UNIT 6 QUANTITATIVE METHOD OF FORECASTING

Objectives

Upon completion of this unit, you will be able to:

- learn the importance of forecasting for decision making
- use forecasting techniques in operations management
- understand different quantitative techniques of forecasting
- know trend analysis, exponential smoothing, decomposition methods, and causal method of forecasting
- find out the suitability of forecasting models
- calculate the errors in forecasting

Structures

6.1 INTRODUCTION

Forecasting is the art and science of predicting the future events. Forecasting was largely an art, but it has now become a science as well. While managerial judgement is still required for forecasting, the manager is added today by sophisticated mathematical tools and methods. While all elements of operations management are important, forecasting is viewed as the key elements in the operations structure. This unit is an excellent overview of quantitative forecasting techniques and models and help recognizing the different models. Also it will help to know their use according to one's needs. It can be highlighted that the qualitative forecasting is discussed in unit 5 of MS53 . The reader has to read this quantitative forecasting in combination with the qualitative forecasting. Then only he or she can have a complete understanding of the subject of forecasting. The needs of the market are changing for us, and we have to respond more quickly than before. To do so, we have placed a higher emphasis on forecasting. Students are required to refer MS"8 for different types forecasting techniques.

6.2 FORCASTING

Forecasting, in general, presents an unresolved philosophical dilemma. `You can never plan the future by the past', said Edmund Burke. But Patrick Henry disagreed: I know of no way of judging the future but by the past'. Operations managers try to forecast a wide range of future events that potentially affect success. Main concerned here is that of forecasting customer demand for product and services. Forecasting may be short-term or long-term by nature.

Forecasting is an essential tool in any decision-making process. It's uses vary from determining inventory requirements for a local shoe store to estimating the annual sales of video games. The quality of the forecast strongly related to the information that can be extracted from past data.

Defining Forecasting

A forecast is an estimate of a future event achieved by systematically combining and casting forward in a predetermined way data about the past. It is simply a statement about the future. It is clear that we must distinguish between forecast per se and good forecasts. Good forecast can be quite valuable and would be worth a great deal. Long-run planning decisions require consideration of many factors: general economic conditions, industry trends, probable competitors actions, overall political climate, and so on.

Prediction, on the other hand, is an estimate of a future event achieved through subjective considerations of managers. This subjective consideration need not occur in any predetermined way.

Forecasts are possible only when a history of data exists. An established TV manufacturer can use past date to forecast the number of picture screens required for next week's TV assembly schedule. A fast-food restaurant can use past data to forecast the number of hamburger buns required for this weekend's operations. But suppose a manufacturer offers a new refrigerator or a new car, he cannot depend on past data. He cannot forecast, but has to predict. For prediction, a good subjective estimates can be based on the manager's skill, experience, and judgement. One has to remember that a forecasting technique requires statistical and management science techniques.

In general, when business people speak of forecasts, they usually mean some combination of both forecasting and prediction. Commonly, forecasting is substituted freely for economic forecasting. It implies for some combination of subjective calculations and subjective judgement. We caution students and operations managers to avoid misunderstanding.

Forecasts are often classified according to time period and use. In general, short-term (up to one year) forecasts guide current operations. Medium term (one to three years) and long-term (over five years) forecasts support decisions on plant location and capacity Forecast are never perfect. Because it deals with past data, our forecasts will be less reliable the further into the future we predict. That means forecast accuracy decreases as time horizon increases. The accuracy of the forecast and the its costs are interrelated. In general, the higher the need for accuracy translates to higher costs of developing forecasting models. So how much money and manpower is budgeted for forecasting? What possible benefits are accrued from accurate forecasting? What are possible cost of inaccurate forecasting? The best forecast are not necessarily the most accurate or the least costly. Factors such as purpose and data availability play important role in determining the desired accuracy of forecast.

6.3 APPLICATION TO DIFFERENT FUNCTIONAL AREAS

Forecasting is one input to all types of business planning and control, both inside and outside the operations function. Marketing uses forecasts to plan products, promotion, and pricing. Finance uses forecasting for managing cash flows and as an input to financial planning. Accountants rely on forecasts of costs and revenues for tax planning. Human resource personnel need forecasts for recruiting.

The main focus of this unit is on forecasting on operations function. It serves as an input for decision on process design, capacity planning, and inventory control. For process design purposes, forecasting is needed to decide on the type of process and the degree of automation to be used. For example, a low forecast of future sales might indicate that little automation is needed and the process should be kept as simple as possible. If greater volume is forecast, more automation and more elaborate process including line flow might be justified. Since process decisions are long-range in nature, they can require forecasts for many years into future. Forecast can measure the variability in demand during lead time that in turn can help carry proper safety stock levels. Appropriate safety stock inventory levels could minimise overall carrying and stockout costs associated with these items.

6.3.1 Forecasting in operations management

In studying forecasting, we must be careful not to be emotional in immersing ourselves in techniques and loose track of the reasons for forecasting. Forecasting is an important component of operations planning. It is absolutely necessary for planning, scheduling, and controlling the system to facilitate effective and efficient output of goods and services.

Forecasting is helpful in operations management as regard to the aggregate demand forecast. It is obtained by estimating expected volumes of sales, expressed in dollars, and then converting the sales dollars into homogeneous production units. Production unit can be subdivided into component parts and converted into labor or material requirements. These resource forecasts are used to plan and control operation subsystems as shown Figure 6.1

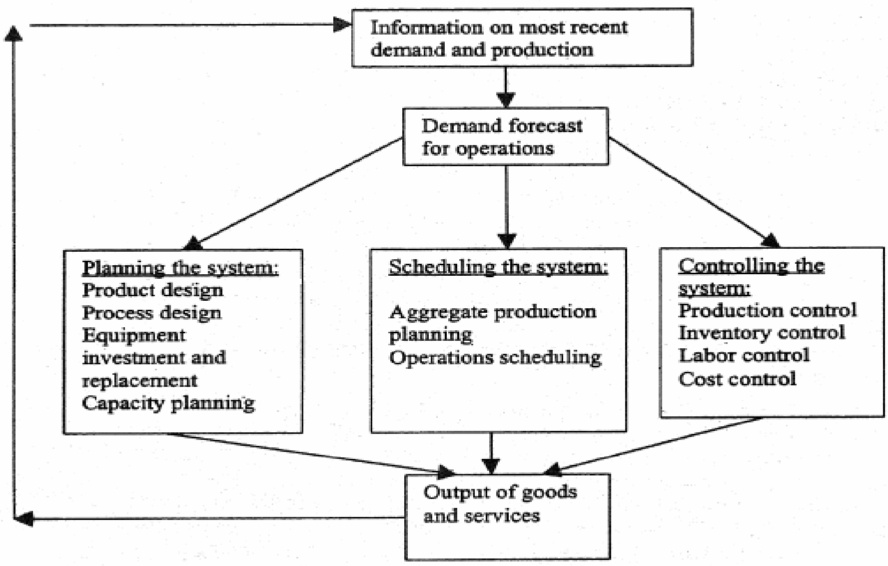

Figure 6.1: Using demand forecasting and production/ operations subsystems

Source: Production and Operations Management by Adam and Ebert, PHI, New Delhi.

Refer Figure 6.1. There are three types of operations sub-functions which need forecasting. These operations sub-functions are planning the system, scheduling the system, and controlling the system. Each one will be discussed below in detail.

Planning the system: Managers need to forecast demands so that they can design or redesign processes necessary to meet demand. Automated, continuous flows facilitate high production volumes; manual or semi-automated, intermittent flows are generally more economical for smaller production volumes. The demand forecast is critical to this design. We have discussed a bit of this at the beginning of the unit. Wide variation between anticipated demand and actual demand can result in excessive operations"costs. Capacity planning utilities forecasting at different levels. A long range forecasting is needed for planning the total capacity of facilities. For medium range capacity decisions, a detailed forecasting will be needed to determine the subcontracting, hiring plans, and equipment utilisation. Shoat-range capacity decisions, including assignment of available people and machines to jobs or activities in the near future, should be detailed in terms of individual products and they should be highly accurate. If capacity is not expanded fast enough, both individual firms and the national economy suffer. On the other hand, too much capacity is burdensome. For example, Jet aircraft, at $20 million each, cannot be purchased and stocked for occasional demand, since the cost of excess capacity is considerable. Boeing, McDonnell Douglas, and airbus- the world's largest commercial aircraft producers- try very hard to have manufacturing plants size a to meet exactly the number of aircraft demanded. If the plants are too large, it will be costly to the firm.

Scheduling the system: Job scheduling in intermittent and continuous operations is more stable if demand forecasts are accurate. Accurate demand forecasting is needed for best utilisation of the existing conversion system. Managers need intermediate run demand forecasts for three months, six months, and a year into the future. Both current and future workforce levels and production rates must be established from these forecasts.

Controlling the system: In regards to controlling inventory, production, labor, and overall costs, managers need accurate demand forecast. Accurate forecasts are needed for the immediate future- hours, days, and weeks ahead. Thus a computerised forecasting system may be needed for these decisions.

In general, there are different types of decisions in operations and different associated forecasting requirement as shown in Table 6.1. A peep into the table indicates that there are two types of forecasting methods in operations management: qualitative methods of forecasting and quantitative methods of forecasting. It is to be reminded that qualitative forecasting has been discussed in unit S. This unit will deal with the quantitative methods of forecasting only.

Table 6.1 Forecasting uses and methods

Uses of forecasting For operations	Time horizon	Accuracy required	Number of products	Management level	Forecasting methods
Process design	Long	Medium	Single or few	Top	Qualitative and causal
Capacity planning Facilities	Long	Medium	Single or few	Top	Qualitative and causal
Aggregate planning	Medium	High	Few	Middle	Causal and Time series
Scheduling	Short	Highest	Many	Lower	Time series
Inventory management	Short	Highest	Many	Lower	Time series

Source: Operations Management by R.G Schroeder, McGraw-Hill.

Qualitative forecasting depends on managerial experience, because they do not use any h specific quantitative models. This method is helpful when past data are not available to the managers or not reliable. Thus different individuals can apply the same qualitative methods hilt can arrive at different forecasting results. Because of non-availability of data, the managers can utilise the forecasting by using the qualitative methods. Some of the well-known qualitative methods are:

- Management judgment
- Consensus
- Writing `scenarios' of the possible events that might occur
- Judgmental methods(Delphi technique)
- Based on the individual's feeling and expert opinion

Some of these methods are discussed in unit

Quantitative forecasting is used when:

- Past data is available, and
- Past data can be fitted into a pattern that can be expected to continue into the future.

Activity A

Think of any organisation of manufacturing or service sector. Evaluate its short-term, medium-term and long-term forecasting techniques. Which of the forecasting technique are in use? '

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6.4 SPECIFIC FORECASTING METHODS

We have been highlighting that forecasting is important in operations and strategic planning. In stead of getting too deeply into specific types of forecasts for varying situations, we summarize that the long-range strategic planning and facilities decisions use less analytical qualitative methods of forecasting, and operational planning in production and inventory control uses more analytical, time series analysis. Causal forecasting techniques are used for a variety of planning situations but are specially in intermediate-term planning.

Table 6.2 summarizes modern forecasting techniques being used in industries. There are three types of forecasting techniques: qualitative forecasting, naive (time series/ forecasting, and causal forecasting. Qualitative and naive models are the most frequently used forecasting techniques in operations management. The causal models are very costly models to implement and are not suitable for short-term forecasting typically needed h• operations managers. Even though qualitative models are very popular, it has been discussed in unit 5, and this unit will limit its discussion to quantitative forecasting.

Table 6.2: Summary of representative forecasting techniques

Model Type	Description
1) *Qualitative models*	
Delphi method	Questions panel of experts for opinions
Historical data	Makes analogies to the past in a judgmental manner
Nominal group technique	Group process allowing participation with forced voting
2) *Naive (Time series) Quantitative models*	
Simple average	Average past data to predict the future based on that average
Exponential smoothing	Weights old forecasts and most recent demand
3) *Causal Quantitative Models*	
Regression analysis	Depicts a functional relationship among variables
Economic modeling	Provides an overall forecast for a variable such as gross national product

Source: Production and Operations Management by Adam, E.E., and Ebert, Ronald J., Fifth Edition, 1997, Prentice-Hall of India, New Delhi.

6.5 MAIN CLASSES OF QUANTITATIVE MODELS

1) Time series models
   i) Trend projection (long-term)
   ii) Decomposition methods (intermediate-term)
   iii) Smoothing methods (short-term)
      a) exponential smoothing
      b) Naive forecasting
2) Causal models
   i) Regression and correlation analysis

ii) Econometric model

iii) Simulation method

iv) Input-Output model

A causal model attempt to relate some quantity to other factors. Time series models attempt to relate some quantity strictly to time. A time-series is a time ordered sequence of observations which have been taken at regular intervals over a period-of time (hourly, daily, weekly, monthly, annually etc.).

Some of the important forecasting techniques are explained below.

6.5.1 Time Series Models

i) Trend Projection

Reasons for studying trends are:

1) The study of secular trends allows us to describe a historical pattern.
2) Studying secular trends permits us to project past patterns, or trends, into the future.
3) In many, situations, studying the secular trend of a time series allows us to eliminate the trend component from the series.

Trend can be a straight line or a curvilinear. Example of straight line trend is increase of pollutants in the environment as shown in Figure 6.1(a) . Common example of curvilinear relationship is the life cycle of a new product. When a new product is introduced, its sales volume is low (i), as the product gains recognition and success, unit sales grow at an increasing rate (ii). After the product is firmly established, the unit sales grow at a stable rate (iii). Finally, as the product reaches the end of its life cycle, unit sales begin to decrease (iv) in Figure 6.1(b).

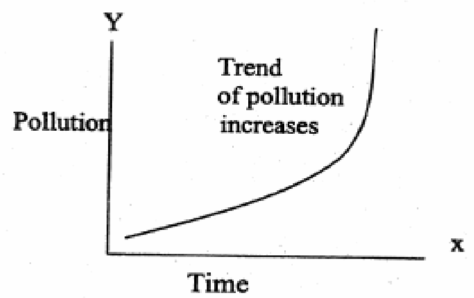

Figure 6.1(a)

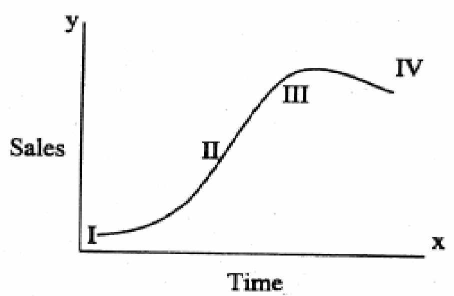

Figure 6.1(b)

Trend reflects the effect of global movements in the time series. Consider the linear trend. $Y_t = b_o + b_l x$

Where T, = estimated value of the dependent variable

b_o = Y- intercept

b_l = slope of the trend line

x = independent variable (time in trend analysis)

Objective is to find out the value of slope(b_l) and intercept(b) from the given set of independent values of x.

We can describe the general trend of many time series using a straight line. But we are faced with the problem of finding the best-fitting line. We can use the least squares methods to calculate the best-fitting line, or equation.

Least-squares Method

Find $b_{,,}$ and b1 such that we get the "best" line which fits the data.

Given (Y_1, Y_2, Y_3, ..., Y_n), find $b_,$ and b_l such that the demand

$$T_t = \sum_{t=1}^{n} (yt - Tt)^2 \quad \text{is minimised} \qquad \text{Eq. (6.2)}$$

Where T_t = estimated demand

Y_t = actual demand

We can show that the estimates via the least squares method are:

$$b_o = \overline{Y} - b_1 \, \overline{t} \qquad \text{Eq. (6.3)}$$

$$b_1 = \frac{n\sum_{t=1}^{n} Y_t - \sum_{t=1}^{n} t \sum_{t=1}^{n} Y_t}{n\sum_{t=1}^{n} t^2 - (\sum_{t=1}^{n} t)^2} \qquad \text{Eq. (6.4)}$$

$$\overline{Y} = \frac{1}{n} \sum_{t=1}^{n} Y_t \qquad \text{Eq. (6.5)}$$

$$\overline{t} = \frac{\sum_{t=1}^{n} t}{n} \qquad \text{Eq. (6.6)}$$

Example: Sales data of a company are given for 14 time periods (t) with respective sales data (y_t). Forecast the sales value for the period 15.

(Adapted from course material, Department of Management Studies, IIT, Delhi).

t	Y_t
1	32
2	28
3	30
4	34
5	30
6	43
7	36
8	42
9	42
10	55
11	47
12	56
13	54
14	57
105	586

Solution: First step is to calculate the value of ($t * Y_t$) and t^2 from the given sales data.

t	Y_t	$t * Y_t$	t^2
1	32	32	1
2	28	56	4
3	30	90	9
4	34	136	16
5	30	150	25
6	43	258	36

7	36	252	49
8	42	36	64
9	42	378	81
10	55	550	100
11	47	517	121
12	56	672	144
13	54	702	169
14	57	798	196
105	586	4927	1015

← **Column total**

Second step is to calculate the column total for each of the four columns.

Here total number of sales data are 14 and its column total is 105, column total of Y_t is 586, that of $t * Y_t$ is 4927 and that of t^2 is 1015.

Now, n = 14

$t = 7.5$

Y = 41.857142, from eq. 6.4

$b_1 = (14(586) - (105)(586)) / (14(1015) - (105)^2) = 2.3384$

from eq. 6.3

$b_o = 41.857142 - 2.3384\ (7.5) = 24.3171$

$T_t = 24.3191 + 2.3384\,t$ => upward trend

How do we forecast future y_t

$Tt = b_o + b_1 t$ based on 'n' observations

Forecast for periods n+1 and n+2, …… are obtained as

$T_{n+1} = b_o + b_1(n+1)$

$T_{n+2} = b_o + b_1(n+2)$ and so on.

In the previous example, we can calculate the value of 'T' for the period 15

$T_t = 24.32 + 2.34\,t$

$T_{15} = 24.32+2.34\ (15) = 59.42$

$T_{15} = 59.42$

ii) Decomposition Methods

These methods ot time-series analysis are based on the belief that the demand can be separately and distinctly broken down into its components, viz. trend, cycle, seasonality and randomness. Thus demand usually is considered as a product of these components:

$$Y_t = T_t \times S_t \times C_t \times R_t \qquad \text{(Eq 6.7)}$$

Where

- T_t = Trend component at period t
- S_t = Seasonal index at period t
- C_t = Cyclical factor at period t
- R_t = Randomness as an index at period t
- Y_t = Demand observed

T_t is measured in units of Y_t but S_t, C_t and R_t are all measured in relative terms. Values of S_t, C_t, and R_t >1 indicate effects above the trend (above the average level).

Steps in decomposition method

Step 1: Identify seasonal factors: For the actual series D, compute a moving average, whose length is equal to the seasonality (e.g. 12 in monthly data, 4 in quarterly data , 7 in daily data). Before calculation of moving average, let's define it.

What is a moving average?

Moving average is an average that is repeatedly updated. As we move into the future and. as new observations become available, the oldest values in the series are discarded, thereby keeping the average current The message given by the moving averages technique is that while history helps to plan the future, only a small fragment of the past is relevant. Thus, if we have the monthly demand data, we may calculate average of only six month's values in the data to get the forecast for the next month and such a moving average is called a six month moving average. The selection of the number of periods in the moving average what is being forecasted and (ii) the characteristics of the demand. Simple n -period moving average forecast for the period (t+l) = (aD_k) x 1/nwhere.k = (t - n+l) tot.

The moving average technique can be made simple by an example. For 4- period moving average, we need four observations to compute.

Example: (Adapted from course material, Management Department, IIT, Delhi)

The following table gives the number of patients in a family practice for last four years. Each year is divided into four quarters and the respective patients being treated are included for forecasting.

Year 1	Quarter	Patients
	1	920
	2	916
	3	895
	4	905
Year 2		
	1	947
	2	930
	3	902
	4	940
Year 3		
	1	996
	2	952
	3	923
	4	960
Year 4		
	1	1201
	2	1142
	3	1106
	4	1163

How do you make forecasts for the period 17 i.e. for the first quarter of 5" year, considering the trend component, seasonal component, cyclical component and random component?

Solution: The following table calculates the 4-period moving averages (MAs) and centered MAs. The sample calculations are shown below. You can calculate the values very fast and accurately with the help of MS-Excel.

Year 1	Quarter	Patients	4-MAs	Centered MAs
	1	920		
	2	916		
	3	895		912.375
	4	905	909	917.50
Year 2				
	1	947	915.75	920.125
	2	930	919.25	925.375
	3	902	921	935.875
	4	940	929.75	944.75
Year 3				
	1	996	942	950.125
	2	952	947.5	952.25
	3	923	952.75	983.375
	4	960	957.75	1032.75
Year 4				
	1	1201	1009	1079.375
	2	1142	1056.5	1127.625
	3	1106	1102.25	
	4	1163	1153	

Here the time period t =1, 2...16

4-period MA = (920+916+895+905) / 4 = 909 (at period 4)

(916+895+905+947) / 4 = 915.75 (at period 5)

and (1201+1142+1106+1163) / 4 = 1153 (at period 16)

from period 6 to 15, you can calculate the values accordingly and cross check with tabulated vales.

Note that the first moving average (= 909) is for the whole first year i.e. it is associated with all the four quarters of the whole year. Therefore it is reasonable to center it. This is the (T_t C_t) component. Now question is where do we center it2 It can be between 2nd and the 3rd quarter.

We can handle this by taking the midpoints of successive moving averages. For example: (909+915.75) / 2 = 912.375 (can be centered at 3' quarter) that is the average of the MA of 4th quarter and MA of 5th quarter.

This indicates that you are using 50 observations and centering at the middle. Similarly, (915.75+919.25) / 2 $^{=}$917.50 (center at 4)

Other values are calculated and tabulated for your reference.

Step 2: From T_t C_t component (centered MA), identify seasonal-random component.

$$S_tR_t = \frac{Yt}{T_tC_t}$$

For Year 1

Q3=> 895 /912.375$^{=}$ 0.981

Q4=> 905 / 912.5 = 0.986

Year 1	Q3	0.981
	Q4	0.986
Year 2	Q1	1.029
	Q2	1.005
	O3	0.964
	Q4	0,995
Year 3	Q1	L048
	Q2	0.997
	Q3	0.939
	Q4	0.930
Year 4	Q1	1.113
	O2	1.013

Note: We loose 4 points, 2 at the beginning (i.e. Ql and Q2) and 2 at the end (Q3 and Q4).

Step 3: Seasonal factor (S_t) of each quarter can be obtained by taking averages from each year.

1st quarter effect = (1.029+1.048+1.113) /3 =1.063
2nd quarter effect = (1.005+0.997+1.013) / 3 =1.005
3rd quarter effect = (0.981+0.964+0.939) / 3 = 0.961
4th quarter effect = (0.986+0.995+0.93) / 3 =0.970

Step 4: Deseasonalize the series now

$$T_t\, C_t\, R_t = Y_t / S_t$$

For year 1

QI 920/ 1.063 =865.475
Q2 916/1.005 = 911.442
Q3 895/0.971 = 931.32
Q4 905/0.970 = 932.99 and so on.

Deseasonalized series

865.475	Year 1
911.442	
931.320	
932.990	
890.874	Year 2
925.373	
938.605	
969.072	
936.971	Year.3
947.264	
960.458	
989.691	
1129.821	Year 4
1136.318	
1150.884	
1198.969	

Now we have calculated the deseasonalized value.

Step 5: Next important point is to identify the trend in the deseasonalized series.

From trend projection, we know that Eq. 6.1 is

$T_t = b_o + b_1 t; t = 1,2,..., 16$

and calculating the values of b_o, b_1 and get the value as $T_t = 825.94 + 19.12\, t$

Step 6: Obtain cyclical component $C_1 R_t = Y_t / (S_t T_t)$ Where (Y_t / S) is deseasonalised series.' Before calculating $C_t R_t$, you must calculate the values of T_t i.e. the trend in the

deseasonalised series for all the 16 periods.

The values are given in the table. Students are required to check the values. Then calculate the values of C_t R_t

Yt/St	Tt	CtRt
865.475	845.06	1.024158
911.442	864.18	1.054689
931.32	883.8	1.053767
932.99	902.42	1.033875
890.874	921.54	0.966723
925.373	940.66	0.983748
938.605	959.78	0.977937
969.072	978.98	0.989879
936.971	998.02	0.938829
947.264	1017.14	0.931301
960.458	1036.26	0.926850
989.691	1055.38	0.937757
1129.821	1074.5	1.051485
1136.318	1093.62	1.039042
211250.8	1112.74	189.8474
1198.969	1131.86	1.059290

It is required to forecast the values for the period 17 i.e. for the first quarter of 5th year.

T_{17}=825.94 + 19.12(17) =1150.98

Considering cyclical factor which is usuall forecasted by using smoothing techniques (explained in this unit) C_{17}= 1.02

Now $T^{17}C_{17}$ = 1150.98 (1.02) = 1174.

Note: Use Ms-Excel to calculate the values.

Forecast =1174 (1.063) =1247.96 (as 1.063 is the seasonal factor for the 1 quarter).
=> Forecast = 1274.96 for 1st quarter of the 5th year.

Activity B

In the Atlanta area, the number of daily calls for repair of speedy copy machines has been recorded as follows:

October	calls
1	132
2	170
3	95
4	110
5	120
6	135
7	190
8	95

a) Prepare a three-period moving-average forecast for the data. What is the error on each day?

b) Prepare a three period weighted-moving-average forecast using weights of WI =0.5, w2 =0.3 ans w3= 0.2.

c) Which of these two forecasts is the best.

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iii) Smoothing Technique

It is suitable for short-term forecasting. It is generally averaged by weighting the past data. The extreme case is 'naïve' forecasting.

a) Exponential Smoothing

Exponential smoothing is an averaging method that exponentially decreases the weighting of old demands. The models are well known and often used in operations management. They are popular because of the fact that (i) they are readily available in standard computer software packages, and (ii) they require relatively little data storage and computation.

Exponential smoothing is distinguished by the special way it weights each past demand. The pattern of weights is exponential in form. Demand for the most recent period is weighted most heavily and the weights decreases exponentially for the old periods. In other words, the weights decreases in magnitude the further back in time the data are weighted, the decrease is nonlinear.

Let F_t = forecast for next period t

As each time period expires, a new forecast is made.

Forecast for next period (F_t) = a* (actual demand for most recent period)

+ (1-a) *(demand forecast for most recent period)

$=> F_t = a\ Y_{t-1} + (1-\alpha)\ F_{t\_1}$ Eq. (6.8)

Where $Y_{t.1}$= the most recent observation

$F_{t.1=}$ the old forecast

α = smoothing parameter, $0 < \alpha < 1$, which indicates that one has to specify the value of α.

Why is this model called exponential smoothing?

The expansion of the above equation (6.8) shows that

$F_1 = \alpha\ Y_1 + (1-\alpha)\ F$

Also $F_{t-1} = \alpha\ Y\ t_{.2} + (1-\alpha)\ F_t z$, putting the value of F_{t-1} in eq.6.8, we get

$F_t = \alpha\ Y_{t\_1} + (1-\alpha)\ [\alpha\ Y_{t-2} + (1-\alpha)\ F_{t-2})$

or $F_t = a\ Y_{.,} + (1-\alpha)\ \alpha\ Y_{t\_2} + (1-\alpha)^2\ F_{t.2}$ Eq. (6.9)

Similarly $F_{t-2} = \alpha\ Y\ \alpha + (1-\alpha)\ F_{t-3}$ and substituting the value of F_{t-2} Eq. (6.9)

We get $F_t = a\ Y_{t-1+(1-\alpha)}\ a\ Y_{t-2} + (1-\alpha)^2\ [\alpha\ Y_{t-3} + (1-\alpha)\ F_{t-3}]$

$=> F_{t=}\alpha\ [Y_{t-1} + (1-\alpha)\ Y\ t\text{-}2 + (1-\alpha)2Y\ as\ ] + (1-a)^3\ F\ t\text{-}"s$

$=> F_t = a\ [(I-\alpha)^\circ\ Y_{t-1} + (1-\alpha)^1\ Y_{t-2} + (1-\alpha)^2 Y_{t-3}. + (1-\alpha)^3\ F_{t-3}$ Eq. (6.10)

The expansion of Eq. (6.10) can be continued further, but it will not help us. Since $0 < a < 1$, the term $a(l-a)^\circ$, $a\ (I-a)'$, $a\ (1-a)^2$ and so forth are successively smaller in the equation. More specifically these weights decrease exponentially. Weights decrease as we consider past observations. The most recent observation has the highest weight and the weights are decreasing exponentially(or geometrically) as we go to past observations. Thus exponential smoothing is also called the exponentially weighted moving average.

Rearranging and rewriting Eq. (6.8)

$F_t^= F_{t-1} + \alpha\ (Y_{t-1} - F_{t-1})$

Where the term ($Y_{t-1} - F_{t-1}$) is called the error term: (e t-1)

The form of the forecasting equation above is known as the `error correction form' of the ES model.

The higher the value of a, the faster the system responds to current data: As a increases, there is more correction.

If $\alpha = 1 => F_t = Y_{t-1}$(Naive forecast)

$\alpha = 0 => F_t = F_{t-1}$ (Forecast never changes and data has no role)

These are the two extreme cases.

The limitation of ES is that (i) it is not suitable for data that include long-term upward or downward movements(trend) in data.(ii) the use of ES would produce forecast that were too low for upward trend and too high for downward trend(i.e. either overestimated or underestimated).

Example: Sales data for 10 periods are given. Forecast the data for the next period i.e. for the 11' period. Assume a =0.4 and F_1= 700 (initial value)

t	Yt
1	700
2	724
3	720
4	728
5	740
6	742
7	758
8'	750
9	770
10	775

Solution: We can use the equation (6.8) F_t= F + a (Y_{t-1}- $F_{t-l)}$ for solving the problem.

t	**Yt**	**Ft**
1	700	700
2	724	700
3	720	709.6
4	728	713.76
5	740	719.46
6	742	727.67
7	758	733.4
8	750	743.24
9	770	745.95.
10	775	755.57
11		763.34

For calculating F_t (i.e. forecasting for the period t) we need the value of $F_{t-1,}$,. For simplicity we can assume the first value of F_t equal to 700 i.e. the value equal to Y_t.

Next value can be forecasted as: $F_t = F_{t-1}$ + a ($Y_{t.4}$ - $F_{t-1.}$)

Where F_{t-1} = 700 (just previous value), Y_{t-1} = 700 (just previous -value) and a = 0.4

=> 700 + 0.4 (700-700) =700

For next forecasting: $F_{t-1=}$700, Y_{t-1} =724 and a = 0.4

=> 700 + 0.4(724-700) = 709.6

The last forecasting is F_{11} = 755.57 + 0.4(775-755.57) $^{=}$763, 34

Similarly rest of the calculation can be carried out. The calculated values are tabulated in the table above.

Activity C

Using the data in activity B, prepare exponentially smoothed forecasts for the following cases:

a) a =0.land Ft =130

b) a = 0.3 mid ft =130

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Activity D

Compute the errors of bias and absolute deviation for the forecasts in activity c. Which of the forecasting models is the best?

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Advantages: Simple exponential smoothing and other exponential smoothing models share the advantage of requiring very few data points be stored. To update the forecast from period to period, you need only a, last period demand, and last period forecast. Remember this model incorporates in the new forecast all past demands. The model is easy to understand and easily computerised for thousands of part numbers, supply items and inventory items. The model is helpful in both the manufacturing and service sectors.

Difference Between the Decomposition Method and Smoothing Method

1) The decomposition methods assume that the seasonality for a particular period of the seasonal cycle can be treated as constant, whereas smoothing method constantly update the seasonal ratios.
2) The decomposition methods determine a particular linear or non-lenear trend in the data and use this relationship in all future forecasting, the smoothing methods constantly track and update the trends (up and down) in every period.
3) The decomposition methods attempt to determine the cyclical factor and use it across the board.
4) The basic difference between the decomposition methods and the smoothing methods is that, the later (i) continually tracks and modifies the demand components and (ii) does not isolate the demand components but gives the correctness in the procedure.

If the demand pattern is more stable over a long period of time, the decomposition method is suitable, whereas if the demand pattern is fluctuating more frequently, the smoothing techniques may be more useful.

Decomposition method, as explained earlier, is the forecasting required for intermediate range forecasting. This type of forecasting is helpful in' production planning. Decomposition methods may also use smoothing in their analysis.

6.5.2 Causal Models

i) Regression Analysis

It is a causal forecasting model in which, from historical data, a functional relationship is established between variables and then use to forecast dependent variable values. We consider a simple regression equation here for two variables and their linear relationship.

$F_t=a+bX_t$

Where is the forecast for period t, given the value of the variable X in period t. The coefficient a and b are the intercept and slope of the line. The coefficient a and b are computed by the following two equations: [4]1?

$$b = \frac{n(\sum X_t D_t) - (\sum X_t)(\sum D_t)}{(\sum X_t^2) - (\sum X_t)^2}$$

$$a = \frac{\sum D_t - b\sum X_t}{n}$$

Where D= a + bX and n is the number of periods.

Example: (Adapted from Production/Operations Management by Adam and Ebert). A paper box company makes carryout pizza boxes. The operations planning department knows that the pizza sales of a major client are a function of the advertising dollars the client spends, an account of which they can receive in advance of the expenditure. Operation planning is interested in determining this relationship between the client's advertising and sales. The amount of pizza boxes the client will order, in dollar volume, is known to be a fixed percent of sales.

Quarterly advertising and sales

Quarter	Advertising ($ 100,000)	Sales ($1,000,000)
1	4	1
2	10	4
3	15	5
4	12	4
5	8	3
6	16	4
7	5	2
8	7	1
9	9	4
10	10	2

Forecast for the next period i.e. for the period 11.

Solution: Computing b and then a, where advertising is X_t

For quarter t, sales are D_t for quarter t , sales are D_t for quarter t, and forcast is F_t for future period t

Quarter(t)	Xt	Dt	Xt *Xt	Xt *Dt
	Advertising	Sales		
1	4	1	16	4
2	10	4	100	40
3	15	5	225	75
4	12	4	144	48
5	8	3	64	24
6	16	4	256	64
7	5	2	25	10
8	7	1	49	7
9	9	4	81	36
10	10	2	100	20
Sum	96	30	1060	328

b= [10(328) -96 (30)] / [(10(1060) - 96* 96] = 0.29

a= [30-0.29(96)] / (10) = 0.22

Thus, the estimated regression line, the relationship between future sales F_t and advertising X_t is F_t= 0.22 + 0.29X_t

The operations planner can now ask for planned advertising expenditures , and from that sales can be forecasted.

Now substitute 11 for X_t into the equation above gives

F_t = 0.22+0.29(11) = 3.41

So sales are forecast as $3,410,000.

Note: A variety of regression models are discussed in block 5 of MS-8. Students have to read this unit in combination with block 5 of MS-8 so as to get a clear understanding of the regression models for forecasting.

Activity E

Describe the uses of qualitative, time series and causal forecasting in your organisation.

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6.6 FORECAST ERROR

Whether it is a simple smoothing or more advanced smoothing, an estimate of forecast error should be computed along with the smoothed average. This error estimate might be used for several purposes: .

i) To set safety stocks or safety capacity and thereby ensure a desired level of protection against stockout.

ii) To monitor erratic demand observations.

iii) To determine when the forecasting method is no Ionger tracking actual demand and needs to be reset.

In forecasting, a commonly used forecasting error is mean absolute deviation or MAD. It is a forecast error measure that is the average forecast error without regard to direction; calculated as the sum of the absolute value of forecast error for all periods divided by the total number of periods evaluated. It is mathematically defined as:

$$\text{MAD}=\frac{\text{Sum of the absolute value of the forecast error for all periods}}{\text{Number of periods}}$$

$$\frac{\sum(\text{Forecast demand - actual demand})}{n}$$

Where n is the number of periods

The above expression is just the average error observed, without regard to sign, over all the past periods of forecasting. The MAD is similar to standard deviation, except we have not squared the errors for each period and then taken the square root of them. This is called Mean square error (MSE). Mathematically it can be written as:

$\text{MSE} = (1/n) \times S\ (Y_t - F_t)^2$

$\text{MAD} = (1/n) \times S\ ½\ Yt - Ft\ _{1/2}$

MAD is used to determine whether the forecast is tracking with the actual time-series values. To determine this, a tracking signal is computed, as follows:

$$\text{Tracking signal} = \frac{\text{Running sum of forecast error}}{\text{MAD}} = \frac{\text{RSFE}}{\text{MAD}}$$

Tracking signal is thus a computation of bias in the numerator divided by the most recent estimate of MAD. Bias is a less commonly used error measure.

Bias = (sum of forecast error for all periods) / number of periods

$= \sum$ (forecasted demand - actual demand) / n

In computerised forecasting systems it is extremely important to incorporate error controls of the type discussed above. This will ensure that the system does not run out of control.

Example: Read the table below Forecast values and the actual values are given. Calculate RSFE, Forecast error, cumulative error, MAD. Calculate also the tracking signal (TS).

Quarter	Forecast demand	Actual demand
1	100	90
2	100	95
3	100	115
4	110	100
5	110	125
6	110	140

Solution: The tracking signal is calculated in the table below which is self explanatory.

Quarter demand	Forecast demand	Actual	error	RSFE error	Forecast error	Cumulative	MAD	TS
1	100	90	-10	-10	10	10	10	-1
2	100	95	-5	-15	5	15	7.5	-2
3	100	115	15	0	15	30	10	0
4	110	100	-10	-10	10	40	10	-1
5	110	125	15	5	15	55	11	0.5
6	110	140	30	35	30	85	14.2	2.5

For forecast to be 'in control' 89% of the 'errors' are expected to fall within. ± 2 MADs , 98% within 3MADs, or 99% within 4MADs.

6.7 SELECTING A SUITABLE FORECASTING METHOD

In this section we will present a framework for selecting a suitable qualitative, time-series, and casual methods of forecasting. The framework is based in large part on the survey conducted by Wheelwright and Clarke (1976), who identified factors that companies consider important when they select a forecasting method. The most important factors are indicated as follows:

i) User and System Sophistication: How sophisticated are the managers who are expected to use the forecasting results? It has been found that the forecasting method must be matched to the knowledge and sophistication of the user. Generally speaking, managers are reluctant to use results from techniques they do not understand. Another factor is the status of forecasting systems currently in use. Wheelwright and Clarke found that forecasting systems tend to evolve toward more mathematically sophisticated methods; they do not change in one grand step. So the method chosen must not be too advanced or sophisticated for its users or too far advanced beyond the current forecasting system

ii) Time and Resource Available: The selection of a forecasting method will depend on the time available in which to collect the data and prepare the forecast. This may involve the time of .users, forecasters and data collectors. The time required is also closely related to the necessary resources and the costs of the forecasting method. The preparation of a complicated forecast for which most of the data must be collected may take several months and cost thousands of money Value. For routine forecasts made by computerized systems, both the cost and the amount of time required may be very modest

iii) Use or Decision Characteristics: The forecasting method must be related to the use or decisions required. The use is closely related to such characteristics as accuracy required, time horizon of the forecast, and number of items to be forecast. For example, inventory and scheduling decisions require highly accurate short range forecasts for a large number of items. Time-series methods are ideally suited to these requirements. On the other hand, decisions involving process and facility planning are long range in nature; they require less accuracy for, perhaps, a single estimate of total demand. Qualitative or casual methods tend to be more appropriate for those decisions. In the middle time range are aggregate planning and budgeting decisions which often utilize time-series or casual methods.

iv) Data Availability: The choice of forecasting method is often constrained by available data. An econometric model might require data which are simply not available in the short run; therefore another method must be selected. The Box-Jenkins time-series method requires about 60 data points (5 years of monthly data).

v) Data Pattern: The pattern in the data will affect the type of forecasting method selected. If the time-series is flat, a first order method can be used. However, if the data show trends or seasonal patterns, more advanced methods will be needed. The pattern in the data will also determine whether a time-series method will suffice or whether casuals model are needed. If the data pattern is unstable over time, a qualitative method maybe selected. Thus the data pattern is one of the most important factors affecting the selection of a forecasting method.

Another issue concerning the selection of forecasting methods is the difference between fit and prediction. When different models are tested, it is often thought that the model with the best fit to historical data (least error) is also the best predictive model. This is not true. For example, suppose demand observations are obtained over the last eight time periods and we want to fit the best time-series model to these data. A polynomial model of degree seven can be made to fit exactly through each of the past eight data points. But this model is not necessarily the best predictor of the future.

Finally, an interesting question concerning model selection is the accuracy of qualitative human forecasting versus quantitative model based forecasting. Ebert (1976) compared humans to model for a variety of underlying time-series demand patterns. He found that, when the data included a great deal of random noise or non-linear seasonal patterns, models did better than humans provided that care was taken in fitting the models. However, when simple (first order) exponential models with an arbitrary value of a were used, the humans often performed better than the models. This research indicates that quantitative models do not always provide better forecasts than humans.

6.8 SUMMARY

The unit illustrated the concept of business forecasting in operations management and define forecasting as the use of the past data to future events. Prediction, on the other hand, refers to subjective estimates of the future. Skill, experience, and sound judgement of a manager are required for good predictions.

We studied the tradeoffs between cost and accuracy in forecasting approaches. Generally, less expensive forecasting approach leads to less accurate results. We discussed two basic groupings of forecasting techniques: naive (time-series) models i.e. trend analysis, decomposition method and exponential technique, and casual models i.e. regression technique. Research results illustrated that forecasting model depends upon the length of the forecast period, the level of noise, the measure of forecast error, and, the demand pattern. Often, forecasts are not made with statistical models but individuals can and do intuitively use past data to forecast future events.

6.9 SELF-ASSESSMENT EXERCISES

1) How does a quantitative forecasting differ from a qualitative forecasting?

2) What do you understand by time series analysis? How would you go about conducting such an analysis for forecasting the sales of a product in your firm?

3) Compare time series analysis with other methods of forecasting? Briefly summarize the strength and weakness of various methods.

4) A test was conducted on a given process for the purpose of determining the effect of an independent variable X (such as process temperature) on certain characteristics of a finished product Y (such as density). Twenty observation were taken and the following results were obtained:

$X = 5.0 \sum(X_1 - \overline{X}) = 160, \sum(X_1 - \overline{X})(Y_1 - \overline{Y}) = 80$

$Y = 3.0 \sum(Y - \overline{Y})^2 = 83.2$

Assume a model of the type Y = b_o +b_l X +I

a) Calculate the fitted regression equation

b) Prepare the analysis of the variable table

c) Determine 95% confidence limits for the true mean value of y when

   1) X=5.0

   2) X=9.0

5) A survey of used car sales in a city for the 10 year period 1976-85 has been made. A linear trend was fitted to the sales for month for each year and the equation was found to be Y= 400 + 18t, where t=0 on January 1, 1981 and t is measured in 1/2 year (6 monthly) units.

   a) use this trend to predict sales for June, 1990.

   b) If the actual sales in June, 1987 are 600 and the seasonal index for June sales is 1.20, what would be the relative cyclical, irregular index for June, 1987?

6) What would be the considerations in the choice of a forecasting methods?

7) Find the 4-quarter moving average of the following time series representing the quarterly production of coffee in an Indian state.

Production (in tones)				
Year	Quarter I	Quarter II	Quarter III	Quarter IV
1983	5	1	10	17
1984	7	1	10	16
1985	9	3	8	18
1986	5	2	15	19
1987	8	4	14	21

8) Given below is the data of production of a certain company in lakhs of units.

Year	1991	1992	1993	1994	1995	1996	1997
Production	15	14	18	20	17	24	27

   a) Compute the linear trend by the method of least spa re.

   b) Compute the trend values of each of the year.

9) Contrast forecasting and prediction and give an example of each.

10) Which would use in evaluating a forecast, MAD or Bias? Why?

11) Present evidence that suggests forecasting is an important tool in the service sector.

12) Describe the cost \ accuracy tradeoffs associated with both sophisticated statistical models and intuitive forecasting.

13) An ice creme parlor experiepced the following demand for ice cream last month. The current forecasting procedure is to use last year's corresponding weekly sales as this year's forecast.

Week	Forecasted demand (in gallons)	Actual demand (in galloons)
June 1.	210	200
June 8	235	225
June 15	225	200
June 22	270	260

Calculate MAD and bias and interpret each

14 A manufacturer of mole and gopher poison has experienced the following monthly demand for an environmentally sound pesticide poison.

Month	Actual demand (in cases)
February	620
March	840
April	770
May	950
June	1000

a) Using a simple average, what would the forecast have been for May and June.

b) What would be the three-month simple moving average have been for May and June?

c) Which forecasting method would you recommend? Why?

6.10 FURTHER READINGS

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