the heading of replacement of unit machines or single component equipment, to differentiate it from multi-component equipment.

14.2.1 Replacement of Capital Equipment

The decision to replace an expensive capital equipment involves the investment of a large sum of money. Since the capital equipment has a useful life of many financial years, the costs, such as the running and maintenance costs and the benefits accruing from the investment in the form of revenues earned from the product and services produced continue for a number of years. Moreover, since money can earn interest at a certain rate through its investment for a period of time, it is important to recognize that a sum of Rs.1,000 received at some future date is not worth as much as Rs. 1,000 in hand at the present. This relationship between interest and time leads to the concept of 'the time value of money'* . Thus the time value of money implies that equal sums of money received at different points in time do not have equal value if the interest rate is greater than zero. Engineering economics is concerned with decision-making relating to engineering operations covering acquisition, use and retirement of assets with a view to achieve optimal economy in terms of money. Thus engineering economic analysis is concerned with the evaluation of alternative decisions with regard to an investment and these alternatives are described by indicating the amount and timing of estimated future receipts and disbursements that will result from each decision. The final choice is made based on the analysis of the estimated consequences of the feasible alternatives and for a proper comparison, the estimated consequences should be expressed in identical monetary terms. There must, therefore, be a basis of comparison, or a measure of equivalence, which summarizes the significant differences between investment alternatives. A basis of comparison, in this case, is an index containing particular information about a series of receipts and disbursements resulting from the investment alternative and the most common bases for comparison used in replacement decision - making are the present worth amount, the annual equivalent amount or the equivalent uniform annual cost, and the future worth amount.

Activity A

Explain Time Value of Money. Find out its relevance in engineering operations covering acquisition, use and retirement of assets relating to own organisation.

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14.2.2 Present Worth, Interest Formulae and Equivalence

Let **P** represent the principal amount, **n** the number of years, or the number of annual interest periods, for which it has been invested/borrowed, and **i** the annual interest rate in percent. Then the interest earned by the investor, or due from a borrower, is P.n.i./ 100, and the total sum due is the principal plus the interest, and that is:

F, sum resulting from the initial investment P

$$= P + P.n.i/100 = P(1 + \frac{i}{100}.n) \quad (14.1)$$

Thus if Rs. 1,000 is invested for a period of 2 years at a rate of interest of 10 percent, then the simple interest earned is Rs. 200 and the total amount due after 2 years is Rs. 1,200. When the investment is made for a length of time equal to several interest periods,

* Inflation also results in a reduction in the value of money meaning that the purchasing power of Rs. 1,000 at some future date, as for example, five years hence, will be much less than it is at the present.

provision is made that the earned interest is due at the end of each interest period and since the borrower is allowed to keep the earned interest until the deposit becomes due, the deposit/loan is increased by the amount equal to the interest due at the end of each interest period. Thus, if P is invested for n annual interest periods and the relevant interest rate is i percent per annum, payable annually, then after n years the sum F resulting from the initial investment is:

$$F = P \qquad (14.2)$$

Thus, if P = Rs. 1,000, i = 10 percent, compounded annually and n = 2 years as considered earlier, the amount due after 2 years is Rs. 1,210.The present day value, or the present worth, of a sum of money to be spent or received in the future is obtained by doing the reverse calculation. Thus the present worth of F is:

$$P = F \qquad (14.3)$$

where r = is termed the discount rate. Thus the present worth of

Rs. 1.210 to be received two years from now is P = 1210 = Rs. 1000. Till now, we have assumed that the interest rate is paid once every year. Interest rate may, in fact, be paid weekly, monthly, quarterly, or semi-annually, and when this is the case the formulae (14.2) and (14.3) has to be modified accordingly. If the interest rate of **i** percent per annum is paid **m** times per year, then in **n** years the value F of an initial investment P is:

$$F = P \qquad (14.4)$$

Also the present value P of the money to be spent or received **n** years in the future is:

$$P = F \qquad (14.5)$$

Further, it is possible to assume that the interest rate is paid continuously. This is equivalent to letting **m** in equation (14.4) tend to infinity. When this is the case,

$$F = \quad P \quad = P \exp \qquad (14.6)$$

and the appropriate present worth formula is:

$$P = F \exp \qquad (14.7)$$

In practice, in replacement problems it is usually assumed that interest rates are paid once per year and thus equation (14.3) is used in present worth calculations. Continuous discounting is sometimes used either for mathematical convenience or because it is felt that it reflects cash flows more accurately and when this is the case, equation (14.7) is used. Moreover, it is usually assumed that the interest rate **i** is given as a decimal and not in percentage. Equations (14.3) and (14.7) then become:

$$P = F \qquad (14.8)$$

$$P = F \exp [- i\, n] \qquad (14.9)$$

Let us now consider an illustrative example of an investment decision in which three equally effective machine tools are available and their purchase price, cost of installation (which is the same in all the three cases), operating and maintenance cost for succeeding three years and the salvage value at the end of three years are as given in Table 14.1. Let us also assume that the appropriate discount rate, **r**, is 0.9 (or, i = 11 per cent) and the operating and maintenance costs are paid at the end of the year in which

Table 14.1: An Illustrative Investment Decision Problem

Machine Tools	Purchase Price (Rs.)	Installation & commissioning cost (Rs.)	Total cost of acquisition (Rs.)	Operating and maintenance cost (Rs.)			Salvage (Rs.)
				yr. 1	yr. 2	yr. 3	
A	5,00,000	10,000	5,10,000	10,000	10,000	10,000	30,000
B	3,00,000	10,000	3,10,000	20,000	30,000	40,000	15,000
C	6,00,000	10,000	6,10,000	5,000	8,000	10,000	35,000

they are incurred. We have to calculate the present worth of the three investment alternatives to be able to compare them and arrive at the choice, which will achieve optimum economy in terms of money. Now

$\left(\frac{1}{1+i}\right)^n$

P_A, Present worth of machine tool A = $5,10,000 + 10,000\,(0.9) + 10,000\,(0.9)^2 + 10,000\,(0.9)^3 - 30,000\,(0.9)^3$ = Rs. 5,12,520

P_B, Present worth of machine tool B = $3,10,000 + 20,000\,(0.9) + 30,000\,(0.9)^2 + 40,000\,(0.9)^3 - 15,000\,(0.9)^3$ = Rs. 3,70,525 and

P_C, Present worth of machine tool A = $6,10,000 + 5,000\,(0.9) + 8,000\,(0.9)^2 + 10,000\,(0.9)^3 - 35,000\,(0.9)^3$ = Rs. 6,02,755

With the data as given in the table, machine tool B should be purchased since it gives the minimum present value of the costs.

Activity B

Calculate the present worth of the machine in Table below and suggest which machine should be purchased. The appropriate discount rate is 0.8.

Machine tools	Total cost of acquisition	Operating & Maintenance cost (Rs.)			Salvage Value (Rs.)
		1 Yr	2nd Yr	3rd Yr	
A	2, 50,000	5,000	5,000	5,000	15,000
B	3,50,000	3,500	3,500	3,500	20,000
C	2,00,000	4,000	4,000	4,000	12,000
D	1,75,000	4,5000	4,500	4,500	8,000

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We have noted that the cost of ownership of the asset includes not only the cost of acquisition of the asset which is a single payment made right at the beginning, but also the cost of maintenance, which can be viewed as a series of payments spread over a number of years. This is true of the operating cost as well, which in common parlance is used to include both the cost of use and the cost of maintenance. If a series of payments S_0, S_1, S_2, …., S_n are to be made annually over a period of n years, starting from the zeroeth year, then the present value, or the present worth, or such a series is:

$$P = S_0 + S_1\left(\frac{1}{1+i}\right)^1 + S_2\left(\frac{1}{1+i}\right)^2 + ... + S_n\left(\frac{1}{1+i}\right)^n \qquad (14.10)$$

Moreover, if the payments S_j, j = 0,1,2,…,n, are equal, then the series of payments is termed an annuity and is denoted by A. An annuity can either be a receipt or a disbursement, which is made annually, but in replacement decision, we generally deal with payments as opposed to receipts. In the case of an annuity, the present worth of the series is:

$$P = A + A\left(\frac{1}{1+i}\right)^1 + A\left(\frac{1}{1+i}\right)^2 + ... + A\left(\frac{1}{1+2}\right)^n$$

$$= A\left[1 + \left(\frac{1}{1+i}\right)^1 + \left(\frac{1}{1+i}\right)^2 + \left(\frac{1}{1+2}\right)^3 ... + A\left(\frac{1}{1+i}\right)^n\right]$$

The term inside the large brackets is a geometric progression and using the formula for a sum of (n+1) terms of a geometric progression, we get:

$$P = A\left[\frac{1-\left(\frac{1}{1+i}\right)^{n+1}}{1-\left(\frac{1}{1+i}\right)}\right] = A \qquad (14.11)$$

If two or more investment alternatives or decisions have to be compared, then their identifying characteristics must be stated in the same terms or placed on an equivalent basis. In engineering economics, the meaning of equivalence pertains to the value in exchange and is of primary importance. A few examples can be given to explain equivalence, such as the fact that a present amount of Rs. 500 is equivalent to Rs. 1038 received after 7 years if the interest rate is 11 percent per annum. In other words, if a person considers 11 percent to be a satisfactory rate of interest for a long-term investment, he would be indifferent to receiving either Rs.500 now or Rs. 1038, 7 years hence. Similarly, if the rate of interest is 6 percent per annum, then Rs. 1000 received today is equivalent to Rs. 1,791 received 10 years from now, or Rs. 1000 received now is equivalent to Rs. 237.40 received at the end of each year for the next 5 years, or for that matter Rs. 237.40 received at the end of each year for the next 5 years is equivalent to receiving nothing for the next 5 years and then receiving Rs. 317.70 at the end of years 6, 7, 8, 9 and 10. Three factors are involved in the calculation of the equivalence of monetary amounts at different points in time and these are the amounts of the sums, the time of occurrence of the sums and the interest rate or the rate of discounting. To be able to convert present sums to future amounts and annuities, annuities to future amounts and present worth and future amounts to present worth and annuities, interest formulae and interest factors are used. The most commonly used interest formulae are given in Table 14.2. The notations used in the interest formulae are explained below:

i = annual interest rate, in decimal,

n = the number of annual interest periods,

P = the present principal sum, or the present worth or a future sum, F, or the present worth of a series of equal annual payments, A.

A = annuity, or an amount, in a series of n equal payments, made at the end of each annual interest period, and

F = a future sum n annual periods hence, or the compounded amount of a present sum P, or sum of the compounded amounts of a series of equal annual payments, A.

Table 14.2: Interest Formulae

Interest Factor	Use	Symbol	Formulae
Compound amount	To find F, given P	(F/P, i.n)	$(F = P(1 + i)^n$
Present worth	To find P, given F	(P/F, i, n)	P = F
Equal payment series compund amount	To find F, given A	(F/A, i, n)	F =A
Equal payment series present worth	To find P, given A	(P/A, i, n)	P =A
Equal payment series sinking fund	To find A, given F	(A/F, i, n)	P =F
Equal payment series capital recorvery	To find A, given P	(A/P, i, n)	A=P

$$\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

The relative value of several alternatives is usually not apparent from a simple statement of their future receipts or payments until these amounts are placed on an equivalent basis. We shall illustrate this with a simple example. Let us take the case of an engineer who has sold his patent to a company and is offered a sum of Rs. 1,25,000 now or Rs. 16,500 per year for the next 10 years, since 10 years is estimated to be the useful life of the patent to the company. These two alternatives are shown in *Table 14.3*.

Table 14.3: Equivalence - Illustrative Example

End of year	Receipts - Alternative I	Receipts - Alternative II
O	Rs. 1,25,000	0
1	0	Rs. 16,500
2	0	Rs. 16,500
3	0	Rs. 16,500
4	0	Rs. 16,500
5	0	Rs. 16,500
6	0	Rs. 16,500
7	0	Rs. 16,500
8	0	Rs. 16,500
9	0	Rs. 16,500
10	0	Rs. 16,500
Total Receipts	**Rs. 1,25,000**	**Rs. 1,65,000**

The engineer has taken a house-building loan on which he is paying interest at the rate of 6 percent. He obviously wants to use this interest rate for the evaluation of the two alternatives. The equivalent values of these two alternatives for an interest rate of 6 percent have to be found by the use of interest formulae. One-way is to determine the present worth of alternative II for comparison with alternative I. The present worth of alternative II is:

$$P_{\text{Alternative II}} = \text{Rs. } 16{,}500\ (P/A, 6, 10) = \text{Rs. } 16{,}500 \times 7.3601 = \text{Rs. } 1{,}21{,}441.65$$

Therefore, alternative I is more desirable even with a small interest rate of 6 percent, that is with discount rate r = 0.94.

The annual equivalent amount is another basis for comparison that has characteristics similar to the present worth amount. Any cash flow can be converted into a series of equal annual payments by first calculating the present worth of the original series and the multiplying the present worth amount by the interest factor (A/P, i, n). Thus the annual equivalent amount for interest rate i and n years can be written as:

$$AE = PW\ (A/P, i, n), \text{ where}$$

$$PW = \qquad (14.12)$$

and S_{jt} = net cash flow for investment proposal j at time t

An asset such as an equipment or a machine is a unit of capital. Two monetary transactions in the forms of a single payment and a single receipt are associated with the procurement and eventual retirement of a capital asset, namely, its cost of acquisition, or first cost, and the salvage value. From these amounts, it is possible to derive the equivalent annual cost of capital recovery and return of the asset for use in engineering economic studies as follows:

Let P = first cost of the asset,

F = estimated salvage value of the asset, and

n = estimated service life of the asset in years.

Then the equivalent annual cost of capital recovery and return of the asset may be expressed as the equivalent annual fist cost less the equivalent annual salvage value, or

Equivalent annual cost = P(A/P, i, n) - F(A/F, i, n)

But since (A/F, i, n) = (A/P, i, n) - i,

$$\text{So, } P(A/P, i, n) - F(A/F, i, n) = (P-F)(A/P, i, n) + F\,i \qquad (14.13)$$

As an illustration let us consider an asset with a first cost of Rs. 50,000, an estimated service life of 5 years and a salvage value of Rs. 10,000. For an interest rate of 6 percent, the equivalent annual cost works out to (Rs. 50,000 - Rs. 10,000) (0.2374) + Rs. 10,000 (0.06) = Rs. 10,096, since (A/P, 6, 5) = 0.2374 from tables.

In case of capital equipment, in the form of an asset used for production of goods or services, the equivalent annual cost consists of two elements, namely, the equivalent annual cost due to its first cost and the salvage value, which is called its equivalent annual cost of capital recovery and return, and the equivalent annual cost of maintenance, since cost of ownership includes both the cost of acquisition and cost of maintenance over the useful life of the asset.

14.2.3 Determination of the Optimal Replacement Interval, or the Economic Life of a Capital Equipment

In practice, the maintenance cost of capital equipment generally increases with age. The simplest case of increasing maintenance cost is that of linearly increasing annual maintenance cost. We shall illustrate the engineering economic analysis used for the determination of the optimal replacement interval of capital equipment for this general and more practical case of increasing maintenance cost through an example in which the annual maintenance cost is a linearly increasing function of time.

Let us consider the case in which the total cost of acquisition of the capital equipment is Rs. 5,00,000. The capital equipment has an estimated useful life of 8 years and if it is retired at the end of 8 years, its salvage value will be zero. It can, however, be retired earlier and the estimated salvage values at the end of years 1,2,3,4,…,7 are Rs. 2,00,000. Rs. 1,50,000, Rs. 1,00,000, Rs. 80,000, Rs. 70,000, Rs. 60,000 and Rs. 50,000 respectively. These salvage values are fairly representative for such capital equipment. The maintenance costs, assumed to be payable at the end of years 1,2,3,4,….,8, are as given in Table 4.4, starting at Rs. 10,000 for the first year and increasing linearly to Rs. 80,000 for the eighth year. The relevant interest rate, in this case, is 6 percent, or $i = 0.06$. Given the first cost, P = Rs. 5,00,000 and with the salvage values, F, at the end of years 1,2,3,….,8 as given in column (2) of Table 14.4, the equivalent annual cost of capital recovery and return can be calculated by the use of equation (14.13), that is, (P-F)(A/P,6,n) + 0.06.F. The values thus obtained are given in column (3), using the appropriate values of (A/P,6,n) given in column (8) (since this is also required for the calculation of equivalent annual cost of maintenance). The values given in column (3)

Table14.4 :Total Equivalent Annual Cost of a Capital Equipment with Increasing Maintenance Cost.

End of year	Salvage Value (Rs.)	EAC of Capital Recovery & Return (Rs.)	Mainte-nance Cost (Rs.)	Present worth Factor (P/F,6,n)	Present worth of Mainte-nance as of begining of year 1 (Rs.)	Comulative PW of Main-tenance (Rs.)	Capital Recovery Factor (A/P.6,n)	EAC of Mainte-nance (Rs.)	Total EAC (Col. 3+Col. 9) (Rs.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	2,00,000	3,30,000	10,000	0.9434	9,434	9,434	1.06000	10,000.04	3.40,000.04
2	1,50,000	1,99,904	20.000	0.8900	17,800	27,234	0.54544	14,854.51	2,14,758.51
3	1,00,000	1,55,644	30,000	0.8396	25,188	52,122	0.37411	19,499.36	1,75,143.36
4	80,000	1,26,007.80	40,000	0.7921	31,684	84,106	0.28859	24,272.15	1,50,279.95
5	70,000	1,06,282	50,000	0.7473	37,365	1,21,471	0.23740	28,837.21	1,35,119.21
6	60,000	93,078.40	60,000	0.7050	42,300	1,63,771	0.20336	33,304.47	1,26,382.87
7	50,000	83,613	70,000	0.6651	46,557	2,10,328	0.17914	37,678.16	1,21,291.16
8	0	80,250	80,000	0.6274	50,192	2,60,519	0.16104	41,953.98	1,22,473.89

thus constitute one component of the total equivalent annual cost of the equipment. Now coming to the second component, the annual maintenance costs are given in column (4). Using these values, the entries of column (6), the present worth of maintenance cost as of beginning of year 1, are obtained by multiplying the entries of column (4) by (P/F,6,n) values given in column (5). With the entries of column (6) available, the entries of column (7), which represent the cumulative values of present worth of maintenance cost, are obtained by the summation of the entries of column (6) upto and including that year from the beginning. The cumulative present worth values of column (7) are then converted to the equivalent annual cost of maintenance by multiplying by the appropriate (A/P,6,n) factors given in column (8). The equivalent annual cost of maintenance, thus obtained, is given in column (9). The entries of column (9) constitute the second component of the total equivalent annual cost of the equipment, which are obtained by summing the entries of column (3) and column (9). These are given in column (10). The annual maintenance cost, the equivalent annual cost of maintenance, the equivalent annual cost of capital recovery and return and the total equivalent annual cost figures of Table 14.4 have been plotted in Figure 14.1. From the total equivalent annual cost curve of the figure and Table 14.4, it is evident that the minimum total equivalent annual cost is obtained at the end of 7 years. It is also noticed that the total equivalent annual cost curve drops steeply, flattens out gradually and becomes rather flat in the region of minimum. Thus for this capital equipment with the given salvage values and with a linearly increasing maintenance cost the minimum cost life is 7 years given an interest rate of 6 percent. Therefore we may conclude that the optimal replacement internal for given capital equipment is 7 years. This example illustrates the method used for the evaluation of the minimum cost life, or the optimal replacement interval, of capital equipment.

Before we close our discussion of determination of the optimal replacement interval, an important point needs to be brought out. This is concerned with the trend of annual maintenance cost. In the example, we have considered a linear trend, but, in practice, the maintenance cost may turn out to be non-linear (or piece-wise linear), increasing gradually in the initial years of the useful life of the equipment but in later years the increase in the annual maintenance cost can be expected to be more. In the problem, for example, in case the annual maintenance cost in the sixth, seventh and eighth years were Rs. 65,000, Rs. 85,000 and Rs. 1,20,000 in place of Rs. 60,000, Rs. 70,000 and Rs. 80,000 respectively, with the salvage values as given and for an interest rate of 6 percent, the minimum cost life would still work out to 7 years, but the total equivalent annual cost curve would be flatter near the region of the minimum and would also rise more steeply after the seventh year. A truly non-linear form of the maintenance cost instead of the piece-wise linear form would obviously result in a smaller minimum cost life, that is, a shorter optimal replacement interval.

14.2.4 Evaluation of Replacement Alternatives : Defender Vs Challenger

The objective of minimization of future cost of ownership of assets demands that the replacement of an existing capital equipment with possibly a more modern (state-of-the-art) equipment, having lower operating and maintenance costs, should be done when it is most economical to do so rather than when the existing equipment is worn out. In a replacement study, two assets are compared namely, the asset presently owned, that is the defender and its challenger. The economic future of the defender can be represented by a cash flow of estimated receipts and payments. Further, since the economic future of the challenger can also be represented in the same way, the method of analysis used earlier can be used for comparing the cash flows of the defender and the challenger. However, because in a replacement study the alternatives being compared are the existing, or old, asset and its possible replacement, certain characteristics of this type of economic decision problem must be taken into account. They are as follows:

a) Usually the duration and magnitude of cash flows for the defender and the challenger are quite different. The challenger will have high capital cost and low operating and maintenance cost, while the reverse is true for the defender, which is the equipment being considered for retirement. Thus to justify the consideration of replacement, the capital cost of the defender should be lower and decreasing while its operating and maintenance costs should be higher and increasing.

Figure 14.1 : Minimum Cost life of a Capital Equipment

b) The remaining life of the defender is usually short and the future operating and maintenance costs can be estimated with relative certainty. Moreover, a decision not to replace the defender now can always be reversed in the future. This has to be kept in mind and in case a clear superiority of the challenger is not brought out by the analysis, then it may be advisable to defer the replacement decision and review it a year later.

c) Because of tax implications associated with retirement and depreciation of a physical asset, the effect of taxes on various replacement alternatives can sometimes be significant. However, it must be kept in mind that the effect of taxes on a replacement decision does not, in anyway, affect the method of comparing replacement alternatives.

d) In engineering economic analysis of replacement decisions, the role of sunk cost must be understood and treated correctly. Sunk cost is the difference between the present book value of an asset and the present realizable salvage value (market value if the existing asset is sold now) and if incorrect estimates of the market value of the defender are made, then there is always the danger of ending up with a positive sunk cost, which cannot be recovered. For this reason, some analysts recommend that the sunk cost of the defender be recovered by adding it to the cost of acquisition of the challenger. From the point of view of tax, the value of sunk cost is important if either a capital gain (in case of a negative sunk cost) or a capital loss (in case of a positive sunk cost) is involved but this can be taken care of by charging the sunk cost to an other incomes/unrecovered capital account head for the year in which the sunk cost was incurred. Therefore, it is recommended that the sunk cost should not be included in a replacement study and the salvage (or present market) value of the existing equipment should instead be used for the analysis. This is important since the method of treating data relative to an existing asset should be the same as that used for treating data relative to a possible replacement in an engineering economic analysis. This point will be explained with the help of an illustrative example in the following paragraph.

Let us consider the following example

A truck was purchased 3 years ago for Rs. 2,00,000 with an estimated life of 8 years, salvage value of Rs. 20,000 and an annual operating cost of Rs. 50,000. Straight-line depreciation has been used for the truck. A challenger has now been offered for Rs. 2,50,000 and a trade-in value of Rs. 1,20,000 has been offered on the old truck. The estimated life of the challenger is 10 years, with an estimated salvage value of Rs. 25,000 and an annual operating cost of Rs. 40,000. The new estimates of the defender are an estimated remaining life of 6 years, with a realizable salvage of Rs. 20,000 and with the same operating cost. From the statement of the problem we find that a sunk cost is to be incurred on the defender and that is the only reason for specifying the method of depreciation. Using straight-line depreciation, the annual write-off, $D = (2{,}00{,}000 - 20{,}000)/8 =$ Rs. 22,500 and the present book value, $BV_3 = 2{,}00{,}000 - 3 \times 22{,}500 =$ Rs. 1,32,500. Therefore, sunk cost to be incurred = present book value - present realizable salvage value = 1,32,500 – 1,20,000 = Rs. 12,500. But, Rs. 12,500 is not added to the first cost of the challenger, since this action would:

i) try to ‘cover up’ past mistakes of estimation, and

ii) penalize the challenger unnecessarily because the capital to be recovered each year would be higher due to the increased cost of acquisition.

Instead of adding the sunk cost to the cost of acquisition of the challenger, the trade-in value offered for the defender, which represents the present realizable market value, is used and the data which is to be used for evaluation of the replacement alternatives is given in *Table 14.5*.

Table 14.5: Data for Evaluation of Replacement Alternatives - Illustrative Example

	Defender	Challenger
Present value, P	Rs. 1,20,000	Rs. 2,50,000
Estimated life, n	6 years	10 years
Salvage value, F	Rs. 20,000	Rs. 25,000
Annual operating cost, AOC	Rs. 50,000	Rs. 40,000

In the evaluation of replacement alternatives, since the operating costs of the defender and the challenger are generally for the same period in the future and the statement of the problem usually denotes them as constants over the remaining/ estimated lives, these are not discounted and converted to equivalent annual costs. In the defender vs. challenger problems, a term called the equivalent uniform annual cost (EUAC) is used instead of the total equivalent annual cost. EUAC, therefore, is equivalent annual cost of capital recovery and return plus the annual operating cost. Using the truck problem as an illustration and with an interest rate of 10 percent:

$$
\begin{aligned}
EUAC_{Defender} &= (P-F)(A/P,10,6) + 0.10F + AOC \text{ Defender} \\
&= 100,000 \times 0.22961 + 0.1 \times 20,000 + 50,000 \\
&= 22961 + 2000 + 50,000 = \text{Rs. } 74961, \text{ and} \\
EUAC_{Challenger} &= (P-F)(A/P,10,10) + 0.10F + AOC \text{ Challenger} \\
&= 2,25,000 \times 0.16275 + 0.1 \times 25,000 + 40,000 \\
&= 36,618.75 + 2,500 + 40,000 = \text{Rs.}79,118.75
\end{aligned}
$$

Thus, in this case, the analysis reveals that it would be economical not to replace the old truck with the new one offered and thus the decision is to keep the defender and review the decision, if required, in future. Let us consider another true-to-life replacement decision to illustrate the practical kind of replacement problems faced in the industry and also the method used for the evaluation of replacement alternatives.

A E.O.T. crane was installed in an assembly shop 15 years ago at a cost of Rs. 4,00,000. The economic life was estimated to be 20 years with a salvage value of Rs. 80,000. Straight-line method has been used for depreciation accounting. The operating cost for the crane, including labour and maintenance, is Rs. 92,500 per year. Since repairs have not been extensive and preventive maintenance activities were carried out generally as per schedule, the crane is now expected to last another 10 years at the end of which, it is expected to have zero salvage value. When the crane was installed, the products being produced were heavier and bulkier than what are being produced currently. Today the same service can be provided by two fork-lift trucks at one-half the present operating cost. Fork-lift trucks can be purchased for Rs. 60,000 each with an expected life of 8 years and a salvage value equal to one-tenth of their first cost. However, 200 square feet of storage space valued at Rs. 30 per square feet would be lost in providing space for the fork-lift trucks to manoeuver in. Moreover, if the fosk-lift trucks are purchased, the E.O.T. crane will have to dismantled at a cost of Rs. 40,000 and then sold at a firm price of Rs. 40,000. Based on the company's desired rate of return of 10 percent, the replacement study is conducted as follows:

Comparison of Capital Costs

The defender, the crane, has a present value of P = Rs. 90,000 - Rs. 40,000 = Rs. 50,000 and the latest estimates place its remaining life, n = 10 years with the salvage value just equal to the cost of removal, that is, F = 0. It is also noted that its book value, Rs. 4,00,000 – (Rs. 4,00,000 – Rs. 80,000) = Rs. 1,60,000, would reveal a

sunk cost of Rs. 1,60,000 – Rs. 50,000 = Rs. 1,10,000. However, the fact that this sunk cost is irrelevant and is apparent from the estimates of remaining life and salvage figures. Since the sunk cost is derived from 15 year of estimates, it should play no part in the replacement study. Thus considering remaining life of 10 years and an interest rate of 10 percent, the equivalent annual cost of capital recovery and return,

$$EAC_{Defender} = \text{Rs. } 50{,}000\ (A/P,10,10) + 0$$

$$= \text{Rs. } 50{,}000 \times 0.16275 = \text{Rs. } 8{,}137.50$$

In comparison, the pair of fork-lift trucks, the challenger, has an equivalent annual cost of capital recovery and return,

$$EAC_{Challenger} = 2[(P-0.1P)(A/P,10,8) + 0.10 \times 0.10P]$$

$$= 2[\text{Rs. } 54{,}000 \times 0.18744 + \text{Rs. } 600], \text{ since } F = 0.10P$$

$$= \text{Rs. } 21{,}443.52.$$

Comparison of Other Costs

Annual operating cost of E.O.T. crane = Rs. 92,500

Annual operating cost of fork-lift trucks = 1/2 x Rs. 92,500 = Rs. 46,250.

Also, annual cost of lost storage space if fork-lift trucks are used = Rs.200 x 30 =Rs. 6,000

Thus, based on the comparative cost figures,

$EUAC_{Defender}$ = Rs. 8,137.50 + Rs. 92,500 = Rs. 1,00,637.50, and

$EUAC_{Challenger}$ = Rs. 21,443.52 + Rs. 46,250 + Rs. 6,000 = Rs. 73,693.52

Therefore, comparative advantage of the challenger over the defender =

$$EUAC_{Defender} - EUAC_{Challenger} = \text{Rs. } 26{,}943.98 \cong \text{Rs. } 26{,}950 \text{ per year.}$$

Thus, the replacement study shows that the advantage of the challenger is significant. Although, this is the case, it must be realized that the edge given to the challenger probably stems from functional causes, specifically that the products presently produced are lighter and smaller than in the past. The points that must, therefore, be considered in the replacement analysis are with regard to basic functional causes; namely, whether there is any possibility of another change in the products being manufactured, whether the operating of fork-lift trucks in the assembly shop would create any safety hazard, and finally whether the storage space would become more valuable as a result of expansion of company's operations. Such questions surround nearly every replacement study and deserve careful attention in a replacement decision.

Activity C

What is sunk cost? Discuss its relevance in machine replacement in an engineering economic analysis.

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14.3 REPLACEMENT OF UNIT MACHINES AND COMPONENTS

There are essentially two kinds of replacement problems encountered with regard to unit machines and components, namely, one related to the determination of replacement interval for unit machines, or single component equipment, whose operating cost* increases with age, and the other which deals with components wherein there is no age-related increase in operating cost but which are subject to sudden and catastrophic failures. The replacement problems with regard to unit machines and components, therefore, can be classified as either deterministic, in which the timing of equipment replacement, which, in turn, is dictated by the increasing trend of the operating and maintenance cost and also the outcome of the replacement action are assumed to be known with certainty, or probabilistic, in which timing and outcome of the replacement action depend on chance, since it is generally very difficult to predict with certainty when a failure of the component will occur. Jardine calls these two classes of problems with regard to unit machines and components as short-term deterministic and short-term probabilistic respectively to emphasize the fact that the interval between replacements is comparatively short, more likely to be measured in weeks or months rather than in years, as is the case for capital equipment. Thus in case of unit machines and components one need not take into account the effect of inflation, or interest on borrowed capital, since even if the replacement interval is a couple of years or so, the effect of inflation can possibly be ignored for purpose of analysis and therefore, discounting and present worth, equivalent annual cost, or EUAC methods are generally not called for. Thus we find that replacement problems concerning unit machines, which may be considered to be single component equipment for the purpose of replacement analysis as already stated, and components do not require engineering economic analysis since the time value of money is comparatively insignificant and may be ignored. On he other hand, these replacement problems are amenable to quantitative procedures, which are basically optimization procedures and thus can be considered as application of operations research. In this sub-section, we shall first consider the unit machine replacement problem, that is, the case in which the equipment operating cost increases with age, and then we shall discuss the component replacement problem, namely, the case in which there is no age-related increase in operating and maintenance cost but in which the components are subject to sudden failures.

14.3.1 Determination of Optimal Replacement Interval for an Equipment Whose Operating Cost Increases with Use

Let us consider an equipment which is not subject to sudden failures but whose deterioration through use results in increasing operating and maintenance cost as shown in Figure 14.2(a). To reduce the operating and maintenance cost, a replacement can be performed and after the replacement, the trend of the operating cost is known since it is assumed that the replacement always returns the operating cost to that of new equipment. Thus the operating cost trends following each replacement are identical and the interval between replacements is constant and of length t_r. This deterministic trend in operating costs is shown in Figure 14.2(b).

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* The equipment operating cost, in many practical situations is the total cost of operation of the equipment, or unit machine, and includes the cost of use as well as the cost of maintenance. This explains the increasing trend, since the increase with age of the cost of use, in most cases, is not significant.

Figure 14.2 : Operating Cost in a Short-term Deterministic/unit Replacement Problem

The short-term deterministic or unit machine problem may be stated as follows:

From time to time, say every two years or bi-annually, major surveys are performed on equipment as per statutory requirement. Between two consecutive surveys, the operating cost of the equipment increases due to the deterioration of certain parts of the equipment. Some of these deteriorating parts can be replaced to bring down the operating cost of the equipment. However, the replacements cost money in terms of materials and wages, that is, cost of spares and cost of manpower, and thus a balance has to be struck between the money spent on replacements and the savings obtained by reducing the operating cost. This is shown in Figure 14.3(a). Now let c(t) be the operating cost per unit time at time t after replacement, and C_r be the cost of a replacement. The replacement policy is to perform n equally spaced replacements at intervals of t_r between two consecutive surveys, that is, in the interval (O,T) as shown in Figure 14.3(b). The objective is to determine the optimal replacement interval, which will minimize the sum of the operating and replacement costs between two consecutive surveys. Now if we denote the total cost by $C(t_r)$, then

$C(t_r)$ = replacement cost between surveys + operating cost between surveys

Now, replacement cost between surveys = Number of replacement

between surveys x cost of one replacement = nC_r (14.14)

and, operating cost between surveys = Number of intervals between

surveys x operating cost of each interval between surveys

= (n + 1) (14.15)

Therefore, $C(t_r) = nC_r + (n + 1)$ (14.16)

$t_r^2 = \frac{2C_r}{b}$ $\frac{bT}{2}$ $\frac{T}{2}.C_r + \frac{bT}{2} = 0$ $- C_r + \frac{T.b.t_r}{2}$

t_r, we may denote n in terms t_r and T, thus

$(n + 1)\, t_r = T$, and therefore, n = – 1 (14.17)

Therefore, $C(t_r)$ = (14.18)

Equation (14.18) is the model of the problem relating the interval between replacements, t_r, to the total cost between surveys, that is, the sum of the replacement and operating costs between surveys, $C(t_r)$.

To find the optimal value of t_r, we have to differential $C(t_r)$ with respect to t_r and equate it to zero.

Let us consider two possible forms of the operating cost per unit time shown in Figure 14.4 (a) and (b). These two are fairly representative of what one can expect to encounter in practice.

***Case I*:** Linear trend in operating cost, c(t) = a + bt.

=

so, or, or

(a) Trade-off between operating Cost and Replacement Cost

(b) n Equally-spaced Replacement between Two Consecutive Surveys

Figure 14.3 : Short-term Deterministic Problem: Determination of Optimal Replacement Interval

So, t_r = (14.19)

Case II: Modified exponential trend in operating cost,

$c(t) = A – Be^{–kt}$

so,

or

Since the expression within brackets must equal 0, and

Therefore, –

or,

or (14.20)

$\frac{T}{t_r}.C_r + TA + \frac{TB}{t_r.k}e^{-kt_r} - \frac{TB}{t_r.k}$ kt r) = 0

...on (14.20) will give the optimal value of replacement interval, t_r. ...consider the case of an equipment on which annual surveys are performed and whose operating cost per week is of the form $c(t) = 1000 – 750.e^{–0.30t}$, that is A = Rs. 1000, B = Rs. 750, and k = 0.30.

Also, C_r, cost of replacement, as given = Rs. 1000

With these values, equation (6.21) becomes

$e^{–0.30t_r}$

This equation can be solved by a graphical plot method. Solving we get, t_r = 4.59, that is, with $c(t) = 1000 - 750e^{-0.30t}$ and C_r = 1000, the optimal interval between replacements is 4.59 weeks. Also substituting this value of t_r in the total cost equation, we get:

But T = 1 year = 52 weeks,

So, C(4.59) =

$$\frac{52 \times 1000}{4.59} - 1000 + 52 \times 1000 + \frac{52 \times 750}{4.59 \times 0.30} e^{-0.3 \times 4.59} - \frac{52 \times 750}{4.59 \times 0.30}$$

= Rs. 41153.26

(a) Linear Trend in Operating Cost

(b) Modified Exponential Trend in Operating Cost

Figure 14.4 : Unit Replacement Problem: Two Illustrative Trends of Operating Cost per Unit time

Activity D

What is short-term deterministic or unit machine problem? How do you take replacement decisions in case equipment/component whose operating cost increases with use?

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14.3.2 Determination of Optimal Replacement Interval for a Component/Equipment Subject to Sudden Failures

Certain items which should more appropriately be termed as equipment rather than components such as valves, gearboxes, water pumps, chemical metering pumps, electronic control devices etc. also fall in the category of items which work perfectly satisfactorily till they fail and there is no age-related increase in operating and maintenance cost. In this section we shall discuss the available models for the determination of the optimal replacement interval for such items of equipment and components. This presents a problem of decision-making under uncertainty since it is impossible to predict with certainty when a failure will occur, or, stated more generally, when the transition from one state to another will occur. Therefore, as stated earlier, this is a class of short-term probabilistic problem and for this class of problems we need to invoke the assumptions stated in the initial introductory paragraph, namely, that the total cost of replacement after the failure must be greater than before, there are only two possible states, or conditions, of the equipment/component, good or failed, and that the condition, or state, is always known, and finally that the hazard rate of the equipment or component must be increasing, that is, the value of the Weibull shape parameter, β, be grater than one.

In this class of problems, since we are considering items which operate with constant efficiency until sudden failure, there are two preventive replacement procedures which are adopted, namely, the constant interval policy in which the preventive replacement of the equipment is carried out at fixed intervals and the problem is to determine the optimal interval between replacements to minimize the total expected cost per unit time, and the age-based policy wherein instead of preventive replacements at fixed intervals, one determines the optimal replacement age for the item. The first procedure is known as the block replacement policy and the second procedure is known as the age replacement policy.

Activity E

A) What is the difference between equipment and components? How do you take replacement decisions in case of equipment/components subjected to sudden failure?

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B) Critically study the replacement decisions of equipments/components in your organisation or organisation you are acquainted with. Are the decisions at par with what you learnt from this unit?

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14.3.3 Block Replacement Policy — Determination of Optimal Interval Between Preventive Replacements

Since the equipment or component is subject to sudden failure, when failure occurs it has to be replaced. Moreover, since the failure is unexpected, we may reasonably assume that failure replacement is more costly than preventive replacement. This is true in most cases because a preventive replacement can be adequately planned and necessary arrangements made for it to reduce both time and cost, and there is a possibility that a failure may cause damage to other components of an equipment, or other equipment, making the total replacement cost much greater after the failure. Thus, in order to reduce the number of failures and thereby reduce the total expected cost of replacement, preventive replacements can be carried out at specific intervals but for this one has to ensure that the proper balance is obtained between the amount spent on preventive replacements and the resulting benefit from preventive replacements, namely the expected reduction in failure replacements and the expected failure replacement cost. Moreover, since the planning horizon, or the period of time over which the equipment/ component has to be operated/used, is much greater than the intervals between preventive replacements, we may consider only one cycle of operation, as shown in Figure 14.5(a), and develop a model for the total expected cost of replacement for this cycle*. Thus we want to determine the optimum interval between preventive replacements to minimize the total expected cost of replacement per unit time. Given that C_p is the cost of a preventive replacement, C_f the cost of a failure replacement ($C_f > C_p$) and t_p is the fixed interval between preventive replacements, then the total expected cost per unit time for one cycle

$$C(t_p) = \frac{\text{Total expected cost in interval } (0, t_p)}{\text{Length of the interval}}$$

But the total expected cost = cost of a preventive replacement in interval (O, t_p) + Expected cost of failure replacement = $C_p + C_f \cdot H(t_p)$,

Where $H(t_p)$ = expected number of failures in interval (0, t_p)

$$\text{So, } C(t_p) = \frac{C_p + C_f \cdot H(t_p)}{t_p} \qquad (14.21)$$

The problem comes down to the determination of $H(t_p)$, the expected number of failures in an interval of length t_p. In case f(t), the failures density function, is known, the expected number of failures over a large interval, H(t), can be obtained from the

* If the interval between preventive replacements were long we would then be required to discount the costs and also develop a model for a series of cycles, or the entire planning horizon, instead of one cycle since optimization over a long period is not equivalent to optimization per unit time when discounting is included.

expected mean life, or the mean time to failure (MTTF), that is, given that the time to failure can be represented by the negative exponential, or Weibull, or gamma distribution with known parameters, then the expected mean life can be calculated

and H(t) can be estimated therefrom, since then H(t) = , However, this is not

applicable to our problem since the preventive replacement interval, t_p, is short, and therefore, $H(t_p)$ is not the same as H(t), where t is long. Accordingly, for the purpose of determination of $H(t_p)$, renewal theory approach and Laplace transforms have to be used. Now, consider Figure 14.5(b) and let:

N(t) be the number of failures in interval (0, t). Then

H(t) = expected number of failures in interval (0,t)

= E[N(t)], where E [ ] denotes expectation.

Also, let t_1, t_2, .. = intervals between failures 1, 2, ……, and

S_r = time upto the r^{th} failure = $t_1 + t_2 + \ldots + t_r$.

Now, probability that N(t) = r is the probability that t lies between the r^{th} and $(r+1)^{th}$ failure.

So, $P[N(t) < r] = 1-F_r(t)$,

Where $F_r(t)$ = cumulative distribution function of S_r and $P[N(t)>r] = F_{r+1}(t)$.

Also, since $P[N(t)<r]+ P[N(t)=r]+ P[N(t)>r] = 1$

$$P[N(t)=r] = 1-[1-F_r(t)] - F_{r+1}(t) = F_r(t) - F_{r+1}(t)$$

Thus, the expected value of N(t) then is

$$H(t) = \ldots [F_r(t) - F_{r+1}(t)$$

$$\sum_{r=1}^{\infty} \frac{1}{s}[f^*(s)]^r = \frac{f^*(s)}{s}[1 + f^*(s) + f^*(s)^2 + \ldots]$$

or H(t) = (14.22)

On taking Laplace transforms* of both sides of equation (14.22) we get:

H*(s) =

$$= \frac{f^*(s)}{(s)} \cdot \frac{1}{1 - f^*(s)} = \frac{f^*(s)}{s[1 - f^*(s)]}$$

H(t) has to be determined from $H^*(s)$. This is done through a process known as inversion. The table of Laplace transforms of common functions is available and inversion can be done by reference to this table. However, the inversion of H(s) is rather difficult since the problem is that $H^*(s)$ as calculated by equation (4.23) has to be inverted (and not $f^*(s)$) and this is not usually available except for very simple functions none of which, with the exception of $f(t) = \lambda e^{-\lambda t}$, which is the density function for the negative exponential distribution, are applicable in reliability and maintenance management. Thus, in practice, since inversion is not possible except for the case of negative exponential distribution, a discrete approach has to be adopted for the determination of H(t).

(a) One Cycle of Operation

(b) Determination of H (t) – Expected Number of Failures in an Interval of Length t

(c) Use of Discrete Approach for Determination of H (t_p)

Figure 14.5 : Block Replacement Policy for Unit Machines and Components

14.3.4 Age Replacement Policy : Determination of Optimal Preventive Replacement Age

In this case, we consider the same problem, but instead of making preventive replacements at fixed intervals thus incurring the possibility of carrying out a preventive replacement shortly after a failure replacement, the time at which the preventive replacement is performed depends upon the age of the equipment/ component, which is the elapsed time from the previous replacement, as shown in Figure 14.6(a). Thus the policy is to perform a preventive replacement after the equipment has reached a specified age, t_p, with failure replacements being performed whenever necessary. The objective is, therefore, to determine the optimal replacement age of the item to minimize the total expected cost of replacement per unit time. As in the earlier case, let C_p be the cost of a preventive replacement, C_f the cost of a failure replacement (with $C_f > C_p$) and f(t) the failure density function, that is, the probability density function of the times to failure of the item. Now in this case, there are two possible cycles of operation as shown in Figure 14.6(b), namely, one cycle in which no failure takes place and the item reaches its planned replacement age, t_p, and the other cycle in which a failure occurs before the planned replacement age necessitating a failure replacement. Thus the total expected cost of replacement per unit time,

$$C(t_p) = \frac{\text{Total expected replacement cost per cycle}}{\text{Expected length of the cycle}}$$

Where the total expected replacement cost per cycle = cost of preventive replacement x probability of a preventive cycle + cost of a failure replacement x probability of a failure cycle = $C_p.R(t_p) + C_f.[1-R(t_p)]$, and the expected length of the cycle = length of the preventive cycle x probability of a preventive cycle + expected length of the failure cycle $= t_p.R(t_p) + M(t_p)$, where $M(t_p)$ is the expected length of the failure cycle.

$\int_{-\infty}^{t_p} t.f(t).dt.$

0

$$\text{So, } C(t_p) = \frac{C_p.R(t_p) + C_f.[1-R(t_p)]}{t_p.R(t_p) + M(t_p)} \qquad (14.24)$$

For the evaluation of $R(t_p)$, $[1- R(t_p)]$ and $M(t_p)$,, we shall refer to Figure 14.6(C)

which shows the density function given by a unimodal positively skewed probability distribution, which is what may be expected in the case of increasing hazard rate ($\beta>1$ for the Weibull distribution and $\alpha>1$ for the gamma distribution). With f(t) as given in the figure, the probability of a preventive cycle, $R(t_p)$, is the probability of a failure occurring after t_p (since no failure occurs before t_p). Thus $R(t_p)$ is equivalent to the shaded area and $[1\text{-}R(t_p)]$ then is equal to the unshaded area. Now coming to $M(t_p)$, referring to the

figure, the mean life or the mean time to failure of the distribution is but since

the preventive replacement occurs at t_p^*, for the calculation of $M(t_p)$, we are interested

in the mean of the unshaded portion, which is . .

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* Jardine uses the normal distribution to illustrate the calculation of $M(t_p)$. Such a condition is theoretically feasible since the Weibull and gamma distributions can also take near normal shapes with b>1 and a>1, since the skewness reduce with increasing values of b and a. In case we assume

that time to failure are normally distributed, $M(t_p)$.=

(a) Age Replacement Policy – Replacement after a Specified Age T_p Plus Failure Replacements when Necessary

(b) Two Possible Cycles of Operation

(c) Evaluation of $R(t_p)$, $[1 - R(t_p)]$ and $M(t_p)$

Figure 14.6 : Age Replacement Policy for Unit Machines and Components

14.4 REPAIR AND OVERHAUL DECISIONS

As noted at the beginning of the chapter, the overhaul action is taken to restore the equipment to a satisfactory working condition, or as nearly as possible to the 'as good as new' condition, to minimize the incidence of unexpected failures, but it is carried out before the equipment, or system, reaches a defined failed state. A decision to overhaul an equipment may be taken when the equipment operating and maintenance cost is found to be increasing, or alternatively, when the equipment hazard rate shows a significant increase but this decision is taken when the equipment is working and not in a failed state. In some instances, the overhaul action may be preponed for the sake of convenience or the ease of operation, as for example, if the boiler is shut down due to a superheater tube leakage and the elapsed time from the preceding overhaul of the boiler is close enough to the statutory requirement, the power station may decide to carryout an overhaul of the boiler right after the repair of tube leakage.

In the case of continuous process plants, major systems such as boilers or steam generating plants, are required to be overhauled at prescribed intervals as per statutory regulations. Thus in case of such pre-identified systems, there is no decision-making required in the sense that the interval is prescribed and the management action consists of planning of resources necessary for the overhaul of the system required to be carried out at specified intervals. However, an equipment of the system, such as an air heater or a boiler feed pump, may deteriorate resulting in increased operation and maintenance cost. To reduce the total cost, which must include operating cost and cost of periodic overhauls, and/or loss of efficiency, and to minimize the incidence of unexpected equipment failures, the equipment may be overhauled at fixed intervals in between the system overhauls. Thus one of the decision problems involved is the determination of the interval between equipment overhauls and we shall discuss this problem first. There is also another decision problem and this is concerned with the determination of the optimal overhaul cost limit for an equipment. A decision to overhaul is taken to restore the equipment to the 'as good as new' condition but the overhaul of the equipment may or may not restore the equipment to the 'as good as new' condition, whereas the replacement of the equipment will definitely restore it completely. Thus, at that time, a decision has to be taken as to whether to overhaul or replace the equipment. The degree to which an equipment is restored is a function of its age and this decision, namely whether to overhaul or replace, is taken on the basis of its age. The estimated overhaul cost is compared with the overhaul cost limit and if it is less than the overhaul cost limit, it is overhauled, otherwise it is replaced with a new equipment. The overhaul cost limit is the maximum amount of money which should be spent on overhauling an equipment of a given age and the problem then becomes one of determination of the overhaul cost limit which will minimize the total expected cost given that the equipment is required to still operate for another n periods, say years. We will also discuss this problem.

14.4.1 Determination of Optimal Overhaul Interval for Deteriorating Equipment

This problem deals with an equipment of a continuously operated system in which the system is overhauled at fixed intervals as per statutory requirement, and such system overhauls are called surveys. The equipment of the system deteriorate with use or operation affecting the system operating cost and the system efficiency. This equipment can be restored through periodic overhauls within this period between survey and the problem is the determination of the optimal overhaul interval for such equipment, which deteriorate with use. Let us, for example, take the case of a thermal power station in which the surveys on the boiler have to be conducted every fourteen months but within this period, deposits build up on the inside surfaces of the boiler affecting its efficiency. To reduce the loss of efficiency, important parts of the boiler, such as air heater, economiser and superheater, can be thoroughly cleaned periodically. The problem, therefore, is to determine the optimal overhaul interval of the air heater. We will discuss two possible decision bases, namely, one with the assumption that the interval between equipment overhaul is constant in as much as the system output between equipment overhauls is the same (and we shall also discuss a variation in which the equipment of the system are overhauled at fixed intervals) and the other in which this assumption is relaxed and a dynamic programming model is formulated.

The cost of an equipment overhaul consists of costs of labour and material required and can be estimated reasonably well. The equipment overhaul cost can be taken as constant and let the overhaul cost be C_S* . Since the equipment deteriorates with use, the cost of operation increases with use and can be taken to be a function of m, the system output, in terms of kilograms of steam generated etc., and in our case m is the system output upto the previous equipment overhaul. Therefore, the cost of operation or use for a system output of q units between equipment overhauls is

$C_u =$ We now consider an overhaul policy in which Q kgs of steam (or any other kind of system output) is produced between surveys, or system overhauls, with n equipment overhauls equally spaced in terms of system output, that is, the interval between overhauls is the same in each case. We thus assume that the interval between equipment overhauls is constant, that is, $q_1 = q_2 = q_3 = \ldots q_{n+1} = q$ as shown in Figure 14.7(a). Since there are n equipment overhauls in the period between surveys, there are (n+1) equipment overhaul intervals, and we have:

C(q), total cost = total cost of equipment overhauls + total cost between surveys of equipment operation

$$= n.C_S + (n+1)C_u$$

$$= n.\ C_S + (n+1)$$

Now Q = (n+1).q and therefore, (14.25)

So, CCq) = Cs(1) + (14.26)

where Q = system output in the period between surveys,

q = equipment overhaul interval, that is, overhaul after q units of system output(as for example, q kgs of steam produced),

C_S = cost of one equipment overhaul, and

C_u = = cost of equipment operation between equipment overhauls.

Since the equipment of the system deteriorates with use, there is a loss of system efficiency, which, in turn, affects Q, the system output between surveys. Thus Q may vary but past records provide enough information for its estimation, which can be the mean or expected value. Alternatives, two or more representative values of Q, as for example, the maximum, minimum and median/mean values, can be used in equation (14.26), which will then give two or more optimal values of q.

* We have used the subscripts S to denote service (which, in common parlance, is used for overhaul) since the subscript O has been used elsewhere in this unit to denote an optimal cost, quantity or time.

Equation (14.26) is the model of the problem relating the equipment overhaul interval, q, to the total cost between surveys. The optimal value of q is the value of q which minimizes C(q) as given by equation (14.26) and for this we have to differentiate C(q) with respect to q and equate it to zero. As is evident and as we have seen in earlier models discussed in this unit, the optimal value of q will depend on the form of C_u, the equipment operating cost. Herein again we may consider two possible forms, namely:

i) linear trend in equipment operating cost, f(m) = a+ bm, and

ii) modified exponential trend in equipment operating cost, $f(m) = A - Be^{-km}$.

Although the liner trend comes to mind first, herein as well it is felt that modified exponential trend, which we had used earlier will be more appropriate since the equipment operating cost will probably increase with increasing system output, m, upto a point, which is its maximum value, and then tend to level off. In case the linear trend in equipment cost is found to be appropriate, that is, the available data on m and equipment operating cost is found to fit the form f(m), equipment operating cost = a + bm, then the optimal overhaul interval,

$q_0 =$ (see equation 14.19) (14.27)

However, in case the modified exponential trend in equipment cost between equipment overhauls is found to be more appropriate, that is, the available data on m and the equipment operating cost is found to fit the form, $f(m) = A - Be^{-km}$, then q_0, the optimal overhaul interval can be found from

$$\left(\frac{B}{K} - C_S\right) = \left(Bq_0 + \frac{B}{K}\right)e^{-kq_0}$$ (see equation14.20) (14.28)

Before we discuss the model in which the equipment overhaul interval is not necessarily kept constant, let us consider a possible variation of the model discussed above. Now there may be situations is which the system efficiency is not affected and the system continues to produce at its rated capacity but the system operating cost increases due to the increase in the operating cost of its constituent equipment which points to the fact that the equipment operating costs, and, therefore, the system operating cost, can be effectively reduced by carrying out equipment overhauls at fixed intervals of time in between two successive system overhauls, or surveys. Herein please note that in the earlier model, we had considered a system, namely, a boiler, in which there existed a loss of system efficiency as well as an increase of the system operating cost between surveys. We will now consider a system in which there is no loss in system efficiency but the system operating cost increases with increased use and such systems do exist in practice. In this case, therefore, the equipment operating cost is a function of time, or $C_u = f(t)$, and we are required to find the optimal equipment overhaul interval, wherein equipment overhauls are carried out at fixed intervals, of time t_S. We are, therefore, required to determine the optimal value of t_S. For the derivation of the model for this case, referring to Figure 14.7(a), we find that instead of Q we now have T and since the times between equipment overhauls is constant, $q_1 = q_2 = \ldots q_{n+1} = q$ is replaced by $t_{S(1)} = t_{S(2)} = \ldots = t_{S(n+1)} = t_S$. Therefore, in this case,

(a) **Constant Interval between Equipment Overhauls, that is, q, Steam Generated between Equipment Overhauls = $q_1 = q_2 = ... = p_{n+1}$**

(b) **Variable Interval between Equipment Overhauls – Dynamic Programming Formulations**

Figure 14.7 : Determination of Optimal Overhaul Interval for Deteriorating Equipment

$C(t_S)$, total cost = total cost of equipment overhauls + total cost between surveys of equipment operation

$$= n.C_S + (n+1)C_u$$

$$= n.\ C_S + (n+1)$$

Also, $T = (n+1).\ t_S$ and therefore, $n = \quad 1$ (14.29)

So, $C(t_S) = C_S\ (\quad 1) +$ (14.30)

where T = time between surveys,

t_S = equipment overhaul interval, in time units,

C_S = cost of equipment overhaul (as before), and

C_u = cost of equipment operation between equipment overhauls =

Herein if we use the linear trend in operating cost, namely, $f(t) = a + bt$, the optimal equipment overhaul interval,

$$(\frac{B}{K} - C_S) = (B.t_S + \frac{B}{K})e^{-kt_S}$$

$t_S =$ (14.31)

Moreover, with modified exponential trend in operating cost, namely, with $f(t) = A\text{-}Be^{-kt}$, the optimal value of the equipment overhaul interval, t_s can be found from

(14.32)

Now, if we compare equation (14.31) and (14.32) with equations (14.19) and (14.20) respectively, we find that they are identical with t_S in place of t_r. This is obvious since the problem definition is identical with the only difference that overhaul, in place of replacement, is used to restore the equipment.

Next we consider a decision base in which the equipment overhaul interval is allowed to vary, that is, the overhaul intervals are not necessarily kept equal. This is possibly a more realistic alternative in cases where use or operation results in a loss of system efficiency as well as an increase in the system operating cost since the system itself deteriorates with use and influences the system operating cost. This model is also based on the assumption that Q kgs of steam is generated between surveys with the basic difference that the n equipment overhauls need not be equally spaced as shown in Figure 14.7(b). The objective is to determine an overhaul policy, which will minimize the sum of the total costs of equipment overhauls and equipment operation between surveys. Now, let f_a be the sum of the total equipment overhaul cost and total equipment operating cost incurred to generate Q kgs of steam starting with a fully restored system, namely, a boiler, and using an optimal overhaul policy. Further let us consider the case where the final, or the n^{th}, equipment overhaul occurs at a point where m kgs. of steam have been generated by the system as shown in Figure 14.7(b). The problem then becomes one of determining m and $(q_1, q_2, \ldots q_n)$ such that the sum of the total cost of generating m kgs. of steam, the operating cost incurred for generating the remaining (Q-m) kgs. of steam and the equipment overhaul cost is a minimum. Thus we can now write a functional equation, which is of the form:

(14.33)

where f_m = sum of the costs of equipment overhauls and operation for generating m kgs. of steam, or m units of system output,

$= (n-1).C_S + n$ and

$C_S(m) = 0$ at $m = 0$, and otherwise $= C_S$

Also the end condition is f_0 (that is, with m=0) = 0. Using the functional equation

(14.33) and inserting derived values of given, for example, that f(m) $= A\text{-}Be^{-km}$ as before, and $C_S(m)$ enables a numerical solution to the problem to be found. For this, given a value of Q, the various possible values of q need to be considered at every recursion.

The models discussed in this section are based on the fundamental assumption that the overhaul restores the equipment fully, that is, at least to as good as it was at the beginning of the period. However, an equipment overhaul may only restore the equipment partially and the degree to which it is restored is a function of its age, that is, the elapsed time from the previous overhaul/replacement. Thus one has to decide whether to overhaul the equipment or to replace it and this decision is based on the overhaul cost limit, and we shall discuss the determination of the optimal overhaul cost limit in the next section.

14.4.2 Determination of Optimal Overhaul Cost Limit for Deteriorating Equipment

Whenever the owner of a vehicle takes his vehicle for a major overhaul, he always compares the estimated cost of overhauling the vehicle with some sort of a overhaul cost limit, which is essentially his bound, or his notion of the maximum amount of money that should be spent on overhauling the vehicle. Also, the cost of overhauling the vehicle is a function of its age and at the same time, his decision on whether to overhaul or replace the vehicle depends on (i) the period of time, or the number of years in the future, for which the vehicle is required to be operational, and (ii) the cost of acquisition of a new vehicle. Thus the overhaul cost limit is determined so that the total cost of operation and overhaul over a fixed period of time in the future is minimized. Therefore, given the estimated overhaul cost for equipment of different ages and also the fixed future period of time over which the equipment is required for use, the problem is to determine the optimal overhaul cost limit. For the derivation of the model, let:

n be the number of periods for which the equipment is required for use,

I the age of the equipment at the beginning of the period,

J the age of the equipment at the end of the period and referring to Figure 14.8, note that J=I+1 if the equipment is overhauled and otherwise J=1 since the decision then is to replace it,

f_I © the probability density function of the estimated overhaul cost for equipment of age I.

L_I the overhaul cost limit for equipment of age I, and

P the cost of acquisition of a new equipment, that is, the cost of replacement.

From Fig. 14.8, we see that there are two possible decisions, namely, that given an estimated overhaul cost c, there is a probability $P_{I,\ I+1}$ + 1 that $c \le L_I$ in which case the decision is to overhaul the equipment, and a probability $P_{I.1}$ that $c > L_I$, and if $c > L_I$, the equipment is replaced at the end of period I. Also please not that as per the statement of the problem we are assuming that the time required to effect an overhaul, T_S, or a replacement, T_r, can be neglected. This is a reasonable assumption since the model is being formulated for a problem of whether to overhaul or replace an equipment given an estimated cost of overhauling the equipment. Now that we have defined f_I © and L_I, we can define $m_I(L_I)$, which is the mean overhaul cost of equipment of age I with an overhaul limit of L_I given that $c \le L_I$.

Therefore, $m_I(L_I) =$ (14.34)

Next, we define $f_n(I)$ as the minimum expected total cost of overhauling and replacing the equipment over n periods in the future starting with an equipment of age I. Thus the objective is to determine overhaul cost limits L_I such that the minimum expected total cost $f_n(I)$ is achieved. Let $C_n(I,J)$ be the expected cost of the first decision with n periods still left and starting with equipment of age I. Therefore,

$$\int_0^{L_I} f_I(c)dc = F_I(L)$$

$$P(1 - \int_0^{L_I} f_I(c)dc)$$

$$\int_0^{L_I} f_I(c)dc$$

verhaul x probability that the overhaul cost is less than + cost of replacement x probability that the overhaul cost is greater than the overhaul limit.

(14.35)

For convenience of notation, let

Therefore, $c_n\ (I, J) = m_I(L_I).F_I(L) + P\{1\text{-}F_I(L)\}$ (14.36)

Now we define $f_{n-1}(J)$ as the minimum expected total cost over the remaining (n-1) periods, and we get,

$f_{n-1}(J) =$ minimum future cost if equipment is of age (I+1) × probability that the overhaul limit was not exceeded at time n+ minimum expected future cost if equipment is of age 1 × probability that the overhaul limit was exceeded at time n.

$= f_{n-1}\ (I+1).F_1(L) + f_{n-1}(1)\{1 - F_I(L)\}$ (14.37)

Therefore, the expected total cost over n periods in the future starting with an equipment of age I.

$= C_n\ (I, J) + f_{n-1}(J)$ (14.38)

Moreover, since the objective is to minimize this total cost by the selection of appropriate overhaul limits, $f_n(I)$ can be written in the form of a recurrence relation as follows:

$f_n(I)$ = $\}$ =

for $n \geq 1$ (14.39)

with $f_0(I) = 0$ for all values of I (since n=0)

Figure 14.8 : Determination of Optimal Cost Limit

From the recurrence relation of equation (14.38), given the probability density function of the estimated overhaul cost for an equipment of age I, I = 1, 2, 3,…, (n-1) and the cost of acquisition of a new equipment, one can determine the optimal overhaul cost limits for the equipment. The dynamic programming model discussed above is taken from Jardine and provides a very useful basis for decision-making. In practical situations, however, there are a few other considerations, which also affect decision-making. One very important consideration is the availability of funds. Sometimes the cost of replacement and consequently difficulties encountered in making adequate budget provisions is an important hurdle. In such instances, it may be worthwhile to consider the reconditioning and retrofitting of the available equipment since one can reasonably expect that the cost of reconditioning and retrofitting will be significantly lower than the cost of a new equipment and if a good job of reconditioning and retrofitting is done, the equipment, in most instances, will be as good as a new equipment. Accordingly, then the decision is whether to overhaul (in house) or to recondition and retrofit the equipment, and P, in that case, is the cost of reconditioning and retrofitting the available equipment.

Activity F

Analyze the overhauling decisions taken for the equipments in your organisation. How do you fix the limit of overhauling costs?

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14.5 SUMMARY

In this unit, replacement, and overhaul and repair decisions have been discussed. All the available models used for decision-making with regard to replacement of capital equipment and also unit machines and components, and overhaul of systems and equipment have been presented and discussed with the help of real-life industrial examples. Repair has been included under overhaul (which incidentally also goes by the name of capital repair), since there is no decision-making involved in repair as a breakdown maintenance, or emergency repair, action. Accordingly, economic considerations are involved only in case of repair when it forms a part of overhaul, which is essentially a preventive maintenance action.

In this unit, the mathematical models used are simple and this has been done purposely. For these models, only a basic knowledge of calculus is needed as a pre-requisite.

14.6 REFERENCES

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